ASSESSMENT OF THE LUCK ASSOCIATED WITH SETTLEMENT PREDICTIONS THAT ARE BASED ON ELASTIC THEORY

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ABSTRACT

This paper deals with quantifying the uncertainty associated with settlement predictions that are based on elastic theory. Because of spatial soil variability, deterministic solutions cannot provide information about the likely error of the prediction. Accordingly, it is only through luck that a particular deterministic prediction coincides with the observed settlement.

INTRODUCTION

Settlement prediction is undertaken as part of either design of new structures or assessment of the performance of existing structures. Although both design and assessment deal with performance, the underlying concepts of the two approaches to settlement prediction are different. The design process is concerned with ensuring that excessive settlements do not occur and as such, the use of conservative soil parameters is common. If the predicted settlement is greater than the observed settlement, the design is deemed to be satisfactory since larger settlements were incorporated in the design, less settlement should not cause any problems.

On the other hand, in the assessment of performance the aim is to ensure that the prediction is as close as possible to the actual field observation. The use of conservative values of soil parameters is no longer acceptable because the measure of success is how close a prediction is to the observed performance. Accurate settlement prediction is very important in staged construction. It is also important in the assessment of existing defective structures, or where additional loads are proposed.

Settlement prediction, within a few percentages of the observed settlement is considered impossible. The difficulties faced by engineers revolve around the uncertainties associated with the loading conditions, the soil response or site conditions, and the prediction process adopted. The impact of variable loading conditions is more readily appreciated compared to the other two issues. It is instructive to evaluate the impact of the idealisations and simplifications that are part and parcel of the prediction process. Unfortunately this is not always done with the rigour that it deserves.

Vanmarcke (1) enumerated three major sources of uncertainties concerning soil response characterisation. These are:

1. Natural heterogeneity of the soil, which arises from variations in mineral composition and stress history of the site.

2. Measurement errors, where measured values differ from actual field values due to sample disturbance, test imperfections and human factors. These errors can be reduced by careful experimentation.
3. Uncertainty attributed to the limited availability of information from site investigations. Soil profile characteristics therefore have to be inferred from limited field testing or laboratory investigation of a limited number of soil samples.

The problems of soil heterogeneity can only be minimised by thorough testing and modelling of the spatial variability. However, not enough attention is paid to the effects of these uncertainties in settlement prediction. The shortcomings of using a deterministic approach led to the development of probabilistic methods in the late sixties most notably by Wu and Kraft (2). However, the probabilistic approach is not widely used in practice because of the statistical terminology and concepts that are not familiar to most geotechnical engineers. Moreover, it is perceived that adopting probabilistic analyses would require an inordinate amount of data, time and effort.

VARIABLE SOIL CONDITIONS MEANS VARIABLE OUTCOMES

Minimal site investigations, due to perceived time and cost constraints often result in limited subsurface data, leading to problems during the design stage. The resulting time delays and budget blowouts invariably exceed any savings on site investigation. There is no doubt that settlement prediction based on inadequate knowledge is risky and fundamentally unreliable. Soil parameters vary spatially and hence a single prediction is as useful as a single measurement of a soil parameter for the entire soil profile. As soil is naturally variable, there ought to be a range of likely predictions. As an example, consider a circular footing 2 m in diameter supporting 100 kPa average pressure, at a site underlain by soft clay. The settlement of the footing can be evaluated based on elastic theory (Equation 1) to assess the performance of the structure.

$$\rho = \frac{qdI_\rho}{E_s}\left(1-\nu^2\right) \tag{1}$$

where ρ = vertical displacement; q = applied pressure; d = diameter of footing; E = Young's modulus; and ν = Poisson's ratio.

Consider two values of Young's modulus E_s = 2 MPa and 6 MPa, obtained from results of two boreholes drilled close to the location and from laboratory testing. Based on a value of influence factor, I_ρ = 1.05, based on Poulos and Davis (3),and Poisson's ratio ν = 0.2, the predicted settlements are summarised in Table 1.

Table 1: Possible settlement predictions for the example

Basis	Young's Modulus (MPa)	Predicted settlement (mm)
Borehole 1	2	50
Borehole 2	6	17
Average value	4	25

It can be seen that there can be three possible outcomes of predicted settlement. It is not readily apparent which value will be close to the observed value, and as such the reliability of the estimated settlement cannot be quantified.

