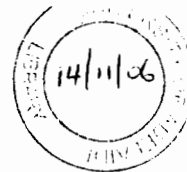


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# Landau-gauge Green's functions in unquenched lattice QCD

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A Thesis presented for the degree of  
Doctor of Philosophy



Centre for the Subatomic Structure of Matter  
University of Adelaide  
Adelaide  
2006

# Abstract

Most previous lattice calculations have been done in quenched QCD, where the effects of fermion vacuum fluctuations are neglected in the creation of the gluon field configurations. The sea-quark masses may be thought of as infinite in the quenched approximation. Due to computational resource issues, the quenched approximation provided the basis for the vast majority of lattice calculations until relatively recently. In order to meaningfully test QCD against experimental observation, realistic simulations with the dynamical sea-quarks are needed. Computing resources becoming available now are powerful enough to treat up, down and strange quarks dynamically.

A major component of this work consists of full lattice QCD calculations of some of the fundamental quantities of QCD. We are computing the gluon and quark propagators in full QCD and investigating the effects of finite sea-quark masses on these. We have used the configurations generated by the MILC collaboration to do nonperturbative simulations with  $2 + 1$  flavors of dynamical quarks. We have performed extensive simulations with realistic quark vacuum polarization (quark loops) to evaluate the effect of unquenching.

We use an improved staggered fermion action “AsqTad” in the dynamical simulations. Current simulations with dynamical staggered quarks have the benefits of both good chiral properties at moderate lattice spacing and being computationally inexpensive. A highlight of this study is the first results for gluon and quark propagators in Landau gauge with  $2+1$  flavors of dynamical quarks. A comparative study of quenched and unquenched results for both quark and gluon propagators, which probes the effects of dynamical sea-quarks is a significant part of this work.

In the second part of this thesis, we study the scaling behavior of the quark and gluon propagator in Landau gauge with  $2 + 1$  flavors of dynamical quarks on two lattices with different lattice spacings and similar physical volumes in order to test whether we are close to the continuum limit for these lattices. We compare the mass function and wave renormalization function for two different lattice spacings and find them to be consistent within errors. The Asqtad quark propagator shows good scaling behavior as does the gluon propagator for the lattice spacings considered. The work carried out in this thesis provides a clear understanding of the role of dynamical sea-quarks on the Landau-gauge Green’s functions.

Testing of the violation of positivity of the gluon propagator comes in the third part of this thesis. Correlation functions play an important role in the non-perturbative studies of the gluon propagator and confinement. Violation of positivity is considered to be a sufficient condition for confinement. An infrared suppressed propagator always violates reflection positivity and we found explicit evidence for this in our study.

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