Meson properties from Lattice QCD

John N. Hedditch

Supervisors: D. B. Leinweber and A. G. Williams

Centre for the Subatomic Structure of Matter
University of Adelaide
Adelaide
2006

This thesis is dedicated to Kati - you are proof the universe is a wonderful place.

Abstract

Quantum Chromo-Dynamics (QCD) is the part of the Standard Model which describes the interaction of the strong nuclear force with matter. QCD is asymptotically free, so at high energies perturbation expansions in the coupling can be used to calculate expectation values. Away from this limit, however, perturbation expansions in the coupling do not converge.

Lattice QCD (LQCD) is a non-perturbative approach to calculations in QCD. LQCD first performs a Wick rotation $t \rightarrow -it_E$, and then discretises spacetime into a regular lattice with some lattice spacing a. QCD is then expressed in terms of parallel transport operators of the gauge field between grid points, and fermion fields which are defined at the grid points. Operators are evaluated in terms of these quantities, and the lattice spacing is then taken to zero to recover continuum values.

We perform computer simulations of Lattice QCD in order to extract a variety of meson observables. In particular, we perform a comprehensive survey of the light and strange meson octets, obtain for the first time exotic meson results consistent with experiment, calculate the charge form-factor of the light and strange pseudoscalar mesons, and determine (for the first time in Lattice QCD) all three form-factors of the vector meson.

Statement of Originality

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying, and further consent to it’s reproduction as a musical or theatrical work.

John N. Hedditch

Acknowledgements

- Thank you first of all to Derek Leinweber and Tony Williams, my supervisors, for their tremendous patience, wisdom, and calm, all of which are attributes I would dearly love to posess myself.

- Big Ups to Ben Lasscock for his organisational genius and committment to being a team-player(TM).

- My Hat Off and great thanks to Sara Boffa and Sharon Johnson who have saved me from ruin many a time.

- Three Cheers to James Zanotti, Ross Young, Waseem Kamleh, Alex Kalloniatis, Tony Thomas, Marco Ghiotti and Marco Bartolozzi, Mariusz Hoppe, and all those others who have made working at the CSSM one of the most enjoyable times of my life.

- Thank you to Ramona Adorjan and Grant Ward for computing support, and sense of humour under pressure.

- The Australian Partnership for Advanced Computing, the South Australian Partnership for Advanced Computing, and the National Facility for Lattice Gauge Theory provided the computational muscle for this project, which would have been quite impossible otherwise.
- Finally, a big thankyou to all of my family, and Kati's family, for their support in this endeavour. It's been a wild ride.

Contents

Abstract v

Statement of Originality vii

Acknowledgements viii

1 Introduction 1
- 1.1 Quantum Chromodynamics 1

2 Lattice QCD 4
- 2.1 Introduction 4
- 2.2 Discrete symmetries 5
  - 2.2.1 Symmetries of Correlation functions 5
  - 2.2.2 Generalisation 7
  - 2.2.3 Proofs 8

3 Mesons from LQCD 9
- 3.1 Introduction 9
- 3.2 Meson correlation functions at the hadronic level 9
  - 3.2.1 Lorentz Scalar fields 9
  - 3.2.2 Lorentz Vector fields 10
- 3.3 Analysis 11
- 3.4 Meson correlation functions at quark level 13
  - 3.4.1 Mesonic operators from the naive quark model 13
- 3.5 Hybrid Mesons 14
  - 3.5.1 Introduction 14
  - 3.5.2 Method 15
  - 3.5.3 Results 17
  - 3.5.4 Summary 25
- 3.6 Exotic Mesons 30
  - 3.6.1 Introduction 30
  - 3.6.2 Physical Predictions 32
  - 3.6.3 Summary 36

4 Source dependence of Hybrid and Exotic signal 41
4.1 Introduction 41
4.2 Method 43
4.3 Results 43
  4.3.1 Hybrid Pion 43
  4.3.2 Exotic 53
4.4 Discussion and Summary 58

5 Meson form factors 59
5.1 Introduction 59
5.2 Three-point function with current insertion 59
  5.2.1 π-meson case 61
  5.2.2 Spin-1 case 61
  5.2.3 Extracting static quantities 64
5.3 Method 66
5.4 Results 67
5.5 Conclusions 88

6 Conclusions 89

A Data pertaining to the calculation of meson effective masses 94

B Obtaining the form of $\langle r^2 \rangle$ 104

C Source dependence results for the SU(3) $\beta = 4.60$, $20^3 \times 40$ lattice 105

D Quark-level calculations 110
D.1 Two-point function 110
D.2 Electromagnetic current insertion 111

E REDUCE script for calculating ratios of three to two-point functions 112

F Data pertaining to the calculation of meson form-factors 114

G Papers by the author 121

List of Figures

1.1 Quark-flow diagrams for three-point and two-point meson vertices. 1

1.2 The scalar meson octet and singlet η'. Image courtesy of WikiImages . . . 2

3.1 Author's sketch of a quark-model meson vs a hybrid meson 14

3.2 Effective mass for standard pseudovector interpolating field, for equal (left) and unequal (right) quark-masses. Results are shown for all eight masses. 17

