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Joint statistics between temperature and its dissipation rate components in a round jet

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The joint statistics between the temperature fluctuation θ and all three components of the temperature dissipation rate ϵ_θ are investigated in the self-preserving region of a slightly heated turbulent round jet. The main factors which determine the correlation between θ and ϵ_θ are the asymmetry of $p(\theta)$, the probability density function (PDF) of θ, and the anisotropy of the small-scale turbulence. The assumption of statistical independence between θ and ϵ_θ appears to be more closely approximated in this flow than in a turbulent plane jet. Relatedly, the assumption of local isotropy is also more closely satisfied in the round jet than in the plane jet. When θ is in the range ± 2 standard deviations, the expectations of all components of ϵ_θ, conditioned on θ, are approximately equal in the fully turbulent part of the flow; the magnitude of the conditional expectation is consistent with the independence assumption. © *1995 American Institute of Physics.*

I. INTRODUCTION

The instantaneous scalar dissipation rate ϵ_θ $[\equiv \alpha(\theta_{,1}^2 + \theta_{,2}^2 + \theta_{,3}^2)$, where $\theta_{,i} \equiv \partial\theta/\partial x_i$ and α is the thermal diffusivity$]$ and its dependence on the scalar fluctuation θ are important in turbulent combustion modeling.[1,2] The mean scalar dissipation rate $\langle\epsilon_\theta\rangle$ (hereafter, angular brackets denote time averages) features in second-order models of turbulent flows: e.g., through the time scale ratio $(\langle q^2\rangle\langle\epsilon\rangle^{-1}/\langle\theta^2\rangle\langle\epsilon_\theta\rangle^{-1})$, where $\langle q^2\rangle$ is the average turbulent energy and $\langle\epsilon\rangle$ is the average dissipation rate of $\langle q^2\rangle$. The dependence between θ and ϵ_θ reflects the dependence between large-scale and small-scale motions. The joint probability density function (JPDF) of θ and ϵ_θ can be correlated to the average rate of creation or destruction of chemical species in both premixed[3] and diffusion[4] flames since the average reaction rate is proportional to the expectation of ϵ_θ conditioned on the stoichiometric value of θ.[1]

Measurements of $\langle\epsilon_\theta\rangle$ have been made in nonreacting turbulent flows: grid turbulence,[5,6] a quasihomogeneous shear flow,[7] a self-preserving plane jet,[8] a self-preserving plane wake,[9] a turbulent boundary layer,[10] a developing round jet,[11,12] and a self-preserving round jet.[13,14] By contrast, however, more limited experimental data are available for the joint statistics of θ and ϵ_θ. Iso-JPDF contours and correlations between the temperature fluctuation θ and an approximation to the temperature dissipation rate ϵ_θ were obtained by Anselmet and Antonia[15] in the self-preserving region of a slightly heated turbulent plane jet. They provided approximate support for the assumption of independence between θ and ϵ_θ. This assumption allows the JPDF, $p(\theta,\epsilon_\theta)$, of these two quantities to be written as a product of the individual or marginal PDFs, viz.

$$p(\theta,\epsilon_\theta) = p(\theta)p(\epsilon_\theta). \tag{1}$$

Anselmet and Antonia[15] noted that Eq. (1) seemed to be more reasonably supported by the data as the distance from the jet centerline increases. Recently, using measurements of $p(\theta,\theta_{,i}^2)$ [unless otherwise mentioned, repeated subscripts do not imply summation] in a turbulent boundary layer and $p(\theta,\theta_{,t}^2)$ $[\theta_{,t} \equiv \partial\theta/\partial t]$ in the developing region $(x_1/d = 3\sim 15)$ of a turbulent round jet, Anselmet *et al.*[16] pointed out that Eq. (1) is strictly valid only when $p(\theta)$ is symmetrical. For a symmetrical PDF, all odd-order moments are zero; in particular, the skewness $S_\theta \equiv \langle\theta^3\rangle/\langle\theta^2\rangle^{3/2}$ is zero. These authors also noted that intermittency (as measured, for example, by the intermittency factor γ or fraction of the time for which the flow is turbulent) plays a much smaller role than the asymmetry of $p(\theta)$ in determining the correlation between θ and ϵ_θ.

While Anselmet *et al.*'s[16] conclusion seems reasonable, it appears to be somewhat at variance with the data of Ref. 11. Using Raman scattering, these authors measured the CH_4 mass fraction (identified, for the present purpose, with θ), two components (axial and radial) of ϵ_θ in the developing region $(5 \leqslant x_1/d \leqslant 17)$ of an isothermal methane round jet. On the jet axis, $\langle\theta_{,1}^2\rangle$ was approximately equal to $\langle\theta_{,2}^2\rangle$. Since $\langle\theta_{,2}^2\rangle$ and $\langle\theta_{,3}^2\rangle$ are equal on the axis, it follows that $\langle\epsilon_\theta\rangle$ should conform approximately with isotropy there. Also, the correlation $\langle\theta\epsilon_\theta\rangle$ was very nearly zero on the axis, i.e., in apparent support of (1) since $\langle\theta\epsilon_\theta\rangle = \langle\theta\rangle\langle\epsilon_\theta\rangle = 0$ if θ and ϵ_θ are independent. Yet, $p(\theta)$ is not symmetrical on the axis since the available data indicate that S_θ is negative. Values of S_θ were not reported in Ref. 11 but previous measurements,[17–19] also in a round jet, show that S_θ varies typically between -1 $(x_1/d \approx 3)$ and -0.3 $(x_1/d \approx 60)$ along the axis. As the distance (x_2) from the axis increases, S_θ crosses zero and becomes positive. Yet, Namazian *et al.*[11] found that the magnitude of $\langle\theta\epsilon_\theta\rangle$ is significantly different from zero when $x_2 > 0$, especially at $x_2 \approx R_u$ (where R_u is the jet half-radius); for $x_1/d < 17$, their data indicated a more pronounced departure of $\langle\epsilon_\theta\rangle$ from isotropy as the radial distance increased.

The preceding observations suggest that it is not clear whether the degree of correlation between θ and ϵ_θ is caused mainly by the departure of $p(\theta)$ from symmetry or by the deviation of $\langle\epsilon_\theta\rangle$ from isotropy or whether it is the result of both those factors. To resolve this ambiguity, we have examined the effects on $\langle\theta\epsilon_\theta\rangle$ of the asymmetry of $p(\theta)$ and the anisotropy of $\langle\epsilon_\theta\rangle$ in the nearly self-preserving region of a slightly heated turbulent round jet. In this flow, Antonia and Mi[13] have found that local isotropy of the temperature field is closely satisfied, in the context of either the variances or spectra of $\theta_{,i}$, on the axis; also, the symmetry of $p(\theta)$ is expected to be a reasonable approximation at the point between $x_2=0$ and $x_2=R_u$ where $S_\theta\approx 0$. Another objective of this paper is to investigate the expectation of $\theta_{,i}^2$ conditioned on particular values of θ, since such statistics are required for closing the equation for the evolution of the PDF of a passive scalar θ in turbulent flows (e.g., Refs. 20–23). We emphasize that, for the present work, all three components of ϵ_θ are measured. In particular, $\theta_{,1}$ was measured directly, thus circumventing the use of Taylor's hypothesis. The present joint statistics of θ and ϵ_θ are therefore likely to be more reliable than if only $\theta_{,t}^2$ were available. It should be noted that the basic data set used for this paper is the same as in Ref. 13.

II. EXPERIMENTAL SETUP AND TEST CONDITIONS

The jet rig has an axisymmetric nozzle with a 10:1 contraction ratio. The air supply was heated by an electrical fan heater (2.4 kW) located at the blower entrance. To obtain a uniform and symmetrical (about the jet axis) mean temperature profile at the nozzle exit of diameter $d=25.4$ mm, the complete tunnel was insulated (25 mm thick insulating foam with a metallic foil overlay). At the nozzle exit, the temperature T_j (≈ 32 °C above ambient) was uniform within $\pm 1\%$. The exit velocity U_j was 11 m/s and the Reynolds number $R_d \equiv U_j d/\nu$, where ν is the kinematic viscosity, was about 1.9×10^4.