DESCRIPTION OF SOIL VARIABILITY

Inadequacy of the Deterministic Approach

In the conventional method, analyses are based on the idealisation of the soil profile as a series of uniform soil layers, each soil layer being represented by a single characteristic value of the soil parameter. This characteristic value may be determined from laboratory testing, in-situ measurements or a combination, plus experience. It is basically engineer-dependent. Figure 1 illustrates the idealisation of the soil profile based on cone tip resistance data from a CPT test. The soil profile is subdivided into 5 uniform soil layers, with each layer being represented by a single value of cone tip resistance. It should be noted that the characteristic values are engineer-dependent where conservative engineers would use lower values while less conservative engineers may use the average values.

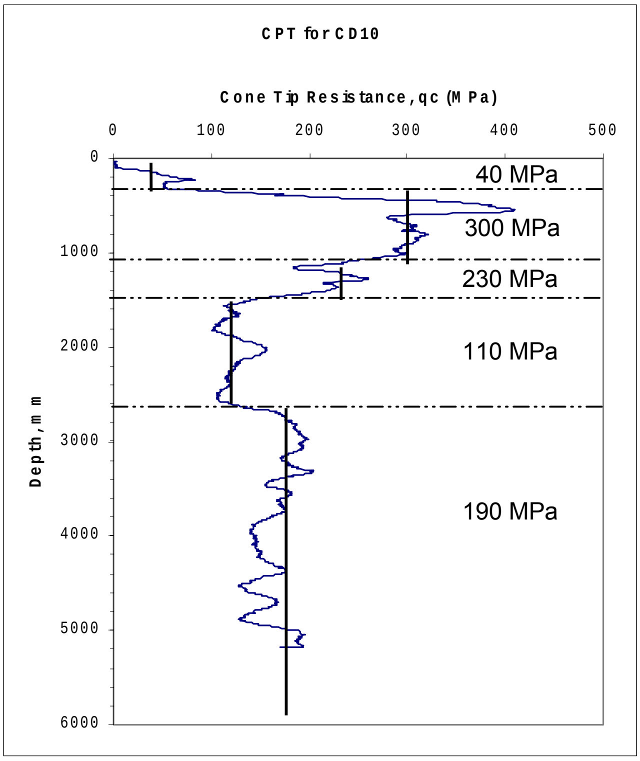

Figure 1: Idealisation of soil profile as a series of uniform layers based on CPT test data

Application of Statistical Concepts

The spatial variation of any soil parameter, v_z can be represented by a mean or trend function t_z , which corresponds to the average value, and a random variation from the trend, ζ_z , as shown in Equation 2.

$$v_z = t_z + \zeta_z \tag{2}$$

According to Vanmarcke (1), the spatial randomness ζ_z, is described by the standard deviation σ, which measures the intensity of fluctuation, and the scale of fluctuation δ, which measures the distance within which a soil property shows relatively strong correlation, as shown in Figure 2.

It is wrongly assumed that a lot of testing is required in order to derive the mean and standard deviation of a soil parameter. This is not always necessary, however, because there is extensive published data on soil parameters. There are numerous empirical correlations based on world-wide studies, as well as data derived from local experience, to help estimate the mean value of a particular parameter. The standard deviation can be estimated from local data by approximating the standard deviation as one-sixth of the range of available data.

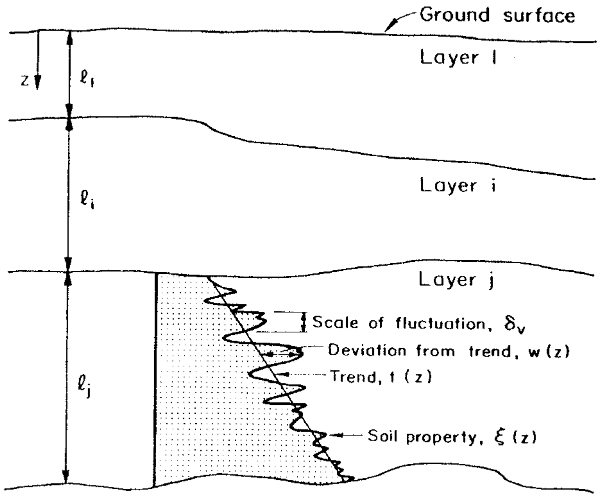

Figure 2: Characterisation of spatial variability of a soil parameter within a soil layer (Phoon and Kulhawy 4)

IMPLICATIONS OF SOIL VARIABILITY ON PREDICTED SETTLEMENTS

The computer program FLEA5 developed by Small and Booker (5), based on finite layer analysis of elastic materials, was used to investigate the effects of spatial variation of Young's modulus on the vertical displacement at the centre of a flexible circular footing. The flexible circular footing was supported by a clayey soil 8 m thick. A pressure of 100 kPa was applied to the footing. The soil layer was subdivided into 49 sub-layers (Figure 3), with each layer assigned a random value of Young's modulus, determined from Monte Carlo formulation to simulate a normal distribution function with specified mean and standard deviation.

