



APPLICATION OF MONTE CARLO
METHODS TO SOME PROBLEMS IN
HIGH ENERGY ASTROPHYSICS

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Abstract

This thesis describes the methodology and results from the investigation of three areas of interest in High Energy Astrophysics. These areas are, firstly the structure of the ejecta of SN 1987A, secondly the propagation of cosmic rays through our galaxy with energies greater than 1×10^{18} eV, and thirdly the detection of these same extremely energetic cosmic rays. The method used to investigate each of these areas is by computer simulation using Monte Carlo methods.

The principle of the Monte Carlo approach is to trace out a large number of special cases for the problem being investigated and from the analysis of these infer a general solution. The problem being treated may have many possible branchings and the decision to follow a certain path is made by sampling from the appropriate probability distribution. The iterative approach of this method makes it an ideal application for high speed computers (for a review of Monte Carlo theory see James (1980)) and the first use of Monte Carlo methods was in the simulation of neutron scattering for the design of the first nuclear reactors in the 1950's using the first primitive computers.

Chapter 1 provides an introduction to SN 1987A. It describes the generation of radioactive isotopes within the supernova ejecta, their subsequent decay and production of γ -rays. Chapter 2 details the Monte Carlo code used to treat the propagation and energy degradation of γ -rays throughout the supernova ejecta. Chapter 3 introduces the treatment of clumping of the elements within the supernova ejecta and examines the effects on the X-ray and γ -ray lightcurves of SN 1987A. The results of the simulations are then compared with experimental data (Syunyaev *et al.*, 1990), theoretical calculations (Arnett, Fryxell and Muller, 1989) and another simulation also investigating the structure of the ejecta of SN 1987A (Kumagai *et al.*, 1989).

A general summary of cosmic rays is contained within Chapter 4. This serves as an introduction to Chapter 5 which investigates the propagation of cosmic rays with energies greater than 1×10^{18} eV. In Chapter 5 the method for simulating cosmic ray propagation is outlined including the effects of galactic magnetic field turbulence. From

these simulations predictions for cosmic ray anisotropy can be calculated. These results are compared with experimental results (e.g. Fichtel and Linsley (1986)) and other treatments of the same problem (e.g. Giller *et al.* (1993) and Berezhinsky *et al.* (1991)).

As an introduction to Chapter 7, Chapter 6 outlines the theory of Extensive Air Showers (EAS) which are generated when cosmic rays interact with the earth's atmosphere. Chapter 7 investigates different detector designs for the detection of cosmic rays with energies greater than 1×10^{18} eV. The method used to investigate each of the detector designs is by the use of Monte Carlo methods and these are outlined in some detail. Results from these simulations are then used to decide on an optimum detector configuration.

Chapter 8 summarises the work presented in this thesis and discusses how the work presented here could be further extended in the future.