All measurements were made at $x_1/d=30$ (self-preservation was approximately reached at $x_1/d\approx 15$, e.g., Ref. 19) and were restricted to the nearly fully turbulent region ($0\leqslant x_2/R_u \lesssim 1$) to avoid flow reversal and high local turbulence intensities. On the axis, the mean velocity U_0 and mean temperature T_0 were 2.1 m/s and 4.8 °C (relative to ambient) respectively. The turbulence Reynolds number R_λ, based on the Taylor microscale λ $(=U_0^{-1}\langle u_1^2\rangle^{1/2}/\langle u_{1,t}^2\rangle^{1/2})$, was approximately 150 and the Kolmogorov length scale η $[\equiv(\nu^3/\langle\epsilon\rangle)^{1/4}]$ was about 0.17 mm. The Péclet number P_λ $(\equiv\langle u_1^2\rangle^{1/2}\lambda_\theta/\alpha$, where $\lambda_\theta=\langle\theta^2\rangle^{1/2}/\langle\theta_{,1}^2\rangle^{1/2})$ was equal to 83. The half-radii R_u and R_θ, defined on the basis of mean velocity and mean temperature profiles, were 75 and 90 mm, respectively. The ratio Gr/R_0^2($\mathrm{Gr}\equiv gR_u^3T_0/\nu T_a$ is the Grashof number, T_a is the absolute ambient temperature, $R_0\equiv U_0R_u/\nu$ is the local Reynolds number) is about 0.0027, indicating that the effect of buoyancy is negligible and justifying the use of temperature as a passive contaminant.

Spatial instantaneous derivatives $\theta_{,i}$ of the temperature fluctuation θ were obtained using two parallel cold wires. Wollaston (Pt-10% Rh) wires of nominal diameter $d_w\approx 0.63$ μm were operated by in-house constant current circuits supplying 0.1 mA to each wire. For this value of electrical current and for the experimental conditions outlined above, the velocity contamination of the temperature signal had a negligible effect on the statistics presented in this paper. The wires, with a nominal length l_w of about 0.4 mm, were perpendicular to the flow direction. Each wire was carefully checked under a microscope for straightness immediately prior to the experiments. Care was taken to ensure that the etched portion of each wire was central and parallel so as to minimize the uncertainty in the measurement of Δx_i, the separation in the x_i direction between the wires. Following a detailed investigation[24] of the effect of Δx_i on $\theta_{,i}$, the separation Δx_i was chosen equal to about 3η since, for this value of Δx_i, the correction which had to be applied to obtain reliable values of $\langle\theta_{,i}^2\rangle$ was relatively small. Also, this value of Δx_i is sufficiently large to avoid the large uncertainty due to the electronic noise.[25]

The diameter d_w and length l_w of the wires were chosen so that the ratio l_w/d_w (≈ 700) was sufficiently large to avoid possible attenuation at low wave numbers[26] while the ratio l_w/η was as small as practicable. At $x_1/d=30$ and $x_2/R_u=0$, the value of l_w/η (≈ 2.6) was small enough to avoid making a wire length correction (Wyngaard's[27] calculations show that for a wire length of about 3η, $\langle\epsilon_\theta\rangle$ is attenuated by about 10%). Only the central part of the Wollaston wires was etched to avoid difficulties associated with fully etched wires. For a given wire length, Paranthoen *et al.*[26] found that the signal for a fully etched wire is more attenuated than that from a partially etched wire. Estimates of the temperature coefficient of the cold wires were made by mounting both wires at the jet exit using a 10 Ω platinum resistance thermometer operated in a Leeds and Northrup 8087 bridge (with a resolution of 0.01 °C).

The cutoff frequency f_c of the low-pass filter was selected by viewing the time derivative spectra on the screen of a real-time spectrum analyser (HP3582A). Special attention was given to the degree of correlation between the two signals and to the time derivative spectra of these signals. If the spectra looked different, one or both of the wires were replaced by newly etched ones until there was no discernible difference. The values of f_c were identified with the frequencies at which the derivative spectra were about 2–3 dB higher than those corresponding to the frequencies at which electronic noise first became important. These settings were determined at each measurement location and found to be the same for the two wires. At $x_2=0$ and $x_2=R_u$, f_c was 2.2 and 1.2 kHz, respectively, while the Kolmogorov frequency f_k $(=\langle U_1\rangle/2\pi\eta$, $\langle U_1\rangle$ is the local mean streamwise velocity) was about 2.0 and 0.8 kHz.

After filtering, the signals from the two wires were passed through buck and gain units to offset the DC components and provide suitable amplification prior to digitizing the signals with a 12-bit A/D converter (RC electronics) on a personal computer (NEC 386). A sampling frequency f_s equal to 2 f_c was used in all cases and the record duration was 50 s. The digital data were directly transferred from the personal computer to a VAX 8550 computer using an ETHERNET (fibre optic cable) link. The spatial and temporal derivatives and their squared values were formed on the VAX computer.

TABLE I. Moments of temperature derivatives.

	Round jet (present)			Planet jet (Antonia *et al.*)	Isotropic value
x_2/R_u	0.0	0.53	1.07	0	⋯
K_{21}^2	0.98	1.04	1.15	2	1
K_{31}^2	1.03	1.08	1.2	⋯	1
K_{21}^4	1.03	0.83	0.6	0.4	1
K_{31}^4	0.95	0.85	0.62	⋯	1
S_{θ_1}	−1.1	−1.08	−1.15	−0.85	0
S_{θ_2}	−0.05	−0.86	−1.01	≈0	0
S_{θ_3}	−0.04	−0.1	−0.08	⋯	0

III. DEPARTURE FROM LOCAL ISOTROPY

Local isotropy of the scalar field requires that

$$p(\theta_{,1})=p(\theta_{,2})=p(\theta_{,3})$$

and

$$p(-\theta_{,i})=p(\theta_{,i}).$$

One consequence of this is the equality

$$\langle\theta_{,1}^n\rangle=\langle\theta_{,2}^n\rangle=\langle\theta_{,3}^n\rangle. \quad (2)$$

When n is odd, $\langle\theta_{,i}^n\rangle=0$ (in particular, the skewness $S_{\theta_{,i}} \equiv \langle\theta_{,i}^3\rangle/\langle\theta_{,i}^2\rangle^{3/2} = 0$); when n is even, the following ratios:

$$K_{21}^n\equiv\frac{\langle\theta_{,2}^n\rangle}{\langle\theta_{,1}^n\rangle}$$

and

$$K_{31}^n\equiv\frac{\langle\theta_{,3}^n\rangle}{\langle\theta_{,1}^n\rangle}$$

are unity. These ratios for $n=2$ and 4 and the magnitudes of $S_{\theta_{,i}}$ $(i=1,2,3)$ are presented in Table I for $x_2/R_u=0$, 0.53, and 1.07. As noted in Sec. II, the present data were estimated from the finite difference $\theta_{,i}\approx\Delta\theta/\Delta x_i$ at the optimum separation $\Delta x_i\approx3\eta$. Also shown in the table are the corresponding values obtained by Antonia *et al.*[28] in the self-preserving region $(x_1/d=40)$ of a slightly heated turbulent plane jet. Table I indicates that the isotropic requirement (2) for $n=2$ is very closely satisfied on the axis. It is less adequately satisfied as x_2 increases. Away from the axis, the departure from (2) is more pronounced at $n=4$ than $n=2$. The departure of K_{21}^n from unity is considerably larger for the plane jet, even on the centerline. This implies that local isotropy of the scalar field is more closely satisfied in the round jet than in the plane jet, as previously noted in Ref. 13.

