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A Boundary Layer Growth Model For One-Dimensional Turbulent Unsteady Pipe Friction

M.F. Lambert[1], J.P. Vítkovský[1], A.R. Simpson[1] and A. Bergant[2]

[1]Department of Civil and Environmental Engineering
Adelaide University, Adelaide SA 5005, AUSTRALIA

[2]Litostroj E.I. d.o.o., Litostrojska 40, 1000 Ljubljana, SLOVENIA

Abstract

Unsteady flow in pipe networks is efficiently modelled using a one-dimensional flow approximation. It is general practice in engineering to assume a quasi-steady state approximation of the friction for unsteady pipe flows. The result of this approximation is an under-estimation of the damping during fast transient events. To remedy this shortcoming, an unsteady friction model is often employed. Unsteady friction models for laminar flow can be theoretically determined and have been successfully used for many years. However, the same cannot be said for unsteady friction in turbulent flows. A number of empirical unsteady friction models have been formulated, but only perform well for certain unsteady transient event types. This paper presents a new unsteady friction model for turbulent flows based on the growth and destruction of the boundary layer during a transient event.

Introduction

Historically the simulation of unsteady flow events in pipelines and pipe networks has only focussed on the largest pressure response for a transient event. This was to anticipate the maximum and minimum pressures a pipeline would experience when subjected to pump failure or an unexpected fast valve closure. Typically, the long-term prediction of transient events was poor. Recently, leak detection and calibration techniques, such as the inverse transient method, have renewed the interest in accurate simulation of long-term transient events.

Driven by observations, such as more rapid pressure decay and phase shifting from fast transient events, researchers have attempted to model these events accurately. Early researchers noticed that, particularly for fast transient events, the damping of the pressure trace was significantly larger than the steady state friction approximation would suggest. Daily *et al*. [5] suggested that this extra dissipation was larger for accelerating flows than for decelerating flows and an empirical relationship was proposed that related the extra dissipation to the instantaneous local acceleration. A similar relationship was proposed by Carstens & Roller [4]. Other models that have been proposed include those for oscillatory flows (Hino *et al*. [7]), using boundary layer models (Wood & Funk [14]), and turbulence models using mixing length concepts (Pezzinga [9]) or the k-ε model (Eichinger [6]). These models are computationally intensive, especially two-dimensional unsteady flow and CFD models, and not suited to the study of transients in long pipelines or networks experiencing sharp transients.

Zielke [17] determined an analytical method to model fast transient events in pipelines occurring during laminar flow conditions. The Zielke formulation applies weights to past velocity differences. This formulation is computationally intensive and was later made more efficient by other researchers. As laminar flow conditions are unlikely to exist in many pipelines and networks in reality, there is a need to be able to model unsteady events that occur during turbulent flow conditions.

Background

When the pressure and flow in a pipeline change with time its behaviour is said to be unsteady or transient. The behaviour of a fluid in a pipeline can be described using conservation of mass and linear momentum (Wylie & Streeter [16]). The simplified equation of continuity for unsteady pipe flow is

$$\left(\frac{\partial H}{\partial t}+V\frac{\partial H}{\partial x}\right)+\frac{a^2}{g}\frac{\partial V}{\partial x}=0 \tag{1}$$

and the simplified equation of motion for unsteady pipe flow is

$$\frac{\partial H}{\partial x}+\frac{1}{g}\left(\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}\right)+J=0 \tag{2}$$

where H = head, V = average velocity, a = wave speed, t = time, x = distance, g = gravitational acceleration and J = headloss per unit length due to friction. Friction in pipelines has been traditionally modelled using a quasi-steady state approximation,

$$J=\frac{fV|V|}{2gD} \tag{3}$$

where D = pipe diameter and f = Darcy-Weisbach friction factor, which can be determined using the Colebrook-White formula. The quasi-steady friction approximation is valid for gradually varying flow, but becomes increasingly invalid for unsteady flows where it underestimates the frictional dissipation. To compensate, an additional unsteady friction term is added to the quasi-steady friction term. An example of such a term is the Brunone *et al*. [3] unsteady friction model (as modified in Bergant *et al*. [2]),