3.3 Effective mass for axial-vector pion interpolating field, for equal (left) and unequal (right) quark-masses. Results are shown for all eight masses. . . 17

3.4 Effective mass for the hybrid pion interpolating field $i\bar{q}^a\gamma_j B_j^{ab} q^b$, for equal (left) and unequal (right) quark-masses. Results are shown for all eight masses. 18

3.5 Effective mass for the hybrid pion interpolating field $i\bar{q}^a\gamma_j\gamma_4 B_j^{ab} q^b$, for equal (left) and unequal (right) quark-masses. Results are shown for all eight masses. 18

3.6 Ground (triangles) and excited state (circles) masses for the pion, extracted using a 3×3 variational process using the first three pion interpolating fields. Signal is only obtained for the heaviest 3 quark masses. . . 19

3.7 The a_0 scalar meson correlation function vs. time. 20

3.8 ρ-meson effective mass derived from standard $\bar{q}\gamma_j q$ interpolator. Results are shown for both equal (left) and unequal (right) quark-antiquark masses. Results for every second quark mass are depicted. 22

3.9 ρ-meson (left) and K^* (right) effective mass plots derived from interpolator $\bar{q}\gamma_j\gamma_4 q$. Every second quark mass is depicted. 22

3.10 Vector meson effective mass from hybrid interpolator $\bar{q}^a E_j^{ab} q^b$. Every second quark mass is depicted, and results are depicted for both ρ (left) and K^* (right) mesons. 22

3.11 Vector meson effective mass from hybrid interpolator $i\bar{q}^a\gamma_5 B_j^{ab} q^b$. Every second quark mass is depicted, and results are depicted for both ρ (left) and K^* (right) mesons. 23

3.12 Vector meson effective mass from hybrid interpolator $i\bar{q}^a\gamma_4\gamma_5 B_j^{ab} q^b$. Every second quark mass is depicted, and results are depicted for both ρ (left) and K^* (right) mesons. 23

3.13 Effective mass plots for a_1 axial-vector meson interpolator. Results are shown for light (left) and strange-light (right) quark-masses. 24
3.14 b_1 axial-vector meson effective mass. Results are shown for both light (left) and strange-light (right) quark masses. 24
3.15 Summary of results for pion interpolating fields. m_π^2, derived from the standard pion interpolator, provides a measure of the input quark mass. . 25
3.16 Summary of results for K interpolating fields. m_π^2, derived from the standard pion interpolator, provides a measure of the input quark mass. . . . 26
3.17 Summary of results for ρ-meson interpolating fields. m_π^2, derived from the standard pion interpolator, provides a measure of the input quark mass. 27
3.18 Summary of results for K^*-meson interpolating fields. m_π^2, derived from the standard pion interpolator, provides a measure of the input quark mass. 28
3.19 Summary of results for pseudovector-meson interpolating fields. m_π^2, derived from the standard pion interpolator, provides a measure of the input quark mass. 29
3.20 Exotic meson propagator for interpolator χ_2. Results are shown for every 2nd quark mass in the simulation. Lower lines correspond to heavier quark masses. For all but the heaviest mass, the signal is lost after t=12. Pion masses corresponding to each quark mass may be found at the beginning of appendix A 31
3.21 Exotic meson propagator for interpolator χ_3. Results are shown for every 2nd quark mass in the simulation. Lower lines correspond to heavier quark masses. 32
3.22 Effective mass for interpolator χ_2. Plot symbols are as for the corresponding propagator plot. 34
3.23 As for Fig. 3.22, but for interpolator χ_3. Signal is lost after $t = 11$. 35
3.24 Effective mass for the interpolator χ_2 with a strange quark. 36
3.25 As for Fig. 3.24, but for interpolator χ_3. 37
3.26 A survey of results in this field. The MILC results are taken from [10] and show their Q^4, $1^{-+} \to 1^{-+}$ results, fitted from $t = 3$ to $t = 11$. Open and closed symbols denote dynamical and quenched simulations respectively. 38
3.27 The 1^{-+} exotic meson mass obtained from fits of the effective mass of the hybrid interpolator χ_2 from $t = 10 \to 12$ (full triangles) are compared with the $a_1\eta'$ two-particle state (open triangles). The extrapolation curves include a quadratic fit to all eight quark masses (dashed line) and a linear fit through the four lightest quark masses (solid line). The full square is result of linear extrapolation to the physical pion mass, while the open square (offset for clarity) indicates the $\pi_1(1600)$ experimental candidate. . 39
3.28 Extrapolation of the associated strangeness ± 1 $J^P = 1^-$ state obtained from χ_2. Symbols are as in Fig. 3.27. 40

4.1 Fermion-source smearing-dependence of conventional pion signal 43
4.2 Gauge-field smearing-dependence of χ_4 hybrid pion signal. Here $n_{\rm src} = 0$, i.e a point source is used for the quark fields. 44