The nonzero values of the skewnesses $S_{\theta_{,i}}$ for $i=1$ and 2 (off the axis), which seem to invalidate the assumption of local isotropy, are associated with the mean temperature gradient $\langle T\rangle_{,i}$ (e.g., Refs. 29–31). The nonzero value of $S_{\theta_{,i}}$ may not mainly reflect the anisotropy of the small-scale temperature field.[28,32,33] It is more likely associated with the asymmetry of the large-scale motion. Figure 1 shows the cospectrum, $\mathrm{Co}_{\theta_{,i}\theta_{,i}^2}$, between $\theta_{,i}$ and $\theta_{,i}^2$ for $i=1$ and 2 at $x_2/R_u=0$ and 0.53. The cospectrum is normalized such that the area under the curve is equal to $|S_{\theta_{,i}}|$. The major contribution (>65%) to $S_{\theta_{,1}}$ or $S_{\theta_{,2}}$ at $x_2/R_u=0.53$ occurs in the range $k_1^*\leq0.1$, suggesting that the nonzero values of $S_{\theta_{,i}}$ result mainly from the asymmetry of the large-scale motion. As a result of symmetry with respect to x_2, the cospectrum $\mathrm{Co}_{\theta_{,2}\theta_{,2}^2}$ is nearly zero at all values of k_1^* [Fig. 1(b)] on the jet axis. However, when $k_1^*\leq0.2$, $\mathrm{Co}_{\theta_{,1}\theta_{,1}^2}$ is negative both on and off the axis since the large scale streamwise motion is asymmetrical with respect to x_1 across the jet (as reflected by the presence of ramps in the temperature signals). Sreenivasan *et al.*[32] provided more direct evidence of the influence of the large-scale motion on the magnitude of $S_{\theta_{,1}}$ in a round jet. A conditional technique was used for removing the effect of the large-scale motion on θ, the skewness of the x_1 derivative of the remaining signal was nearly zero. It is possible that $S_{\theta_{,1}}$ is not a reliable indicator of local isotropy. The magnitude of $S_{\theta_{,1}}$ is virtually unchanged (≈1.1) at different x_2/R_u (Table I); Sreenivasan *et al.*[32] noted that the magnitude of $|S_{\theta_{,1}}|$ (≈0.8) did not change either for different flows or over several orders-of-magnitude variation in R_λ.

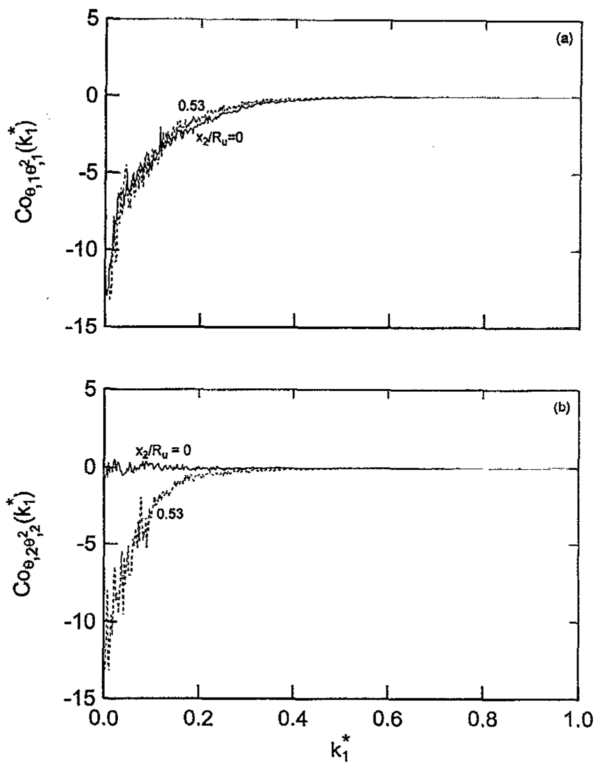

FIG. 1. Cospectra between $\theta_{,i}$ and $\theta_{,i}^2$ $(i=1,2)$ at $x_2/R_u=0$ (—) and 0.53 (---). (a) $i=1$; (b) 2.

IV. CORRELATION BETWEEN θ AND ϵ_θ

The JPDF of $\theta_{,t}^2$ and $\theta_{,i}^2$ $(i=1, 2,$ or 3) is shown in Fig. 2. Clearly, there are important differences between $\theta_{,1}^2$ and either $\theta_{,2}^2$ or $\theta_{,3}^2$. Figure 2(a) implies a relatively high degree of correlation between $\theta_{,t}^2$ and $\theta_{,1}^2$. The correlation coefficient, i.e., $\langle\theta_{,t}^{2*}\theta_{,1}^{2*}\rangle$ [hereafter, the asterisk denotes a variable which is centered and normalized by the root-mean-square (RMS) value], between these two variables is greater than

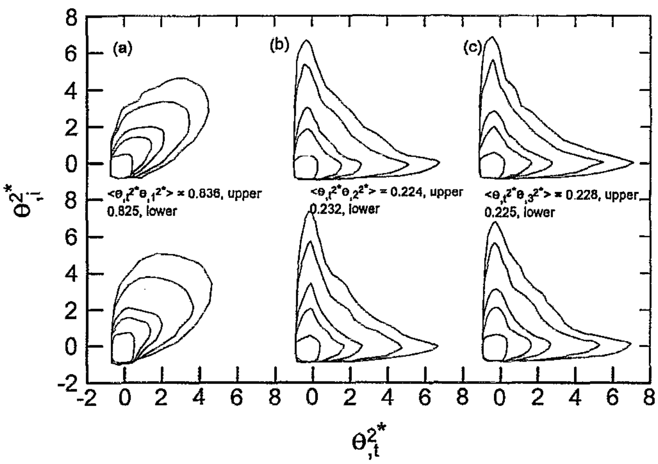

FIG. 2. Normalized JPDF contours of $\theta_{,t}^2$ and $\theta_{,i}^2$. (a) $p(\theta_{,t}^{2*},\theta_{,1}^{2*})$; (b) $p(\theta_{,t}^{2*},\theta_{,2}^{2*})$; (c) $p(\theta_{,t}^{2*},\theta_{,3}^{2*})$. Outer to inner contours: 0.0005, 0.001, 0.005, 0.01, and 0.1. Upper and lower contours are for $x_2/R_u=0$ and 0.53.

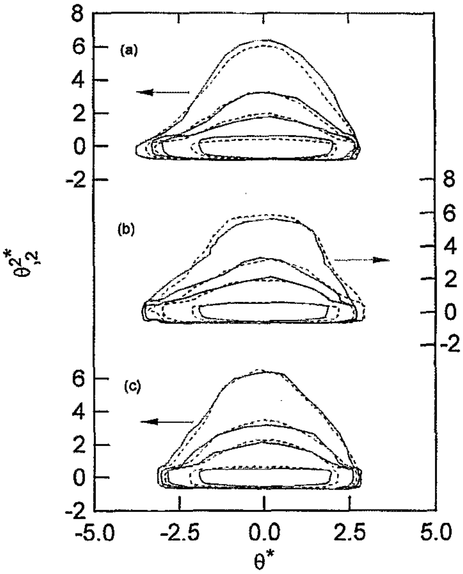

FIG. 3. Independence check for the JPDF of θ and $\theta_{,2}^2$. (a) $x_2/R_u=0$; (b) 0.53; (c) 0.8. ---, $p(\theta^*,\theta_{,2}^{2*})$; —, $p(\theta^*)p(\theta_{,2}^{2*})$. Outer to inner contours: 0.0005, 0.002, 0.006, and 0.05.

0.8 at all x_2 locations. This coefficient should be 1 if Taylor's hypothesis, i.e., $\theta_{,t}=-\langle U_1\rangle\ \theta_{,1}$, applies. Figures 2(b) and 2(c) show a close coincidence between large values of $\theta_{,t}^{2*}$ and small values of $\theta_{,2}^{2*}$ and $\theta_{,3}^{2*}$ and vice versa, indirectly suggesting that periods of strong activity in $\theta_{,2}^{2*}$ and $\theta_{,3}^{2*}$ may occur almost simultaneously and correspond to periods where $\theta_{,1}^{2*}$ is quiescent. An important consequence of this is that the instantaneous behavior of ϵ_θ cannot be inferred solely on the basis of $\theta_{,1}^2$. This should be kept in mind, especially when only the $\theta_{,1}^2$ data are available.

