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Application of the Multi-Modal Integral Method (MMIM) to Sound Wave Scattering in an Acoustic Waveguide.

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1 Table of Contents.

2 Abstract.

The current work is devoted to the problem of sound wave scattering by elastic cylindrical objects in a plain acoustic waveguide. The Multi-Modal Integral Method (MMIM) is proposed, which is based on non-standard representation of the Green's function. It combines advantages of integral equation and eigenfunction methods and provides a quickly converging and highly accurate solution, taking into account all the waveguide modes up to infinite order. As illustrations of application of this method, acoustic diffraction is calculated from a system of several parallel homogeneous cylinders and from an air-filled cylindrical elastic shell.

Numerical solutions are found for various versions of the system of elastic cylinders in a fluid layer with perfectly soft and rigid boundaries. Phase - frequency and amplitude - frequency characteristics are found for modal coefficients of the scattered field. Sharp increase of their amplitudes is found near resonance frequencies of the waveguide and the scattering cylinders. Pictures of the source density on the surface of the cylinders show that the nature of their distribution strongly depends on the frequency and the mutual location of the cylinders in the waveguide. Field structure near the cylinders reveals that higher-order waveguide modes play a significant role in the scattering process.

Spatial distribution of the acoustic power flow near the scattering object is calculated for several frequencies and two sets of elastic properties of the cylinder. It is shown that at the critical frequencies of the waveguide as well as at the internal resonances of the cylinder the acoustic energy flows in closed paths in some regions of the waveguide. Near the internal resonances of the cylinder the closed paths are located in the near vicinity of the scattering object and partially go through its interior. It is suggested that re-radiation of the energy stored in the vortices may contribute to the echo phenomenon.

The integral reflection coefficient is calculated for a system waveguide/shell for different values of wall thickness and distance between the shell and the waveguide bottom. Maxima and minima in the reflection coefficient associated with cut-on frequencies of the waveguide modes and structural resonances of the shell are identified. The calculations show that the conventional definition of target strength in a shallow waveguide is inappropriate.

Different kinds of resonances are identified in frequency and angular dependencies of the velocity amplitude of the shell surface. These resonances belong to the following groups: a) critical frequencies of the waveguide modes, b) structural resonances of the elastic shell, c) resonance oscillations of the gas-filled interior of the shell, d) resonance oscillations of the coupled fluid-shell.

Temporal sequences of pictures showing the spatial structure of the total and scattered fields in the near and far field zones are obtained. It is shown that the incident field produces waves of acoustic pressure propagating along the boundary of the scattering object, which, in turn, generate the scattered acoustic field. In the process of propagation, the waves may interact with each other via the fluid or the scattering object. This leads to significant changes of the structure of the acoustic field and of the amount of acoustic energy reflected from the scatterer. It is also shown that, in most cases, standing waves exist between the scatterer and the waveguide boundaries.

Accuracy of the Multi-Modal Integral Method is discussed. It is shown that the implementation of the method requires few computer resources for good accuracy of the solution.

3 Statement of Originality.

This thesis contains no material, which has been accepted for the award of any other degree or diploma in any university. To the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

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5 Notations.

5.1 Latin Letters.

A

$a_{m\nu}$	amplitude of the m-th cylindrical mode of the incident field with respect to the cylindrical coordinate system with the origin in the centre of ν-th cylinder
$\overline{A}_n$	amplitude of the n-th normal mode of the incident field
$A_{\nu m}$	amplitude of the m-th mode in the representation of the scalar elastic potential in the ν-th cylinder

B

$b_{p\nu}$	p-th Fourier coefficient in the representation of the function $\mu_\nu(\varphi_\nu)$
$B_{\nu m}$	amplitude of the m-th mode in the representation of the z-component of the vector elastic potential in the ν-th cylinder

C

c	sound speed in fluid
c_s	Stoneley wave speed
c_r	Rayleigh wave speed
c_f	Franz wave speed
c_{ls}	speed of symmetric Lamb waves
c_{la}	speed of asymmetric Lamb waves
c_{wg}	Whispering Gallery wave speed
c_l	longitudinal elastic wave speed
c_t	transversal elastic wave speed
$c_{l\nu}$	longitudinal elastic wave speed in ν-th cylinder
$c_{t\nu}$	transversal elastic wave speed in ν-th cylinder
c_g	sound speed in the gas-filled interior of the shell

D

D	depth of the waveguide

E

E	unity matrix

F

$F^{(\nu)}$	z-component of the vector potential of displacement in ν-th cylinder

G

g_n	longitudinal wavenumber of the waveguide mode of order n
G_ν	Green's function associated with the ν-th cylinder

H

H	the distance between the centre of a scattering object and the bottom of the waveguide

I

i	imaginary unit (square root of –1)
I	acoustic intensity vector
ΔI	discrepancy in power flows on both sides of scattering objects

J

J_m	Bessel's function of the first kind of order m

K

k	non-dimensional acoustic wavenumber
k_g	non-dimensional acoustic wavenumber in the gas filled shell
k_A	non-dimensional wavenumber (frequency) of the monopole resonance A
k_B	non-dimensional wavenumber (frequency) of the dipole resonance B
k_n	non-dimensional critical wavenumber of the waveguide mode of order n
k_t	non-dimensional wavenumber of transversal elastic waves
$k_{t\nu}$	non-dimensional wavenumber of transversal elastic waves in ν-th cylinder
k_l	non-dimensional wavenumber of longitudinal elastic waves
$k_{l\nu}$	non-dimensional wavenumber of longitudinal elastic waves in ν-th cylinder
$\bar{k}$	dimensional acoustic wavenumber
$K_{\nu l}$	acoustic field radiated by a point source on the surface of the l-th cylinder and observed on the surface of the ν-th cylinder
$K_{\nu l}^{n}$	n-th term in the series $K_{\nu l}$
$\bar{K}_{\nu l}^{n}$	approximation of the n-th term in the series $K_{\nu l}$
$\bar{K}_{\nu l}$	function describing the contribution of the evanescent modes to the Green's function

