Interaction in Factorial Experiments: Relevance of Curvilinearity of Response

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Abstract

It is proposed that a possible explanation of interaction between independent variables in a factorial experiment is that (a) quantities derived from the independent variables separately add together, but (b) a curvilinear relationship intervenes between their total and the dependent variable observed.

Keywords: Factorial Experiments, Interaction (Statistical), Bad News (Communication of)

Introduction

Factorial experimentation involves manipulating two (or more) x's, and observing y at all combinations of the values of x_1 and x_2. If the effect on y of x_1 depends upon what x_2 is, there is said to be an interaction between the x's. An example from the broad field of management science is in Tan et al. (2003). The dependent variable was predisposition to report bad news about a project going wrong (as measured with a questionnaire). The independent variables were national culture (two levels, individualism (U.S.) versus collectivism (Singapore)) and information asymmetry (also two levels, whether or not bad news could be hidden). The following data are from Table V of Tan et al. (2003) (and refer to the "organizational climate being conducive to reporting bad news" situation).

	Hiding of bad news (Sustainable or not)	
	Sust.	Not sust.
National culture		
Individualism	5.20	5.41
Collectivism	4.53	5.62

There is interaction: the difference of 0.67 in the first column is different from the difference of -0.21 in the second column. (The question of whether interaction is "statistically significant" is not of interest in the present paper.)

As readers may know, there is a vast literature on statistical testing in factorial experiments. But there is surprisingly little about the explanation we might give when interaction is found. In Tan et al. (2003), for example, it is said that "collectivism appeared to amplify the impact of information asymmetry". I suggest this is a description, not an explanation or a theory. The purpose of the present note is to point out that a possible explanation of interaction is that (a) the x's add together, but (b) a curvilinear relationship intervenes between the total of the x's and the dependent variable y observed. A curvilinear relationship should not be regarded as surprising: for some x's, there is an optimum value, and both higher and lower values lead to y being worse (see for example, Patrashkova-Volzdoska et al., 2003).

Theory and Examples

In practice, one has data and wants to find a model. However, it will be easier to appreciate the proposed explanation of interaction by starting from a model and working out what the data will look like. In the following example, round numbers are used, so the calculations will be easy to follow. Consider the curvilinear relationship $y = -x^2 + 1$.

- If x is -0.9, say, y is 0.19. If x increases by 0.8, to -0.1, y is now 0.99, a change of +0.80.
- If x is -0.3, say, y is 0.91. A change of 0.8 in x brings it to 0.5, so y is 0.75, a change of -0.16, which is not the same as +0.80.

Now, suppose those four values of y were observed in a factorial experiment in which two factors (independent variables), x_1 and x_2, were manipulated:

	x_2: Level 1	Level 2
x_1 : Level 1	0.19	0.99
Level 2	0.91	0.75

Having constructed the data in that way, it is obvious that the following model will fit the data. Let α be -0.9 or -0.3, according to

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whether x_1 is at level 1 or level 2. Let β be 0.0 or 0.8, according to whether x_2 is at level 1 or level 2. Add the two contributions together to get the total for a given set of experimental conditions, $x = \alpha + \beta$.

		β	
		0.0	0.8
α:	-0.9	-0.9	-0.1
	-0.3	-0.3	0.5

Then $y = -x^2 + 1$.

Thus the basic model is addition of contributions from x_1 and x_2, as with the main effects model in analysis of variance. The simple pattern is then obscured by the intervention of the curvilinear dependence of y on total x. For more details, see Hutchinson (2004).

As to the data from Tan et al. (2003), suppose α is 0 or -1, and β is 0 or 3.

		β	
		0	3
α:	0	0	3
	-1	-1	2

Further, let $y = -0.13x^2 + 0.47x + 5.2$. The y's are now very close to the observed values:

5.20	5.44
4.60	5.62

On their own, I do not take the parameter values very seriously. It is possible that qualitatively different values would give about as good a fit to the data. What is needed is a suggestion of a suitable variable that is the real driver of y. What has been found is that the relationship is inverted-U in form, the contribution of individualism relative to collectivism is positive, and the contribution of the hiding of bad news not being sustainable relative to this being sustainable is positive.

Why should anyone care about this? In response, I think there are two main contexts for factorial experiments.

1. It may be hoped that interactions are not found, and therefore the whole pattern of means can be explained by main effects only. This will be very useful if the table of data is a large one. For example, if there are 6 levels of one factor and four levels of the other, there are 24 means. But if main effects alone are a good description of the data, only 9 numbers are needed (grand mean, effects of x_1, and effects of x_2).
2. Alternatively, there may be no great interest in the main effects (e.g., they be well known to exist), but it may rather be hoped that interactions are discovered, because they are something new and beyond the existing knowledge.

In case 1, the model proposed may be useful if the main effects model fails to be a good description of the data. At present, the only choices available are the full model with a parameter for every cell of the table, i.e., RC parameters (R = number of levels of x_1, C = number of levels of x_2), and the main effects model having R+C-1 parameters. An extra parameter reflecting the curvature of the dependence of y on total x may give a fit that is appreciably better than the main effects model, with parameters that are appreciably fewer in number than the full model. In case 2, the new model is itself a step towards a theory. And it makes clear what questions we need to answer next. Why are the values of α as they are? Why are the values of β as they are? What is an appropriate name for total x? Why is the nonlinear function as it is? (To expect quantitative answers will usually be unreasonable. But a qualitative explanation may be possible: for example, it might be reasonable to expect a particular ordering of the levels of x_1 in respect of their contributions to arousal, the same might be true of x_2, and an inverted-U dependence of performance on arousal might be considered so frequent a finding that it does not require explanation.) It seems to me that these are much more tractable questions than attempting to explain interaction without any clue as to how this might arise.

Mathematical and Interpretational Questions

Additivity-plus-nonlinearity has been put forward here as a possible explanation for interaction generally, not to argue that it actually is the explanation for the findings in Tan et al. (2003), which in any case was a three-factor experiment, not a two-factor one. (The third factor was whether organisational culture was conducive to reporting bad news.) I suggest two questions in particular as important and to which readers of this journal may already be able to offer partial answers.

What is known about fitting nonlinear models of the type proposed? (Aspects of this question include the following. What algorithms are best? In what circumstances do multiple local optima occur, and in what circumstances do they not? Concerning the dependence of y on x, is some other choice better computationally than a quadratic? Concerning the quantity to be optimised, is some choice better computationally than the sum of squared differences between observed and predicted y's? How can the global optimum be found? How do the answers to these questions vary with the size of the data table?)

By what mechanisms (or via what widely-accepted relationships) do U or inverted-U dependencies arise? (If we had a list, we could look through it and consider whether any of them were plausible with whatever variables were under consideration.) As a start, let me propose three. (a) Measures of performance having an inverted-U relationship to arousal. (b) The dependent variable is the product of an increasing function of x and a decreasing function of x. For example, output = rate of production multiplied by time available. If x both increases the rate of production and also consumes time, dependence of output on x is likely to be inverted-U in shape. (c) There is a normal level of x, to which the system as a whole has adapted. If x is lower than normal, there is some reason why the system's performance is poorer. If x is higher than normal, there is some quite different reason why the system's performance is poorer.

References

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