Estimates of the correlation coefficient between θ and fluctuations of $\theta_{,i}^2$ have been made both for $\theta<0$ and $\theta>0$. A zero value of the coefficient in each case can result from either of the following conditions: (i) statistical independence between θ and $\theta_{,i}^2$, (ii) $p(\theta^*,+\theta_{,i}^{2*})=p(\theta^*,-\theta_{,i}^{2*})$, viz. symmetry of $p(\theta_{,i}^{2*})$. In the present flow, condition (ii) is not satisfied since the skewness of $\theta_{,i}^{2*}$ is in the range 10–20, i.e., $p(\theta_{,i}^{2*})$ is far from being symmetrical. A zero value of the correlation coefficient, when θ is either positive or negative, would therefore be consistent with the notion of independence between θ and $\theta_{,i}^2$. Note that a zero value of the correlation does not necessarily imply independence, e.g., Figure 6.9 of Tennekes and Lumley;[34] strictly, Eq. (1) needs to be verified. Table II indicates that, at $x_2/R_u\leqslant 0.8$, the correlation coefficient is very small, typically less than 0.05, for both $\theta<0$ and $\theta>0$. This implies that there is a very weak dependence between θ and $\theta_{,i}^2$. Since $\epsilon_\theta\equiv\alpha(\theta_{,1}^2+\theta_{,2}^2+\theta_{,3}^2)$, the dependence between ϵ_θ and θ is expected to be also very weak; as shown in Table II, the magnitude of the normalized correlation

$$\frac{\langle\theta\epsilon_\theta\rangle}{\langle\theta^2\rangle^{1/2}\langle\epsilon_\theta\rangle}=\frac{\langle\theta\theta_{,1}^2\rangle+\langle\theta\theta_{,2}^2\rangle+\langle\theta\theta_{,3}^2\rangle}{\langle\theta^2\rangle^{1/2}(\langle\theta_{,1}^2\rangle+\langle\theta_{,2}^2\rangle+\langle\theta_{,3}^2\rangle)},\tag{3}$$

is indeed quite close to zero at all measurement locations. This, in turn, suggests that Eq. (1) is approximately valid; direct checks of the relation $p(\theta^*,\theta_{,2}^{2*})=p(\theta^*)p(\theta_{,2}^{2*})$ (Fig. 3) corroborate this.

Figure 4 indicates that the correlation coefficients $\langle\theta^*\theta_{,i}^{2*}\rangle$ increase slightly with x_2/R_u, as noted in Ref. 15 in the context of a plane jet. In both jets, S_θ varies in a manner similar to $\langle\theta^*\theta_{,i}^{2*}\rangle$, where the data of S_θ for the plane jet ($x_1/d=40$) were taken from Ref. 35. Note, however, that in the range $x_2/R_u\leqslant 1$ or $x_2/L_u\leqslant 1$ (L_u is the plane jet half-width), both S_θ and $\langle\theta^*\theta_{,i}^{2*}\rangle$ are significantly closer to zero for the round jet than for the plane jet. Accordingly, one could conclude, as did Anselmet *et al.*,[16] that the correlation between θ and $\theta_{,i}^2$ [or ϵ_θ] is, to a large degree, related to the

TABLE II. Correlation coefficients between θ and the temperature dissipation components. The normalized correlation between θ and ϵ_θ is also shown.

	$\langle\theta^*\theta_{,1}^{2*}\rangle$		$\langle\theta^*\theta_{,2}^{2*}\rangle$		$\langle\theta^*\theta_{,3}^{2*}\rangle$		
x_2/R_u	$\theta^*<0$	$\theta>0$	$\theta^*<0$	$\theta^*>0$	$\theta^*<0$	$\theta^*>0$	$\langle\theta^*\epsilon_\theta\rangle/\langle\epsilon_\theta\rangle$
0	−0.06	0.017	−0.065	0.016	−0.061	0.012	−0.086
0.27	−0.066	0.022	−0.032	0.003	−0.036	0.004	−0.063
0.53	−0.038	0.033	−0.032	0.009	−0.054	−0.001	−0.047
0.80	−0.017	0.032	−0.018	0.009	−0.032	0.01	−0.028
1.07	−0.059	0.086	−0.018	0.043	−0.047	0.049	0.038

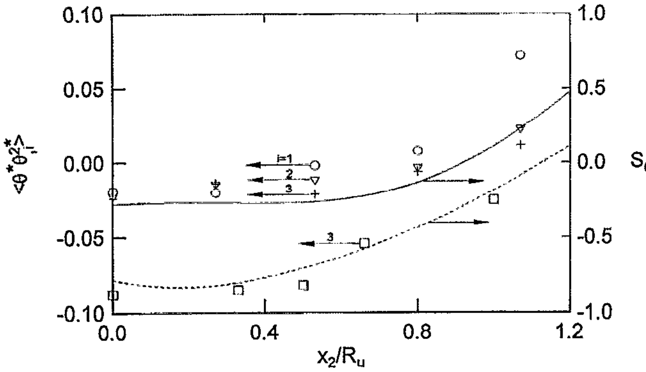

FIG. 4. Distributions of $\langle\theta^*\theta_{,i}^{2*}\rangle$ and S_θ. Round jet (present): ○, $\langle\theta^*\theta_{,1}^{2*}\rangle$; △, $\langle\theta^*\theta_{,2}^{2*}\rangle$; +, $\langle\theta^*\theta_{,3}^{2*}\rangle$; —, S_θ. Plane jet: □, $\langle\theta^*\theta_{,3}^{2*}\rangle$ (Anselmet and Antonia[15]); ---, S_θ (Browne *et al.*[35]).

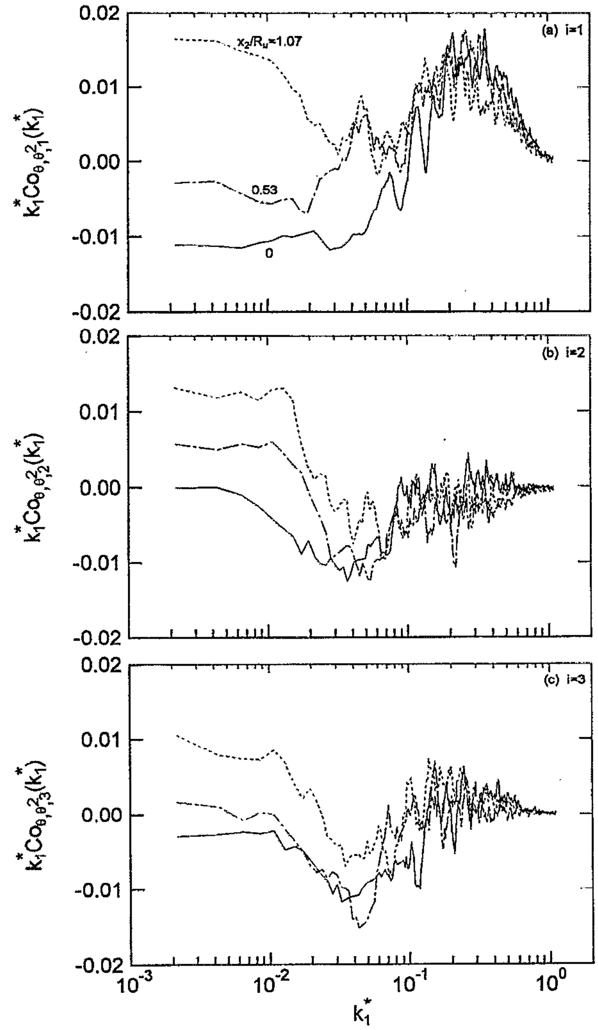

FIG. 5. Cospectra of θ and $\theta_{,i}^2$ at three values of x_2/R_u. (a) $i=1$; (b) 2; (c) 3; —, $x_2/R_u=0$; ---, 0.53; ···, 1.07.

