



Constructions with Bundle Gerbes

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Abstract

This thesis develops the theory of bundle gerbes and examines a number of useful constructions in this theory. These allow us to gain a greater insight into the structure of bundle gerbes and related objects. Furthermore they naturally lead to some interesting applications in physics.

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