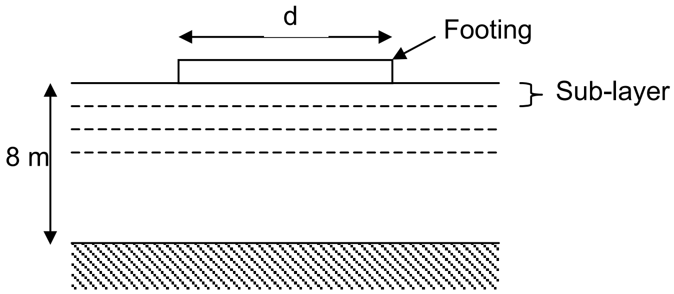

Figure 3: Schematic of model problem analysed using program FLEA5

The spatial variation of Young's modulus is described by the coefficient of variation COV (E), the ratio of standard deviation σ, to mean value μ. Monte Carlo simulation was adopted in the analyses and 200 realisations, or runs, were performed using FLEA5 for each coefficient of variation of Young's modulus. The Young's modulus profiles were generated using a proprietary random number generator. Each profile contains 49 independent random data points that are uniformly distributed and generated based on a mean of 0.5 for COV (E) = 5%, 10%, 15%, 20%, 25%, and 30%. Only Case I and Case III of Figure 1 were used to study the influence of variation of Young's modulus on the vertical displacement of shallow foundation. The effect of variations in Poisson's ratio on settlement prediction was not examined as Fraser and Wardle (6), Cambou (7), Baecher and Ingra (8), and Fenton and Griffith (9) showed that uncertainty in vertical displacement is relatively insensitive to Poisson's ratio. The analyses also investigated the effect of varying the width to depth ratio on the settlement. In the analyses values of depth to radius ratio, (h/a) = 2, 4, 8 and 20, were used. In order to avoid introducing effects due to variations of scale of fluctuation, the depth of each sub-layer was kept constant while the radius of the foundation was varied.

It should be noted that the use of 200 runs implies errors of estimates of the order of 7% for the mean, and 10% for the standard deviation, of the predicted settlement. Bearing these errors in mind, there are two effects of spatial variability on settlement. The first effect is on the expected (mean) settlement, expressed by the settlement influence factor in Equation 1.

This effect is shown in Figure 4. It can be seen that the settlement influence factor increases as the variability of the soil increases. Another way of representing this effect is to define a variability factor, I_v, shown in Figure 5, defined by:

$$\rho_m = I_V . \rho \qquad (3)$$

where, ρ_m is the mean settlement for a specific value of COV (E) and ρ is the settlement of a uniform layer with constant Young's modulus.

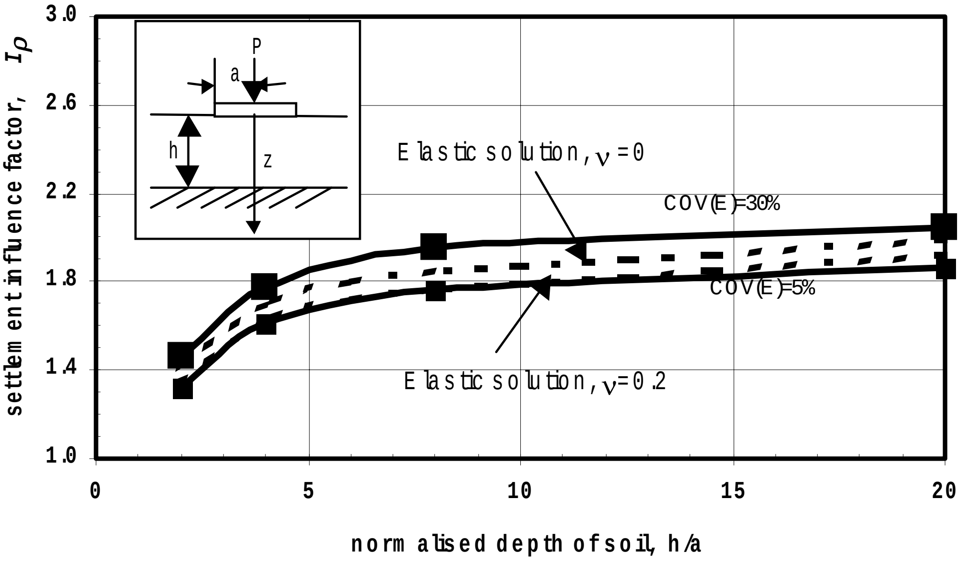

Figure 4: Settlement influence factor for COV (E) = 5% and 30%, compared with solutions of Meyerhof (10) for a uniform soil profile