magnitude of S_θ. It is unlikely, however, that S_θ is the only factor, since the location where S_θ is zero does not coincide with that where $\langle\theta^*\theta_{,i}^{2*}\rangle$ is zero [also see Fig. 4(b) of Ref. 16]. Figure 3 suggests that Eq. (1) is as closely validated on the axis [$S_\theta \approx -0.3$ and the departure of $p(\theta)$ from symmetry is not negligible] as at $x_2/R_u=0.8$ (S_θ is negligible there). Yet, the data presented in Table I indicate that local isotropy is more closely satisfied on the axis. Similarly, the lack of correlation between θ and $\theta_{,i}^2$ ($i=1,2$) obtained by Namazian *et al.*,[11] on the axis in the developing region ($x_1/d \leqslant 17$) of a round jet, should be associated with a non-negligible value of S_θ, on the basis of the data shown in Ref. 19 in the region $2.5 \leqslant x_1/d \leqslant 30$ of a round jet. Perhaps more importantly, the data of Refs. 11 and 12 support the isotropy of $\langle\epsilon_\theta\rangle$, at least approximately, on the jet axis. In a slightly heated round jet, Anselmet *et al.*[16] also obtained a very small value (≈ 0.05) of $\langle\theta^*\theta_{,1}^{2*}\rangle$ at $x_1/d=15$ and $x_2=0$, where $S_\theta \approx -0.5$. Moreover, consistently with the relative magnitudes of $\langle\theta^*\theta_{,i}^{2*}\rangle$ in Fig. 4, the departure from local isotropy is significantly smaller in the round jet than in the plane jet (Sec. III). These results are not unreasonable if one interprets local isotropy as reflecting a lack of dependence between the small-scale motion (which contributes significantly to $\langle\epsilon_\theta\rangle$) and the large-scale motion (which dominates the magnitude of $\langle\theta^2\rangle$). The above discussion strongly supports that the correlation $\langle\theta\epsilon_\theta\rangle$ is influenced not just by the symmetry of $p(\theta)$ but also by the level of departure from local isotropy.

Jayesh and Warhaft[36] presented estimates of the correlation coefficient $\rho \equiv \langle\theta^{2*}\epsilon_\theta^*\rangle$ and the normalized correlation $\rho_P \equiv (\langle\theta^2\epsilon_\theta\rangle/\langle\theta^2\rangle\langle\epsilon_\theta\rangle - 1)$ for decaying grid turbulence with and without a mean lateral temperature gradient $\langle T\rangle_{,2}$. In their case, ϵ_θ was inferred from $\theta_{,t}^2$ via Taylor's hypothesis. With an imposed constant $\langle T\rangle_{,2}$, ρ and ρ_P were typically 0.1 and 0.4 at the last measurement station, suggesting a significant correlation between the scalar and its dissipation rate ϵ_θ. At this station, $p(\theta)$ is quite symmetrical (we estimated a value of 0.03 for S_θ). In the same flow, Tong and Warhaft[37] showed that, when $\langle T\rangle_{,2} \neq 0$, the departure of $\langle\epsilon_\theta\rangle$ from isotropy is significant (typically $K_{21}^2 = 1.52 \pm 0.2$). This is expected since an imposed mean temperature gradient can act as a source of temperature anisotropy in grid turbulence. When $\langle T\rangle_{,2} \approx 0$, ρ and ρ_P were significantly reduced to about 0.03 and 0.1, respectively. Yet, a discernible departure from symmetry can be detected in $p(\theta)$ (see their Fig. 16), the value of S_θ being approximately 0.2. One would expect ϵ_θ to satisfy isotropy roughly in this case. Note that, for the present experiments, ρ and ρ_P are less than 0.01 on the axis, implying a much smaller degree of correlation between the scalar and its dissipation rate than for decaying grid turbulence.

V. STATISTICAL DEPENDENCE BETWEEN θ AND ϵ_θ

In order to gain further insight into the dependence between θ and $\theta_{,i}^2$, cospectra of θ and $\theta_{,i}^2$ are shown in Fig. 5, in the form $k_1^*\mathrm{Co}_{\theta,\theta_{,i}^2}$ vs $\log_{10} k_1^*$ (where $k_1^* \equiv k_1\eta$), at three radial locations ($x_2/R_u=0$, 0.53, and 1.07). Relatively large magnitudes occur in the range $0 < k_1^* \lesssim 0.1$, reflecting the importance of the large-scale motion. For $k_1^* > 0.1$, the

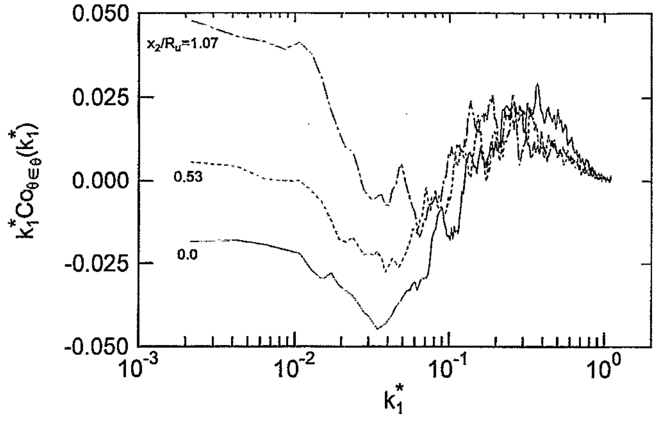

FIG. 6. Cospectra of θ and ϵ_θ at three values of x_2/R_u. —, $x_2/R_u=0$; ---, 0.53; ···, 1.07.

magnitude of $\mathrm{Co}_{\theta,\theta_{,1}^2}$ remains discernible (typically less than 0.015), while $\mathrm{Co}_{\theta,\theta_{,2}^2}$ and $\mathrm{Co}_{\theta,\theta_{,3}^2}$ are almost zero at all wave numbers. This indicates that the statistical dependence between θ and $\theta_{,i}^2$ is weak for $i=2$ and 3 but relatively strong for $i=1$. Although $\theta_{,i}^2$ for $i=1$, 2, and 3 were not obtained simultaneously, the cospectrum between θ and ϵ_θ, i.e., $\mathrm{Co}_{\theta,\epsilon_\theta}$, can be estimated approximately using the data for $\mathrm{Co}_{\theta,\theta_{,i}^2}$. Figure 6 shows data for $\mathrm{Co}_{\theta,\epsilon_\theta}(k_1^*)$; note that

$$\frac{\langle \theta\epsilon_\theta\rangle}{\langle\theta^2\rangle^{1/2}\langle\epsilon_\theta\rangle}=\int \mathrm{Co}_{\theta,\epsilon_\theta}(k_1^*)dk_1^*.$$

As x_2 increases, the main variation in the cospectrum occurs at $k_1^* \lesssim 0.1$:$\mathrm{Co}_{\theta,\epsilon_\theta}(k_1^*)$, which is negative on the axis, becomes positive when $x_2/R_u \gtrsim 0.5$. This suggests that, on average, low temperatures ($\theta<0$), which are associated with the entrained partially-mixed (cooler) fluid, would be related to relatively high dissipation rates ϵ_θ (greater than $\langle\epsilon_\theta\rangle$) in the central flow region.