$$J=\frac{fV|V|}{2gD}+\frac{k_3}{g}\left(\frac{\partial V}{\partial t}+a\phi_V\left|\frac{\partial V}{\partial x}\right|\right) \tag{4}$$

where k_3 = unsteady friction coefficient, $\partial V/\partial t$ = temporal acceleration, $\partial V/\partial x$ = convective acceleration and ϕ_V = a velocity sign operator (= +1 for $V \geq 0$ and –1 for $V < 0$).

Equations (1) to (4) represent a set of nonlinear hyperbolic partial differential equations that are most efficiently solved using the method of characteristics (MOC). The MOC re-orientates the partial derivatives in the direction that a disturbance would propagate in, such that equations (1) and (2) are transformed into two ordinary differential equations in time that are valid along positive and negative characteristics. Solution of the ordinary differential equations is performed on a characteristic grid. In addition, the solution of equations (1) and (2) is subject to initial and boundary conditions. In a pipe network the initial conditions might be a steady state solution and the boundary conditions might be pipe junctions, reservoirs, valves, pumps, etc.

The use of an empirical formula, such as equation (4), to describe the unsteady turbulent friction requires a number of coefficients that are typically determined experimentally. In many cases it is impossible to calibrate for these empirical unsteady friction coefficients. Formulas for k_3 exist for flows with sudden wave fronts, such as Carstens & Roller [4], Shuy & Apelt [12] and Vardy & Brown [13], however, none have been verified for turbulent flows with Reynolds number greater than ≈15,000. Also, empirical unsteady friction models, such as equation (4), perform badly for some unsteady flow cases (Vítkovský [14]). Given these problems with empirical models, a more physically-based model might provide better results.

Boundary Layer Growth Model

The behaviour of friction in a pipeline during a transient event can be visualised using boundary layer concepts. Figure 1 shows an idealised diagram of the growth or development of the boundary layer in a pipe, where x represents the boundary layer development length and δ is the boundary-layer thickness. V and U are the average and maximum (or core) velocities for a cross-section of the pipeline, respectively.

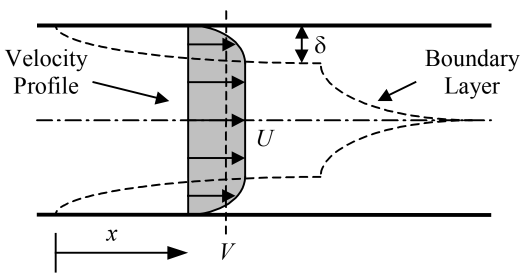

Figure 1. Velocity distribution and boundary layer growth in a pipe.

The coefficients for empirical unsteady friction models, such as that shown in equation (4), are typically based on the Reynolds number of the initial flow or the instantaneous flow. The Reynolds number is defined as

$$\mathbf{R} = \frac{VD}{\nu} \tag{5}$$

where ν = the kinematic viscosity. The transition between laminar and turbulent flow in steady pipe flow occurs between Reynolds numbers of 2,000 and 4,000. Basing an unsteady friction model on the Reynolds number can cause problems, for example, when laminar behaviour persists to higher Reynolds numbers for accelerating flows and transition to turbulent behaviour is postponed (Lefebvre & White [8]). Additional problems may occur due to relaminarisation of a turbulent flow causing high dissipation in decelerating flow (Shuy [11]). Both behaviours can be better explained using boundary layer concepts. A more useful friction parameter, especially for a boundary-layer model, is the plate or distance-based Reynolds number

$$\mathbf{R}_x = \frac{Ux}{\nu} \tag{6}$$

The transition between a laminar and turbulent boundary layer occurs at a plate Reynolds number of approximately 500,000.