The second effect of variable Young's modulus is on the variability of the predicted settlement, expressed in terms of the coefficient of variation of normalised settlement, COV (ρ_i/ρ_m). This is shown in Figure 6. It can be seen that the coefficient of variation of settlement increases with increasing variation of Young's modulus.

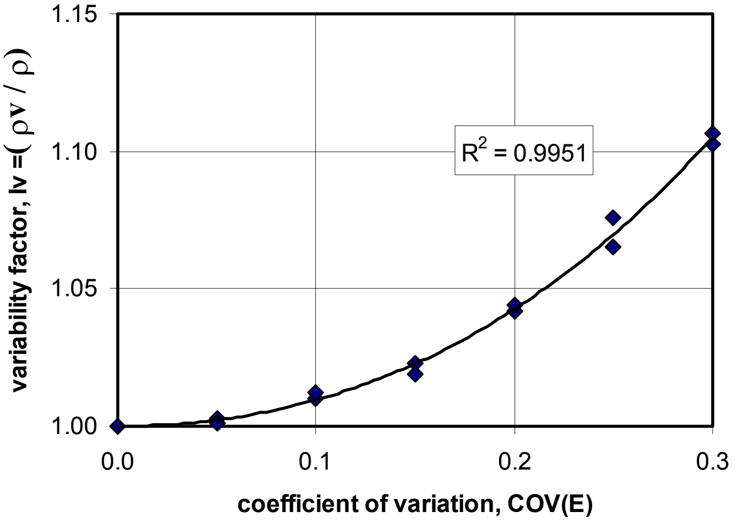

Figure 5: Variation of variability factor of settlement with variability of Young's modulus using (h/a) = 2 to 20

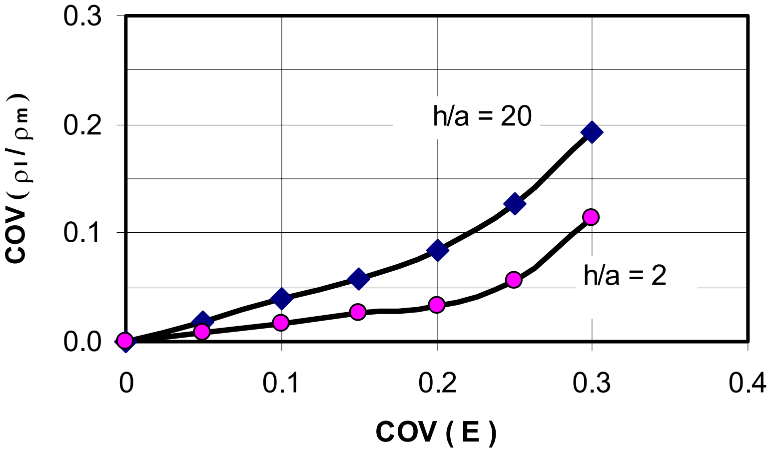

Figure 6: Relation between the coefficients of variation of settlement and Young's modulus for a circular footing on a uniform soil profile

EVALUATION OF PROBABILITY OR 'LUCK' IN SETTLEMENT PREDICTION

The results shown in Figures 5 to 7 can be used to evaluate the likelihood that the observed settlement will lie within a specified range of predicted values. By incorporating probability into the prediction process, a clearer picture emerges about the likely range of settlements. This is very helpful in decision-making, thereby quantifying the risks inherent in the prediction. The method is illustrated using the following example.

A flexible circular footing is to be constructed over a site, with 30-metre deep variable soil layer overlying bedrock. The width of the footing is fixed at 3 m. The footing is to carry a pressure of 50 kPa. Based on past experience, the Young's modulus of the alluvium is variable but the average value is 5 MPa. Laboratory testing of soil samples from the site shows that the Young's modulus of the soil at site can be as high as 9.5 MPa. At a nearby location, the lowest value of Young's modulus obtained from laboratory testing is 3.5 MPa.