Further evidence of the dependence between $\theta_{,i}^2$ and θ can be obtained from measurements of $\langle\theta_{,i}^2|\theta\rangle$, the expectation of $\theta_{,i}^2$ conditioned on individual values of θ. This expectation is defined (e.g., Ref. 21) as follows:

$$\langle\theta_{,i}^2|\theta\rangle=\int_{-\infty}^{\infty}\theta_{,i}^2 p(\theta_{,i}|\theta)d\theta_{,i}, \tag{4}$$

where

$$p(\theta_{,i}|\theta)=\frac{p(\theta,\theta_{,i})}{p(\theta)}$$

is the conditional PDF of $\theta_{,i}$ ($i=1,2,3$) for particular values of θ. Estimates of $\langle\theta_{,i}^2|\theta\rangle$ were made at $x_2/R_u=0$, 0.53, and 1.07; the normalized distributions of $q_i(\theta)\equiv\langle\theta_{,i}^2|\theta\rangle/\langle\theta_{,i}^2\rangle$ are shown in Fig. 7. For all values of i, $q_i(\theta)$ exhibit a peak (not well-defined) near both the lower and upper limits of the temperature range. High-temperature dissipation rates can occur at the interface—where there is a high-temperature difference and therefore strong mixing—between the large-scale warmer fluid and the entrained cooler fluid; this interface is reflected by a relatively large jump in the temperature signal.

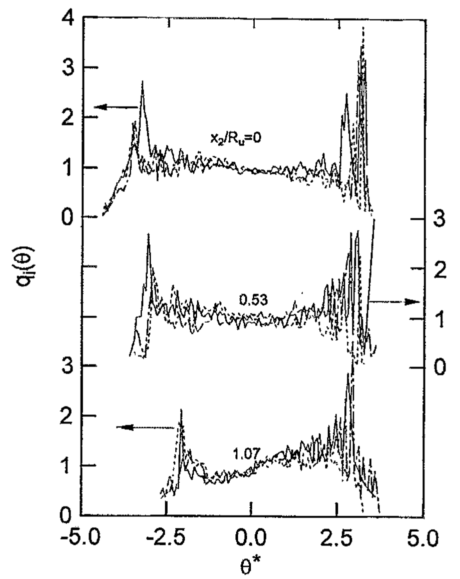

FIG. 7. Distributions of $q_i(\theta)$ at $x_2/R_u=0$, 0.53, and 1.07. —, $i=1$; ---, 2; ···, 3.

Since the cooler entrained fluid comes in contact with warmer fluid at the interface, high values of $\theta_{,i}^2$ may be associated with either low or high values of θ^*, thus leading to the two observed peaks of $q_i(\theta)$. The largest values of $|\theta|$ are associated with the smallest values of $q_i(\theta)$. This is reasonable since temperatures associated with the coolest unmixed or partially mixed fluid and the warmest well-mixed (fully turbulent) fluid are unlikely to be dissipated rapidly. Further, note that, at $x_2/R_u=0$ and 0.53, the left peak in $q_1(\theta)$ is stronger than that in either $q_2(\theta)$ or $q_3(\theta)$, the latter two being nearly equal. The right peak appears to be of equal strength for all i, although the jitter and likely lack of convergence of the data make conclusions difficult. A possible explanation is that the interface in the central flow region may, on average, be approximately normal to the x_1 direction, so that a relatively large streamwise temperature change occurs. When conditioning is on the spatially coherent temperature jump, a very sharp increase is observed in $\langle\theta_{,1}^2/\theta\rangle$ (see Fig. 9 of Ref. 13) near the jump location, where θ is generally negative and thus associated with the cooler entrained fluid. Increases in both $\theta_{,2}^2$ and $\theta_{,3}^2$ also occur at almost the same location but their magnitude is smaller than that of $\theta_{,1}^2$. The jitter and poor convergence of $\langle\theta_{,i}^2/\theta\rangle$ near the high-temperature side requires comment. While much longer record durations would reduce the fluctuations, it is unlikely that they would yield a better definition of $q_i(\theta)$. Kailasnath *et al.*[38] found that, even for very long record durations ($\simeq 15$ h) obtained in the wake of a heated cylinder, $q_1(\theta)$ did not converge at the uppermost temperature end (unpublished wake data obtained in our laboratory corroborate this). The authors suggested that this lack of convergence reflects the infrequent arrival at the measurement station of very high-temperature fluid (in the present case the upper temperature limit would correspond to the temperature at the jet exit).

For $|\theta^*|\lesssim 2$, $q_i(\theta)\approx 1$ at $x_2/R_u=0$ and 0.53. This indicates negligible statistical dependence between θ and $\theta_{,i}^2$ for

$|\theta^*|\lesssim 2$. For statistical independence between θ and $\theta_{,i}^2$, $\langle \theta_{,i}^2|\theta\rangle = \langle \theta_{,i}^2\rangle$ or $q_i(\theta) \equiv \langle \theta_{,i}^2|\theta\rangle / \langle \theta_{,i}^2\rangle = 1$. The probability that θ^* falls in the range $|\theta^*|>2$ is quite small ($\lesssim 5\%$). This negates the effects of the extrema of $q_i(\theta)$ at large $|\theta^*|$ so that $\langle \theta^* \theta_{,i}^{2*}\rangle$ is nearly zero when $x_2/R_u \lesssim 0.5$ (Sec. IV). At $x_2/R_u = 1.07$, except near the two limits of θ^*, $q_i(\theta)$ increases noticeably as θ^* increases. At this location, the departure from local isotropy is more significant than on the axis (Sec. III). Consistent with this, Southerland and Dahm[14] observed that, in a round water jet at $x_1/d \approx 235$ and $x_2/x_1 = 0.11$, the conditional dissipation rate $\langle \epsilon_\theta|\theta\rangle$ is generally an increasing function of θ (in this case, θ should be identified with the dye concentration fluctuation). These authors found a non-negligible departure from local isotropy (e.g., $K_{21}^2 \approx 1.1$ and $K_{31}^2 \approx 1.5$) at the same off-axis location (unfortunately, no information was given on the axis).

In decaying grid-generated turbulence, the statistical behavior of $q(\theta) \equiv \langle \epsilon_\theta|\theta\rangle / \langle \epsilon_\theta\rangle$, inferred from Taylor's hypothesis, was investigated by Jayesh and Warhaft[36] at $\langle T\rangle_{,2} = C$ (a nonzero constant) and $\langle T\rangle_{,2} = 0$. When $\langle T\rangle_{,2} = C$, $q(\theta)$ depends strongly on θ and exhibits a rounded V shape (symmetrical about $\theta = 0$) at all measurement stations x_1/M ($\lesssim 160$), where M is the grid mesh length. In this case, as mentioned in Sec. IV, the departure of $\langle \epsilon_\theta\rangle$ from isotropy is significant.[37] When $\langle T\rangle_{,2} \approx 0$, the distribution of $q(\theta)$ shows a strong peak near the upper limit of θ at $x_1/M = 62.4$ but becomes much flatter, especially for $\theta^* \lesssim 0$ where $q(\theta) \approx 1$, at $x_1/M = 82.4$. Although these authors were not able to measure $q(\theta)$ at further downstream distances, the trend suggests that $q(\theta)$ tends to unity, i.e., $\langle \epsilon_\theta\rangle$ tends to be statistically independent of θ, as x_1 increases. We believe that $\langle \epsilon_\theta\rangle$ should tend to isotropy in this case. Another interesting difference in $q(\theta)$ for the two cases is that, when $\langle T\rangle_{,2} \approx 0$, like the present results for the round jet, the largest temperature fluctuations, either positive or negative, are associated with the lowest values of ϵ_θ; when $\langle T\rangle_{,2} = C$, however, the largest fluctuations are related to the highest dissipation rates.