An example of how the boundary layer thickness changes during an unsteady flow event is demonstrated for the pipeline shown in figure 2. The transient event is generated by the fast closure of a valve that is located at the downstream end of a tank-pipe-valve-tank system.

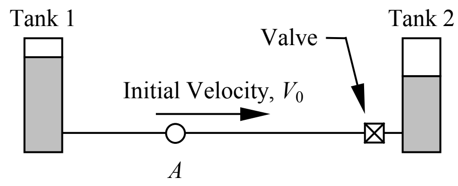

Figure 2. Layout of the simple tank-pipeline-valve-tank system.

The initial flow in the pipeline is steady and fully developed, thus the boundary layer has extended to the centre of the pipe. When the valve closes a water hammer wave propagates from the valve at the wave speed. The wave brings a pressure increase, which is dependent on the compressibility of the fluid and elasticity of the pipe wall material, and a fluid velocity of zero. When the wave reaches tank 1 it is reflected bringing a pressure decrease to the pressure in tank 1 and a flow reversal. This flow reversal must result in a new boundary layer being grown. When the wave returns to the valve, the flow is stopped and the process is repeated with flow in the opposite direction. The result is the successive destruction and growth of the boundary layer as shown in figure 3.

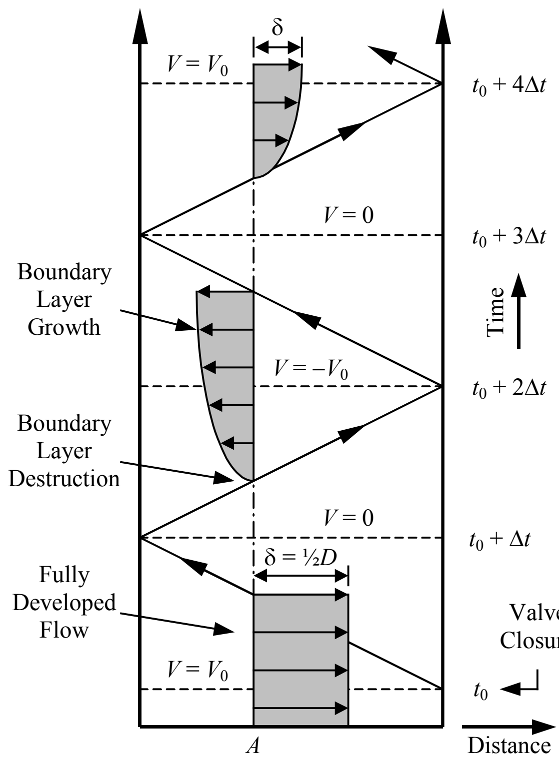

Figure 3. Characteristic diagram for a fast valve closure of the simple tank-pipeline-valve-tank system (shown in figure 2) displaying boundary -layer thickness variation at point A over time.

The higher frictional dissipation (above that of quasi-steady friction) is due to the high shear stresses that occur in the initial stages of the boundary layer growth.

The boundary-layer growth method used in this paper is based on steady boundary-layer growth formulae (Schlichting [10]). The laminar boundary layer thickness is a function of development length

$$\delta = 4.65\sqrt{\frac{\nu x}{U}} \quad (7)$$

The shear stress at the pipe wall, τ, due to the laminar boundary layer is

$$\tau = 0.322\rho\sqrt{\frac{\nu U^3}{x}} \quad (8)$$

where ρ = density. The turbulent boundary layer, in this case, is based on Prandtl's one-seventh-power law for a smooth pipe and thus is only accurate up to Reynolds numbers of 100,000; however, logarithmic laws could be used to extend this range and applicability to rough pipes. The turbulent boundary-layer thickness is

