# Nomenclature

## Decorations

| | |
|---|---|
| $\hat{\square}$ | Fourier coefficient (eq. 2.25) |
| $\tilde{\square}$ | Chebyshev or Chebyshev-Fourier coefficient (eq. 2.33) |
| $\overline{\square}$ | Quantity averaged in azimuthal and axial directions and over time |
| $\underline{\square}$ | Quantity averaged in axial direction and over temporal and azimuthal intervals |
| $\langle\square\rangle$ | Ensemble average |
| $[\square]$ | Argument of a function |
| $((u, v))$ | Inner product of functions $u$ and $v$ (eq. 2.122) |
| $\lvert\square\rvert$ | Absolute value of a real or complex quantity |
| $\lVert\square\rVert$ | Magnitude of a vector |
| $\square^{+}$ | Indicates normalisation in the wall variables $u_\tau$ and $\nu$ |
| $\square'$ | Root-mean-square value of a quantity |
| $\square_1, \square_2$ | Scalar or vector field on regions $R_1, R_2$ |
| $\square_{m,k}$ | Scalar or vector field corresponding to azimuthal Fourier mode $m$ and axial Fourier mode $k$ |
| $\square_r, \square_\theta, \square_z$ | Radial, azimuthal and axial components of a vector |
| $\boldsymbol{V}$ | Vector $V$ |

## Greek Symbols

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| $\alpha_j, \beta_j, \gamma_0$ | Coefficients for stiffly stable time integration scheme (eq. 2.54) |
| $\beta$ | Yaw angle between cylinder and free-stream (fig. 2.1) |
| $\delta$ | Boundary layer thickness based on $0.99V_\infty$ |

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| $\delta^*$ | Boundary layer displacement thickness (eq. 4.8, 7.1) |
| $\Delta x$ | Indicates change or increment of a quantity $x$ |
| $\epsilon$ | Rate of energy dissipation by viscosity per unit volume of fluid (eq. 4.27, viscous dissipation rate) |
| $\zeta$ | Trigonometric argument (eq. 2.35) |
| $\eta$ | Shape parameter for radial coordinate mapping (eq. 2.34) |
| $\theta$ | Azimuthal (spanwise) coordinate (fig. 2.1) |
| $\theta^*$ | Boundary layer momentum thickness (eq. 4.9, 7.2) |
| $\hat{\nabla}^2$ | Decoupled viscous operator (eq. 2.52) |
| $\lambda_p$ | Pressure-source (eq. 2.111, 5.6) |
| $\mu$ | Dynamic viscosity |
| $\nu$ | Kinematic viscosity $\mu/\rho$ |
| $\xi$ | Chebyshev abscissa (eq. 2.34) |
| $\mathbf{\Pi}$ | Cross-product of $\boldsymbol{U}$ and $\mathbf{\Omega}$ (eq. 2.57) |
| $\rho$ | Density |
| $\tau$ | Total shear-stress (eq. 3.9) |
| $\tau_w$ | Mean wall-shear-stress (eq. 4.1) |
| $\varphi_v$ | Temporal spectrum of variable $v$ |
| $\Phi$ | Velocity potential |
| $\omega$ | Angular frequency |
| $\mathbf{\Omega}$ | Vorticity field |
| $\overline{\mathbf{\Omega}}, \boldsymbol{\omega}$ | Mean and fluctuating components of vorticity field |
| $\mathbf{\Omega}_h$ | Solution of time integrated vorticity transport equation with homogeneous boundary condition (§2.4.4) |
| $\mathbf{\Omega}_i$ | Solution of time integrated vorticity transport equation with homogeneous initial condition and inhomogeneous boundary condition (§2.4.4) |
| $\Omega_p$ | Linear combination of azimuthal and axial vorticity components (eq. 2.49) |

## Roman Symbols

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|---|---|
| $a$ | Cylinder radius (fig. 2.1) |

