



**MATHEMATICAL MODELLING OF WIND  
EFFECTS ON CLOSED LAKES**

by

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## TABLE OF CONTENTS

	<u>page</u>
SUMMARY	iii.
SIGNED STATEMENT	v.
ACKNOWLEDGEMENTS	vi.
CHAPTER 1: THE EQUATIONS FOR GEOPHYSICAL FLOWS	
1.1 Derivation Of The Basic Equations	1.
1.2 The Depth Integrated Equations	4.
1.3 The Three Dimensional Equations	8.
CHAPTER 2: CORIOLIS FORCE AND THE CONNECTED LAKE PROBLEM	
2.1 Introduction	12.
2.2 Rotating Rectangular Basin	13.
2.3 The Connected Lake Problem	38.
2.4 The Collocation And Galerkin Techniques	41.
CHAPTER 3: BOUNDARY INTEGRAL SOLUTION	
3.1 Derivation Of The Solution	52.
3.2 Comparison With Analytic Solutions	57.
3.3 Connected Lakes	61.
CHAPTER 4: THE CONNECTED LAKE PROBLEM	
4.1 The Connected Lake System	67.
4.2 Convergence Of The Galerkin Technique	68.
4.3 Convergence Of The Collocation Technique	70.
4.4 The Boundary Integral Method	70.

CHAPTER 5: NUMERICAL MODELS	
5.1 Two Numerical Models	77.
5.2 A Finite Difference Model For Equation (5.1.1)	79.
5.3 Comparison With Analytic Solutions	88.
5.4 A Model For Lake Albert	90.
5.5 A Finite Difference Model For Equation (5.1.2)	95.
5.6 Comparison Between The Finite Difference Models	109.
CHAPTER 6: EDDY VISCOSITY - A REVIEW	
6.1 Experimental Observations	114.
6.2 The Vertical Eddy Viscosity Used In Mathematical Models	124.
6.3 Turbulent Closure Schemes	137.
CHAPTER 7: WIND DRIVEN FLOW IN A CHANNEL	
7.1 The Equations For Wind Driven Flow In A Channel	145.
7.2 $N$ Constant, viz. $N(\eta) = N_0$	149.
7.3 $N$ Linear, viz. $N(\eta) = N_0 + (N_1 - N_0)\eta$ , $N_1 \neq N_0$	149.
7.4 $N$ Quadratic, viz. $N = (\sqrt{N_0} + (\sqrt{N_1} - \sqrt{N_0})\eta)^2$ , $N_1 \neq N_0$	153.
7.5 $N$ Quadratic, viz. $N = (N_0^2 + (N_1^2 - N_0^2)\eta)^{1/2}$ , $N_1 \neq N_0$	155.
7.6 $N$ Quadratic, viz. $N = N_s\eta^2 + N_t\eta + N_0$	160.
7.7 $N$ Composite Linear	165.
7.8 A Turbulent Energy Closure Scheme	171.
7.9 Results	179.
7.10 Conclusion	204.
APPENDIX A: A SECOND ORDER METHOD FOR THE NEUMANN CONDITION WITH CURVED BOUNDARIES	
A.1 Introduction	206.
A.2 Solution	207.
A.3 Conclusion	229.
BIBLIOGRAPHY	231.

## SUMMARY

This thesis investigates general methods with which the circulation induced by a wind blowing over the surface of a closed basin may be calculated. Firstly, the linear, depth integrated equations describing such motions are used to model the motions induced by an oscillating wind blowing over a system of connected lakes each of constant depth. The techniques used to model this problem are the Collocation and Galerkin methods, which assume the basins are of rectangular shape, and a Boundary Integral Technique, which models basins of arbitrary contour. The performance of each of these methods is analysed.

By way of developing the above methods, the effect of the Coriolis force on the motion in rectangular lakes of various dimensions is also discussed. Results from the Boundary Integral Technique are also compared with analytic solutions available for simple geometries.

Various numerical methods of solving the depth integrated equations are also developed. Some of these methods can be used to calculate the effect of a wind blowing over a basin of arbitrary depth and contour. As part of this section of the thesis, finite difference approximations are developed which enable derivatives at a point near a curved boundary, along which a Neumann boundary condition applies, to be modelled with second order accuracy. Results from all the numerical models are compared with each other as well as with analytic solutions of problems with simple boundaries.

Finally, the depth variation of horizontal velocity in such flows is considered. Several analytic solutions, applicable to channel flow, are developed. These are compared with experimental observations. A turbulent energy closure scheme is also used to examine the vertical profile of velocity and comparisons are made between the results from this model and the analytic solutions as well as some experimental observations.