Finite Difference Methods for Advection and Diffusion

Alice von Trojan, B.Sc.(Maths.Sc.)(Hons.)

Thesis submitted for the degree of
Doctor of Philosophy

Department of Applied Mathematics
University of Adelaide

April 2001

Contents

Abstract

Transport phenomena are governed by the processes of advection and diffusion. This work concerns the development of high-order finite-difference methods on a uniform rectangular grid for advection and diffusion problems with smooth variable coefficients. The initial and boundary conditions are assumed to be given with sufficient smoothness to maintain the order of convergence of the scheme under consideration. High-order finite-difference methods for constant coefficients usually degenerate to first or, at best, second-order when applied to variable-coefficient problems. A technique is developed whereby the convergence rate can be increased to the constant-coefficient rate. This modification procedure is applied to finite-difference methods for both the non-conservative and conservative forms of the variable-coefficient advection equation, and to the variable-coefficient diffusion equation. Since the conservative form of the advection equation may be considered as an equation in which the two processes of non-conservative advection and decay (or growth) are taking place simultaneously, it is shown that the conservative process may be split into two separate processes, thereby simplifying the solution procedure. It is also observed that the decay may be identified as a sink term, so methods developed for the solution of the conservative equation may also be applied to the non-conservative advection equation in the presence of sinks. Likewise, the variable-coefficient diffusion equation is noted to be a special case of the variable-coefficient transport equation, so high-order methods developed to solve the former equation may also be applicable to the latter equation. Finite-difference methods can readily be extended to problems involving two or more dimensions using locally one-dimensional techniques. This is demonstrated by application to two-dimensions for the non-conservative advection equation, and to a special case of the diffusion equation. The new modified methods are particularly apt for problems involving smooth initial and boundary data, where they outperform the base methods considerably.