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| $b$ | Outer radius of simulation domain (fig. 2.1) |
| $C_f$ | Skin friction coefficient (eq. 4.5) |
| $E_v[k_x]$ | Spatial spectrum of variable $v$ as a function of wave-number $k_x$ |
| $\hat{G}$ | Green's function for the Poisson equation for pressure (§5.2) |
| $h$ | Plane-channel half-width or radial width $(b-a)$ of cylindrical domain |
| $\boldsymbol{H}$ | Coupled term of vorticity transport equation (eq. 2.53, 2.64) |
| $H_p$ | Linear combination of azimuthal and axial components of $\boldsymbol{H}$ (eq. 2.53) |
| $i$ | Unit imaginary number $\sqrt{-1}$ |
| $I_n$ | Modified Bessel function of the first kind with order $n$ (fig. 2.2) |
| $k$ | Axial Fourier mode-number |
| $k_\theta$ | Azimuthal wave-number (eq. 2.26) |
| $k_z$ | Axial wave-number (eq. 2.27) |
| $K_n$ | Modified Bessel function of the second kind with order $n$ (fig. 2.2) |
| $L_K$ | Kolmogorov length-scale (eq. 3.1) |
| $L_p$ | Length scale for curvature correction of temporal wall-pressure spectrum in mid-frequency range (eq. 5.14) |
| $L_t$ | Duration of temporal intervals used for calculation of temporal spectra and correlation functions (tab. 3.4) |
| $L_T$ | Duration of temporal record (tab. 3.3, 3.4) |
| $L_z$ | Axial period of computational domain (fig. 2.1) |
| $m$ | Azimuthal Fourier mode-number |
| $n$ | Chebyshev polynomial degree |
| $N_i$ | Number of temporal intervals used for averaging of temporal spectra and correlation functions (tab. 3.4) |
| $N_r, N_\theta, N_z$ | Number of spatial grid points in radial, azimuthal and axial directions |
| $N_t$ | Number of samples taken from the temporal interval $L_t$ (tab. 3.4) |
| $N_T$ | Number of samples taken from the temporal record of duration $L_T$ (tab. 3.3, 3.4) |
| $N_\Omega$ | Number of Chebyshev coefficients for radial distribution of vorticity |
| $O$ | Order of magnitude |
| $p_w$ | Wall-pressure fluctuations |
| $P$ | Static pressure field |

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| $\overline{P}, p$ | Mean and fluctuating components of static pressure field |
| $P_T$ | Total (stagnation) pressure field (eq. 2.15) |
| $\overline{q}$ | Turbulence kinetic energy per unit volume (eq. 4.26) |
| $Q$ | Dynamic pressure (eq. 2.117) |
| $r$ | Radial coordinate (fig. 2.1) |
| $R_1$ | Computational domain $a \leq r \leq b$ |
| $R_2$ | External potential flow region $r \geq b$ |
| $R_v[\Delta x]$ | Two-point correlation function of variable $v$ as a function of separation along the spatial or temporal axis $x$ |
| $Re_a$ | Reynolds number based on $V_\infty$ and $a$ |
| $Re_\delta$ | Reynolds number based on $V_\infty$ and $\delta$ (eq. 4.4) |
| $Re_\tau$ | Reynolds number based on $u_\tau$ and $\delta$; equivalent to $\delta^+$ (eq. 4.7) |
| $t$ | Time |
| $t_s$ | Times corresponding to data samples from a simulation (tab. 3.4) |
| $u_\tau$ | Friction velocity (eq. 4.2) |
| $u_{\tau c}$ | Friction velocity adjusted for boundary layer curvature (eq. 4.22) |
| $\boldsymbol{U}$ | Velocity field |
| $\overline{\boldsymbol{U}}, \boldsymbol{u}$ | Mean and fluctuating components of velocity field |
| $\boldsymbol{U}_h$ | Velocity corresponding to $\boldsymbol{\Omega}_h$ (§2.4.4) |
| $\boldsymbol{U}_i$ | Velocity corresponding to $\boldsymbol{\Omega}_i$ (§2.4.4) |
| $U_p$ | Velocity scale for curvature correction of temporal wall-pressure spectrum in high-frequency range (eq. 5.16) |
| $V_\infty$ | Magnitude of free-stream velocity vector |
| $\boldsymbol{V}_\infty$ | Free-stream velocity vector (fig. 2.1) |
| $\boldsymbol{W}$ | Velocity vector field derived from vorticity using arbitrary boundary conditions (§2.5.2) |
| $X$ | Cartesian abscissa (fig. 2.1) |
| $\boldsymbol{X}$ | Non-linear term of vorticity transport equation (eq. 2.47) |
| $y$ | Wall-normal distance |
| $Y$ | Cartesian ordinate (fig. 2.1) |
| $z$ | Axial coordinate (fig. 2.1) |

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