$$\delta = (0.292x)^{4/5}\left(\frac{\nu}{U}\right)^{1/5} \quad (9)$$

The shear stress at the pipe wall due to turbulent boundary layer is

$$\tau = 0.029\rho U^{9/5}\left(\frac{\nu}{x}\right)^{1/5} \quad (10)$$

The boundary-layer model is incorporated into the MOC solution of the governing equations through the unsteady friction term, J, in equation (2). In this case, the unsteady friction in terms of the unsteady shear stress at the pipe wall, τ_U, is

$$J = \frac{4\tau_U}{\rho g D} \quad (11)$$

The shear stress is calculated using the boundary-layer formulae. At each point in the characteristic grid the boundary-layer thickness, pressure and flow are calculated and stored. The frictional effects are evaluated at the base of each characteristic resulting in a first-order-accurate numerical scheme for the friction. Figure 4 shows an example of the boundary-layer thickness and subsequent growth for a positive characteristic in the characteristic grid.

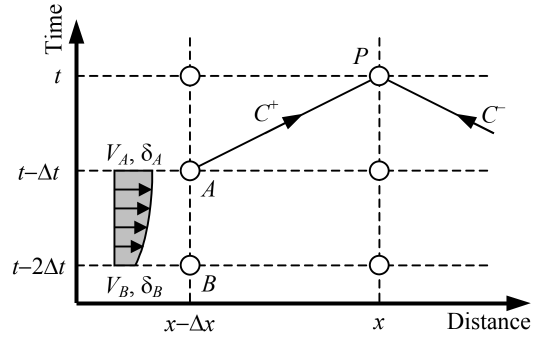

Figure 4. Schematic of the boundary-layer thickness for the positive characteristic in the characteristic grid.

The boundary-layer development length, Δl, is based on the average fluid velocity and the time step of the characteristic grid

$$\Delta l = V_{AVG}\Delta t \quad (12)$$

where V_{AVG} = the average velocity and Δt = the characteristic time step. In figure 3 the average velocity is equal to $\frac{1}{2}(V_A + V_B)$. A new boundary-layer thickness is calculated for point A based on the boundary-layer thickness at B and the development length. The unsteady shear stress is equal to the drag for AB divided by the boundary-layer growth length Δl,

$$\tau_U = \frac{1}{\Delta l}\int_l^{l+\Delta l} \tau dx \quad (13)$$

Equation (13) eliminates any problems caused by very small (or zero) boundary-layer thicknesses that result in near infinite shear stresses.

Additional constraints added into the boundary-layer calculation are:

- The thickness of the boundary layer in a pipe is limited to the radius of the pipe.
- For flow reversals the boundary layer is destroyed and must be re-developed.
- If the flow decelerates too quickly then separation of the boundary layer occurs and the boundary layer must be re-grown.
- Care must be taken to model the transition of the laminar boundary layer to the turbulent boundary layer and vice-versa.
- The ratio of the maximum velocity (required by the boundary-layer formulae) to average velocity (used in the unsteady pipe equations) is calculated based on boundary-layer thickness and type (laminar or turbulent).

The boundary-layer growth model is experimentally tested in the following section.

Comparison with Experimental Data

A versatile laboratory apparatus for investigating water hammer events in pipelines has been designed and constructed (Bergant & Simpson [1]). The apparatus comprises a straight 37.2 m long sloping copper pipe of 22.1 mm internal diameter and 1.63 mm wall thickness connecting two pressurized tanks (similar to figure 2). The estimated relative roughness of the copper pipe walls is 0.0001, which is hydraulically smooth for the flows considered.

Five pressure transducers are located at equidistant points along the pipeline including two that are as close as possible to each of the tanks. Pressure measured at the valve is considered in this paper. The water temperature in tank 1 is continuously monitored and the valve position during closure is measured using a potentiometer that is attached to the valve handle.

A specified pressure in each of the tanks is controlled by a computerized pressure control system. A water hammer event in the apparatus is initiated by closing or opening the ball valve. Fast closure of the valve is normally carried out by a torsional spring actuator (the closure time may be set from 5 to 10 milliseconds). First an initial steady-state velocity condition is established. Second a transient event is initiated by closure of the valve.