The significant difference between the present behavior of $q_i(\theta)$ and that obtained by Kailasnath *et al.*[38] on the axis of a round water jet at $x_1/d = 37$ ($R_d = 3900$) requires comment. On the axis, their distributions of $q_i(\theta)$ ($i = 1$ or 2) increases very significantly with θ (see Fig. 5 of Ref. 38). The rate of increase is several times larger than for the present $q_i(\theta)$ at $x_2/R_u = 1.07$ and Southerland and Dahm's[14] $q_i(\theta)$ at $x_2/R_u \approx 1$. Since $\langle \theta^* \theta_{,i}^{2*}\rangle = \langle \theta^* \langle \theta_{,i}^{2*}|\theta^*\rangle\rangle$, the $q_i(\theta)$ distributions of Kailasnath *et al.* suggest that $\langle \theta^* \theta_{,i}^{2*}\rangle$ should differ significantly from zero, in contrast to the present nearly zero value (Fig. 4). Also, their absolute values of S_θ should be considerably larger than the present value of $|S_\theta|$. The present value of S_θ (≈ -0.3) on the axis is comparable to that obtained by Birch *et al.*[17] in a C_3H_8 air round jet ($S_\theta \approx -0.3$) and Pitts and Kashiwagi[18] in a CH_4 air round jet ($S_\theta \approx -0.4$). The present value is also not very different from that (≈ -0.5) obtained by Anselmet *et al.*[16] on the axis at $x_1/d = 15$. However, in the same jet as used in Ref. 38, Prasad and Sreenivasan[39] obtained a much bigger value (>1) for S_θ in the region $13 \lesssim x_1/d \lesssim 21$. The observed differences between the present data and those of Ref. 38 may be attributed to significant differences in initial conditions between the two flows. This seems plausible as Gao and O'Brien[40] found, using direct numerical simulations of isotropic turbulence, that a nonzero correlation between θ and $\theta_{,i}$ is primarily the result of initial conditions. Similarly, in decaying grid-generated turbulence, Jayesh and Warhaft[36] noted that the residual effect of initial conditions, although not evident in $\langle T\rangle$ and $\langle \theta^2\rangle$, affects $p(\theta)$ and $q(\theta)$ significantly, even at large distances downstream. While the above observations apply to homogeneous turbulence, similar results could be expected for inhomogeneous turbulent shear flows. The effect of initial conditions clearly merits further study.

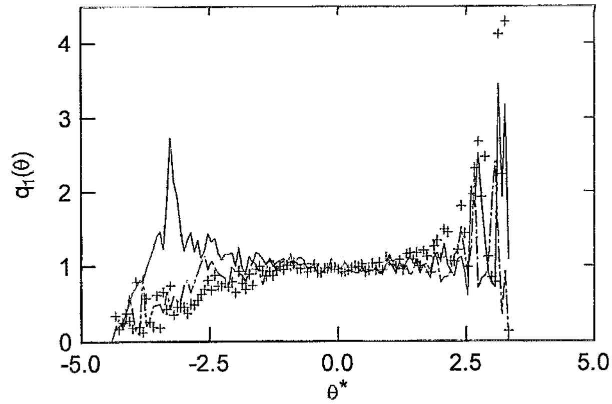

FIG. 8. Comparison between $q_1(\theta)$ and $q_1^t(\theta)$ at $x_2/R_u = 0$. —, $q_1(\theta)$; ---, $q_1^t(\theta)$ for $\theta = \frac{1}{2}(\theta_j + \theta_{j-1})$; +, $q_1^t(\theta)$ for $\theta = \theta_j$.

VI. EFFECT OF TAYLOR'S HYPOTHESIS ON $Q_1(\theta)$

To investigate the effect of using Taylor's hypothesis on the measurement of $q_1(\theta) \equiv \langle \theta_{,1}^2/\theta\rangle / \langle \theta_{,1}^2\rangle$, a comparison is made, in Fig. 8, between the directly measured $q_1(\theta)$ and $q_1^t(\theta)$, inferred from Taylor's hypothesis. The distribution of $q_1^t(\theta)$ is only single-lobed near the high-temperature side; as found by Kailasnath *et al.*[38] Like $q_1(\theta)$, $q_1^t(\theta)$ is quite flat over the range $|\theta^*| \lesssim 2$. However, this plateau is absent in Kailasnath *et al.*'s distribution for $q_1^t(\theta)$, the latter increasing monotonically with θ. A possible reason for this is that there may be differences in the initial conditions between the two jets. Another possibility is as follows. Let $\{\theta_j\}$ ($j = 1,2,\ldots$) represent the discrete time series of θ. An approximation for $(\theta_{,t})_j$ is given by $(\theta_j - \theta_{j-1})/\tau_s$, where $\tau_s = f_s^{-1}$. If $(\theta_{,t})_j$ is the instantaneous derivative at the instant when θ_j occurs, one would expect that $q_1^t(\theta)$ will weight larger values of θ more heavily, since the temperature signal often exhibits ramp-like patterns (a jump followed by a gradual decrease; see Ref. 32). This expectation is corroborated by Fig. 8 and is consistent with the distribution of $q_1^t(\theta)$ obtained by Ref. 38 [we are assuming that they estimated $q_1^t(\theta)$ in this way]. It is worth pointing out that the finite difference $(\theta_j - \theta_{j-1})/\tau_s$ is an adequate approximation to the time derivative when $\theta = (\theta_j + \theta_{j-1})/2$ but not for $\theta = \theta_j$, since the zero value of $\langle \theta\theta_{,t}\rangle [\equiv \frac{1}{2}(\langle \theta^2\rangle)_{,t}]$, which follows from the stationarity of θ, is obtained only when $\theta = (\theta_j + \theta_{j-1})/2$.

The comparison in Fig. 8 indicates that, on the axis, Taylor's hypothesis is approximately valid for estimating $q_1(\theta)$ when $\theta^* \gtrsim -2$ but inadequate when $\theta^* \lesssim -2$. This should not introduce any significant error to the conventional average $\langle \theta_{,1}^2 \rangle$, since the magnitude of $\int_{-\infty}^{-2} p(\theta^*) d\theta^*$ is quite small.[25,41] The breakdown for $\theta^* \lesssim -2$ is readily explained; the magnitude of $\theta_{,t}$ at a point in space depends on how rapidly θ changes with time; this change should be closely related to the instantaneous velocity of the fluid at temperature $(\langle T \rangle + \theta)$. Obviously, the coolest (entrained) fluid should have a streamwise velocity which is much smaller than the local mean streamwise velocity $\langle U_1 \rangle$. Therefore, the use of Taylor's hypothesis, i.e., $\theta_{,t} = -\langle U_1 \rangle \theta_{,1}$, will cause a significant attenuation of $\langle \theta_{,t}^2 | \theta \rangle \langle U_1 \rangle^{-2}$ near the low temperature side; as a result, the low temperature peak in $q_1(\theta)$ is not evident in $q_1^t(\theta)$ (Fig. 6). Recent results obtained by Tong and Warhaft[42] in a round jet corroborate this. It is difficult to understand why Kailasnath *et al.*'s distributions of q_1^t and q_1 are identical on the low concentration side.

VII. CONCLUSIONS

The dependence between θ and ϵ_θ is influenced not only by the symmetry of $p(\theta)$, as was pointed out by Anselmet *et al.*,[16] but also by the departure from local isotropy. The dependence is weaker in a round jet than a plane jet, the difference reflecting the smaller departure from local isotropy for the round jet. The expectation of ϵ_θ, conditioned on θ, depends strongly on θ when the latter is either very positive or negative. This local dependence is due to the close connection which exists between the large-scale motion and the entrainment of unmixed or partially mixed fluid; this connection does not however contribute significantly to the correlation $\langle \theta \epsilon_\theta \rangle$.

In the fully turbulent part of the flow the conditional expectation $\langle \theta_{,i}^2 / \theta \rangle$ is independent of i when $-2 \lesssim \theta^* \lesssim 2$. Within this range, the magnitude of the expectation is nearly 1, implying approximate independence between $\theta_{,i}^2$ or ϵ_θ and θ. For $\theta^* \lesssim -2$, there are significant differences in $\langle \theta_{,i}^2 / \theta \rangle$ between $i=1$ and $i=2$ (or 3). Contrary to the conclusion of Kailasnath *et al.*,[38] our data indicate that estimates of $\langle \theta_1^2 / \theta \rangle$ from $\langle \theta_{,t}^2 / \theta \rangle$ and Taylor's hypothesis are seriously in error near the low-temperature side. On the high-temperature side $(\theta^* \gtrsim 2)$, it is possible that differences exist in $\langle \theta_{,i}^2 / \theta \rangle$ for different i. In this region, the poor convergence of the data which, as noted in Ref. 38, is mainly due to the infrequent arrivals of high-temperature fluid, makes it difficult to draw any conclusions.