Determination of Statistical Parameters

Based on experience and soil testing, the variation of Young's modulus of the soil can be determined by computing the mean and standard deviation, from which COV (E) is determined. The standard deviation can be estimated from the difference between the highest value obtained or expected, and the mean value based on testing and experience. If the two sites are treated independently, then we have two cases. For Case 1 (based on the difference between maximum and average values), σ = 1.5 and COV (E) = 0.3, and for Case 2 (based on difference between average and minimum values), σ = 0.5 and COV (E) = 0.1.

Incorporation of the Effects of Spatial Variability of Young's Modulus

In the example, the width to depth ratio, (B/h) = 0.10 (diameter of 3 m and soil layer thickness of 30 m). The mean settlement ρ_m can be determined using the expected (mean) value of Young's modulus and displacement influence factor, I (e.g. Poulos and Davis, 3). The variability factor I_v, for a COV (E) of 0.1 and 0.3 can be obtained from Figure 6. I_v = 1.02 and 1.11 for COV (E) = 0.10 and 0.30, respectively. Accordingly, the mean settlement, ρ_m = 31 mm for COV (E) = 0.1 and ρ_m = 33 mm for COV (E) = 0.3.

The standard deviation of settlement can be determined from the radius to depth ratio, (a/h) = 0.05 (footing radius of 1.5 m and soil layer thickness of 30 m) and the use of Figure 7. The coefficient of variation COV (ρ_i/ρ_m) = 0.19 for a COV (E) =0.3 and COV (ρ_i/ρ_m) = 0.04 for a COV (E) =0.1. It can be shown that ρ_σ = 1.2 mm for COV (E) = 0.1 and ρ_σ = *6.3 mm* for COV (E) = 0.3. Thus the predicted settlement is *31 ±1.2 mm* for COV (E) = 0.1 and *33 ±6.3 mm* for COV (E) = 0.3.

Evaluation of Probability Associated with Prediction

In order to determine the probability of occurrence of a particular range of values of settlement, the standard normal distribution chart can be used. The mean and standard deviation of the estimated settlement are standardised using $Z = (x-\mu)/\sigma$, where x is the magnitude of settlement of interest.

Suppose we are interested in evaluating the likelihood that the observed settlement will lie between 28 and 30 mm or 25 and 35 mm. Table 2 is a summary of the computations.

Table 2: Summary computation table for probability of settlement falling in given range for the example problem

Settlement range ($\rho_l - \rho_u$) (mm) (1)	COV(E) (2)	Settlement, mean and standard deviation (mm) (3)	Range of values of Z ($Z_l - Z_u$) (4)	Range of probabilities ($p_l - p_u$) (5)	Probability of settlement falling in range (6)	Comment on probability of occurrence (or luck) (7)
28 to 30	0.1	31 and 1.2	–2.500 to –0.833	0.006 to 0.202	0.196	1 in 5 chance
28 to 30	0.3	33 and 6.3	–0.794 to –0.476	0.214 to 0.317	0.103	1 in 10 chance
25 to 35	0.1	31 and 1.2	–5.000 to 3.333	0.00 to 0.9996	0.9996	Extremely likely
25 to 35	0.3	33 and 6.3	–1.270 to 0.317	0.102 to 0.625	0.5225	1 in 2 chance

It can be seen that for the profile with COV (E) = 0.1, there is a 20% likelihood that the observed settlement will be between 28 and 30 mm compared to 10% for COV (E) = 0.3. In other words, if the designer predicted settlements to be between 28 and 30 mm, the likelihood of success (or luck) would be 10 to 20%. On the other hand, if the designer was interested in a less precise estimate, and adopted a wide range for the settlement, say 25 to 35 mm, then for COV (E) = 0.1, this range essentially brackets the likely observations (99.96% likelihood). In the case of COV (E) = 0.3, there is only 52% likelihood that the observed settlement will be within that range.

CONCLUSIONS

It has been shown that the effects of variation of Young's modulus within a soil profile, on the predicted elastic settlements, can be readily quantified and incorporated in routine analyses. Increasing spatial variability increases the expected (mean) settlement and the variation about this mean value. A simple procedure has been presented to enable prediction of the statistical moments of the predicted settlement, and how these can be used to evaluate the likelihood that a particular settlement prediction will eventuate.

NOTATION

COV	coefficient of variation (ratio of standard deviation to mean value)
I_ρ	settlement influence factor
I_v	variability factor for settlement to account for randomly variable soil

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