For the experimental test considered in this paper, the initial velocity was 0.3 m/s, the initial head at tank 1 was 30.0 m and the temperature was 15°C. The Reynolds number of the initial flow was 5,600. The wave speed, equal to 1,290 m/s, was determined from frequency spectrum analysis of the experimental data. Using the logarithmic law of velocity distribution for turbulent flow in a pipeline, the ratio of the maximum velocity to the average velocity was calculated as 1.29. Figure 5 shows the comparison of the experimentally measured head at the valve to the simulated head using both the quasi-steady friction and boundary layer growth models.

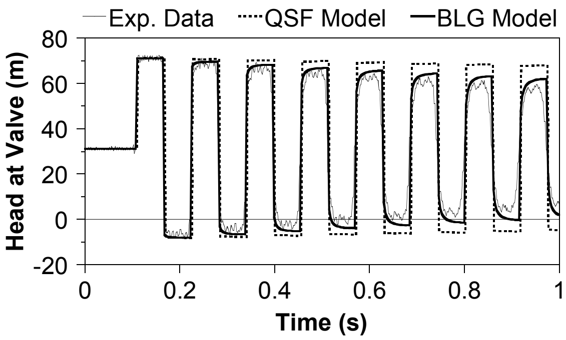

Figure 5. Comparison between experimental data and simulation results using the quasi-steady friction (QSF) and the boundary layer growth (BLG) models.

In figure 5 the quasi-steady friction model under-predicts the damping due to friction as observed in the experimental data. The boundary-layer growth model shows greater frictional damping and a better match with the experimental data. Figure 6 shows the velocity and boundary layer thickness variation at tank 1. Following the initial fully developed flow, a succession of boundary layer growth and separation cycles are observed with separation coinciding with each flow reversal.

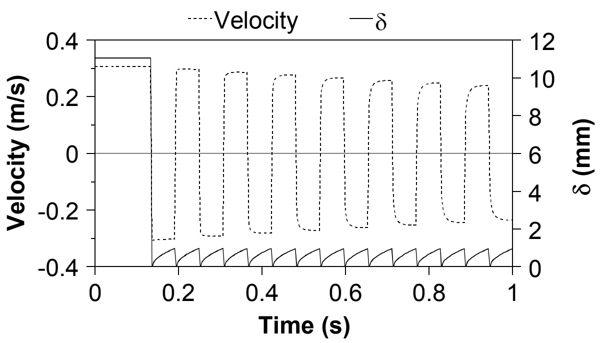

Figure 6. Plot of the velocity and boundary layer thickness at tank 1.

The low Reynolds number for this experiment unfortunately locates the initial friction in the transition zone of the Moody diagram. Therefore, the maximum to average velocity ratio used could be larger than originally calculated, which would produce more damping and an even better match with the experimental results.

Conclusions

The boundary-layer model produces more damping compared to the standard quasi-steady friction model. In this respect the boundary-layer growth model is producing a more realistic unsteady friction effect. It is also noted that for many transient events in turbulent flows, it is the growth of a laminar boundary layer that dictates the friction rather than turbulent friction relationships. This happens because a flow reversal or stopping occurs before the transition to turbulence can be reached.

A feature of the boundary-layer growth model is that unsteady friction is not treated as a separate quantity to the quasi-steady friction (as is performed in current unsteady friction models such as equation (4)). This is advantageous since it is unlikely that, in reality, unsteady friction can be easily separated into quasi-steady and unsteady components.

One problem with the boundary layer model presented in this paper is that steady boundary layer formulae are used. For unsteady flows, unsteady boundary layer models should be used where applicable; however, the model in this paper presents the first generation of a boundary-layer model for unsteady pipe flows, which will be refined. Given the early progress of the boundary-layer model and the insight into the behaviour of friction in fast unsteady flow events, the boundary-layer model shows promise for further study.

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