L

L_ν	external boundary line of the ν-th cylinder
$L_{\nu l}$	radial derivative of the acoustic field of a point source on the surface of the l-th cylinder observed on the surface of ν-th cylinder
$L_{\nu l}^{n}$	n-th term in the series $L_{\nu l}$
$\bar{L}_{\nu l}^{n}$	approximation of the n-th term in the series $L_{\nu l}$
$\bar{L}_{\nu l}$	function describing the contribution of the evanescent modes to the derivative of the Green's function

M

$M_{pm}^{l\nu}$	element of the matrix of the system of linear equations
$\hat{\mathbf{M}}$	scattering matrix

N

n	order of current waveguide mode
N	the number of cylinders in the waveguide

$N_{pm}^{l\nu}$	element of the auxiliary matrix for calculating coefficients $B_{\nu m}$
N_{int}	number of integration points on the interval

P

P	total pressure in the acoustic wave in fluid
P_e	pressure in the incident (external) wave
P^-	pressure in the reflected (scattered back to the source) wave
P^+	pressure in the transmitted (scattered forward) wave
$P_{s\nu}$	pressure in the wave scattered by the ν-th cylinder
P_g	pressure in the gas-filled interior of the shell
P_n	n-th eigenfunction of the boundary value problem in the free waveguide
$\overline{P}_n$	n-th eigenfunction of the conjugate boundary value problem in the free waveguide
P_{pm}^{ν}	coefficients describing the right-hand part of the system of linear equations
P_m	m-th cylindrical component of the acoustic pressure in the exterior fluid
P_m^g	m-th cylindrical component of the acoustic pressure in the interior fluid

R

r_ν	radial polar coordinate with the origin in the centre of the ν-th cylinder
R_ν	radius of the ν-th cylinder
R	radius of a cylindrical scattering object
R_1	outer radius of a cylindrical shell
R_2	inner radius of a cylindrical shell
$\Delta \boldsymbol{r}_\nu$	displacement vector in the ν-th cylinder

S

S^-	reflected wave power flow through the waveguide cross-section
S^+	transmitted wave power flow through the waveguide cross-section
S_e	incident wave power flow through the waveguide cross-section

T

t	time
TS	target strength

V

V	particle velocity amplitude

X

x	vertical Cartesian coordinate
x_ν	x-coordinate of a point on the surface of the ν-th cylinder
x_{l0}	x-coordinate of an integration point on the surface of the l-th cylinder
X_ν	x-coordinate of the centre of the ν-th cylinder

Y

y	horizontal Cartesian coordinate

y_ν	y-coordinate of a point on the surface of the ν-th cylinder
y_{l0}	y-coordinate of an integration point on the surface of the l-th cylinder
Y_ν	y-coordinate of the centre of the ν-th cylinder
Y_0	y-coordinate of the centre of a scattering object

Z

z	Cartesian coordinate normal to the (x,y) plane
z^{ν}_{1m}	dummy variable
z^{ν}_{2m}	dummy variable
$z_{\nu m}$	acoustical impedance of the m-th mode on the surface of the ν-th cylinder

5.2 Greek Letters.

α, A

α_0, α_1	coefficients describing boundary conditions on the waveguide walls

β, B

β_0, β_1	coefficients describing boundary conditions on the waveguide walls

δ, Δ

Δ	Laplace operator
δ	delta-function
δ_{pm}	Kronecker's symbol

ε, E

ε	a small vicinity of a point of singularity

ϕ, Φ

$\Phi^{(\nu)}$	scalar potential of the displacement in the ν-th cylinder
Φ_{nm},	auxiliary functions

γ, Γ

γ_n	coefficient in the eigenfunctions of the conjugate boundary value problem

η, H

η_n	transversal wavenumber of the waveguide mode of order n

φ, ϑ

φ	angle polar coordinate of the observation point
φ_0	angle polar coordinate of the source point

κ, K

$\mathrm{K}_{pm}^{l\nu}$	component of the scattering matrix, determining the acoustic pressure on the surface of the ν-th cylinder

λ, Λ

λ_ν	Lame coefficient of the ν-th cylinder
λ	a) acoustic wavelength; b) Lame coefficient of the shell
$\Lambda_{pm}^{l\nu}$	component of the scattering matrix, determining the radial derivative of the acoustic pressure on the surface of the ν-th cylinder

μ, M

μ_ν	Lame coefficient of the ν-th cylinder
μ_ν^0	source field distribution on the surface of the ν-th cylinder
μ_ν	$\mu_\nu = R_\nu \mu_\nu^0$, source field distribution on the surface of the ν-th cylinder related by the radius of the ν-th cylinder
μ	Lame coefficient of the shell

ν, N

ν	a) sequential number of a cylinder; b) complex mode order

π, Π

π	3.1415926…

θ, Θ

θ_n	phase of the waveguide mode of order n

ρ, P

ρ	density of the fluid
ρ_ν	density of the ν-th cylinder
ρ_g	density of the gas
ρ_s	density of the shell

ω, Ω

ω	angular frequency of the acoustic wave

ξ, Ξ

ξ_0, ξ_1	coefficients describing boundary conditions on the waveguide walls, related to $\alpha_{0,1}$ and $\beta_{0,1}$ as $\xi_{0,1}=\alpha_{0,1}/\beta_{0,1}$

ψ, Ψ

Ψ_n	coefficient in the representation of the incident field as a sum of cylindrical modes