ACKNOWLEDGMENT

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[1] R. W. Bilger, "Turbulent diffusion flames," Annu. Rev. Fluid Mech. **21**, 101 (1989).
[2] R. W. Bilger, "Turbulent mixing and reaction," Proceedings of the Eleventh Australasian Fluid Mechanics Conference, Hobart, pp. 1–9, 1992.
[3] K. M. C. Bray, "Turbulent flows with premixed reactants," in *Topics in Applied Physics: Turbulent Reacting Flows*, edited by P. A. Libby and F. A. Williams (Springer-Verlag, Berlin, 1980), Vol. 44, pp. 115–183.
[4] R. W. Bilger, "Turbulent flows with non-premixed reactants," in *Topics in Applied Physics: Turbulent Reacting Flows*, edited by P. A. Libby and F. A. Williams (Springer-Verlag, Berlin, 1980), Vol. 44, pp. 63–113.
[5] T. T. Yeh and C. W. Van Atta, "Spectral transfer of scalar and velocity fields in heated grid turbulence," J. Fluid Mech. **58**, 233 (1973).
[6] Z. Warhaft and J. L. Lumley, "An experimental study of the decay of temperature fluctuations in grid-generated turbulence," J. Fluid Mech. **88**, 659 (1978).
[7] S. Tavoularis and S. Corrsin, "Experiments in nearly homogeneous turbulent shear flow with a uniform mean temperature gradient, Part 1," J. Fluid Mech. **104**, 311 (1981).
[8] R. A. Antonia and L. W. B. Browne, "The destruction of temperature fluctuations in a turbulent plane jet," J. Fluid Mech. **134**, 67 (1983).
[9] R. A. Antonia and L. W. B. Browne, "Anisotropy of the temperature dissipation in a turbulent wake," J. Fluid Mech. **163**, 393 (1986).
[10] L. V. Krishnamoorthy and R. A. Antonia, "Temperature dissipation measurements in a turbulent boundary layer," J. Fluid Mech. **176**, 265 (1987).
[11] M. Namazian, R. W. Schefer, and J. Kelly, "Scalar dissipation measurements in the developing region of a jet," Combust. Flame **74**, 147 (1988).
[12] R. W. Schefer, A. R. Kerstein, M. Namazian, and J. Kelly, "Role of large-scale structure in a nonreacting turbulent CH_4 jet," Phys. Fluids **6**, 652 (1994).
[13] R. A. Antonia and J. Mi, "Temperature dissipation in a turbulent round jet," J. Fluid Mech. **250**, 531 (1993).
[14] K. B. Southerland and W. J. A. Dahm, "A four-dimensional experimental study of conserved scalar mixing in turbulent flows," Report No. 026779-12, The University of Michigan, 1994.
[15] F. Anselmet and R. A. Antonia, "Joint statistics between temperature and its dissipation in a turbulent jet," Phys. Fluids **28**, 1048 (1985).
[16] F. Anselmet, H. Djeridi, and L. Fulachier, "Joint statistics between a passive scalar and its dissipation in turbulent flows," J. Fluid Mech. **280**, 173 (1994).
[17] A. D. Birch, D. Brown, M. G. Dodson, and J. R. Thomas, "The turbulent concentration field of a methane jet," J. Fluid Mech. **88**, 431 (1978).
[18] W. M. Pitts and T. Kashiwagi, "The application of laser-induced Rayleigh light scattering to the study of turbulent mixing," J. Fluid Mech. **141**, 391 (1984).
[19] L. P. Chua and R. A. Antonia, "The turbulent interaction region of a circular jet," Int. Commun. Heat Mass Transfer **13**, 545 (1986).
[20] E. E. O'Brien, "Turbulent flows with non-premixed reactants," in *Topics in Applied Physics: Turbulent Reacting Flows*, edited by P. A. Libby and F. A. Williams (Springer-Verlag, Berlin, 1980), Vol. 44, pp. 185–218.
[21] S. B. Pope, "PDF Methods for turbulent reactive flows," Progr. Energy Combust. Sci. **11**, 119 (1985).
[22] R. W. Bilger, "Conditional moment closure for turbulent reacting flow," Phys. Fluids A **5**, 436 (1993).
[23] W. E. Mell, V. Nilsen, G. Kosaly, and J. J. Riley, "Investigation of closure models for nonpremixed turbulent reacting flows," Combust. Sci. Tech. **91**, 179 (1994).
[24] R. A. Antonia and J. Mi, "Corrections for velocity and temperature derivatives in turbulent flows," Exp. Fluids **14**, 203 (1993).
[25] J. Mi and R. A. Antonia, "Some checks of Taylor's hypothesis in a slightly heated turbulent circular jet," Exp. Therm. Fluid Sci. **8**, 328 (1994).
[26] P. Paranthoen, C. Petit, and J. C. Lecordier, "The effect of thermal prong–wire interaction on the response of cold wire in gaseous flows (air, argon and helium)," J. Fluid Mech. **124**, 457 (1982).
[27] J. C. Wyngaard, "Spatial resolution of a resistance wire temperature sensor," Phys. Fluids **14**, 2052 (1971).
[28] R. A. Antonia, F. Anselmet, and A. J. Chambers, "Assessment of local isotropy using measurements in a turbulent plane jet," J. Fluid Mech. **192**, 211 (1986).
[29] C. H. Gibson, C. A. Friehe, and O. McConnell, "Structure of sheared turbulent fields," Phys. Fluids Suppl. **20**, S156 (1977).
[30] K. R. Sreenivasan and S. Tavoularis, "On the skewness of the temperature derivative in turbulent flows," J. Fluid Mech. **101**, 783 (1980).
[31] R. Budwig, S. Tavoularis, and S. Corrsin, "Temperature fluctuations and heat flux in grid-generated isotropic turbulence with streamwise and transverse mean temperature gradients," J. Fluid Mech. **153**, 441 (1985).
[32] K. R. Sreenivasan, R. A. Antonia, and D. Britz, "Local isotropy and large structures in a heated turbulent jet," J. Fluid Mech. **94**, 745 (1979).
[33] R. A. Antonia and C. W. Van Atta, "Structure functions of temperature fluctuations in turbulent shear flows," J. Fluid Mech. **84**, 561 (1978).
[34] H. Tennekes and J. L. Lumley, *A First Course in Turbulence* (MIT Press, Cambridge, MA, 1972).
[35] L. W. B. Browne, R. A. Antonia, and A. J. Chambers, "The interaction

region of a turbulent plane jet," J. Fluid Mech. **149**, 355 (1984).
[36]Jayesh and Z. Warhaft, "Probability distribution, conditional dissipation and transport of passive temperature fluctuations in grid-generated turbulence," Phys. Fluids A **4**, 2292 (1992).
[37]C. Tong and Z. Warhaft, "On passive scalar derivative statistics in grid turbulence," Phys. Fluids **6**, 2165 (1994).
[38]P. Kailasnath, K. R. Sreenivasan, and J. R. Saylor, "Conditional scalar dissipation rates in turbulent wakes, jets, and boundary layers," Phys. Fluids A **5**, 3207 (1993).
[39]R. R. Prasad and K. R. Sreenivasan, "Quantitative three-dimensional imaging and the structure of passive scalar fields in fully turbulent flows," J. Fluid Mech. **216**, 1 (1990).
[40]F. Gao and E. E. O'Brien, "Joint probability density function of a scalar and its gradient in isotropic turbulence," Phys. Fluids A **3**, 1625 (1991).
[41]J. Mi and R. A. Antonia, "Corrections to Taylor's hypothesis in a turbulent circular jet," Phys. Fluids **6**, 1548 (1994).
[42]C. Tong and Z. Warhaft, "Passive scalar dispersion and mixing in a turbulent jet," to appear in J. Fluid Mech. (1995).