4.3 Hybrid π- meson (χ_3) effective masses from the $16^3 \times 32$ lattice with $n_{\rm src} = 0$. Results for the heaviest four quark masses are depicted. 45
4.4 Hybrid π- meson (χ_3) effective masses from the $16^3 \times 32$ lattice with $n_{\rm src} = 16$. Results for the heaviest four quark masses are depicted. 46
4.5 Hybrid π- meson (χ_3) effective masses from the $16^3 \times 32$ lattice with $n_{\rm src} = 48$. Results for the heaviest four quark masses are depicted. 47
4.6 Hybrid π- meson (χ_3) effective masses from the $16^3 \times 32$ lattice with $n_{\rm src} = 144$. Results for the heaviest four quark masses are depicted. . . . 48
4.7 Hybrid π- meson (χ_4) effective masses from the $16^3 \times 32$ lattice with $n_{\rm src} = 0$. Results for the heaviest four quark masses are depicted. 49
4.8 Hybrid π- meson (χ_4) effective masses from the $16^3 \times 32$ lattice with $n_{\rm src} = 16$. Results for the heaviest four quark masses are depicted. 50
4.9 Hybrid π- meson (χ_4) effective masses from the $16^3 \times 32$ lattice with $n_{\rm src} = 48$. Results for the heaviest four quark masses are depicted. 51
4.10 Hybrid π- meson (χ_4) effective masses from the $16^3 \times 32$ lattice with $n_{\rm src} = 144$. Results for the heaviest four quark masses are depicted. . . . 52
4.11 Exotic meson effective masses from the $16^3 \times 32$ lattice with $n_{\rm src} = 0$. Results for the heaviest four quark masses are depicted. 54
4.12 Exotic meson effective masses from the $16^3 \times 32$ lattice with $n_{\rm src} = 16$. Results for the heaviest four quark masses are depicted. 55
4.13 Exotic meson effective masses from the $16^3 \times 32$ lattice with $n_{\rm src} = 48$. Results for the heaviest four quark masses are depicted. 56
4.14 Exotic meson effective masses from the $16^3 \times 32$ lattice with $n_{\rm src} = 144$. Results for the heaviest four quark masses are depicted. 57

5.1 Quark-flow diagrams relevant to K^+ meson electromagnetic form-factors. 60
5.2 The up-quark contribution to pion charge form factor. The data correspond to $m_\pi \simeq 830\,\rm MeV$ (top left), $770\,\rm MeV$ (top right), $700\,\rm MeV$ (second row left), $616\,\rm MeV$ (second row right), $530\,\rm MeV$ (third row left), $460\,\rm MeV$ (third row right), $367\,\rm MeV$ (bottom row left), and $290\,\rm MeV$ (bottom row right). For the five lightest quark masses, the splitting between the values for i_κ and $i_\kappa + 1$ is shown. The data are illustrated only to the point at which the error bars diverge. 68
5.3 As in Fig. 5.2 but for the up-quark contribution to kaon charge form factor. 69
5.4 As in Fig. 5.2 but for the strange-quark contribution to kaon charge form factor. 70
5.5 As in Fig. 5.2 but for the up-quark contribution to ρ charge form factor. . 71
5.6 As in Fig. 5.2 but for the up-quark contribution to K^* charge form factor. 72
5.7 As in Fig. 5.2 but for the strange-quark contribution to K^* charge form factor. 73
5.8 As in Fig. 5.2 but for the up-quark contribution to ρ magnetic form factor. We note that for the fifth and sixth quark mass, good $\chi^2/\rm d.o.f$ is achieved for fits including points to $t = 25$, and central value of the fit is not afected significantly. We prefer to focus on regions of good signal. 74

5.9 As in Fig. 5.2 but for the up-quark contribution to K^* magnetic form factor. As for the up contributions, we can achieve a good χ^2 even fitting out to $t = 25$ for the fifth, sixth and seventh quark masses without significantly affecting the central values, but prefer to focus on regions of strong signal. 75
5.10 As in Fig. 5.2 but for the strange-quark contribution to K^* magnetic form factor. 76
5.11 As in Fig. 5.2 but for the up-quark contribution to ρ Quadrupole form factor. 77
5.12 As in Fig. 5.2 but for the up-quark contribution to K^* Quadrupole form factor. 78
5.13 As in Fig. 5.2 but for the strange-quark contribution to K^* Quadrupole form factor. 79
5.14 Mean squared charge radius for each quark sector for pseudoscalar (left) and vector (right) cases. u_π and u_ρ symbols are centred on the relevant value of m_π^2, other symbols are offset for clarity. 80
5.15 Strange and non-strange meson mean squared charge radii for charged pseudoscalar (left) and vector (right) cases. Symbols are offset as in fig. 5.14 80
5.16 Strange meson mean squared charge radii for neutral pseudoscalar (left) and vector (right) cases. 81
5.17 Ratio of mean squared charge radius for a light quark in the environment of light and heavy quarks. Pseudoscalar (left) and vector (right) results are shown for comparison. 81
5.18 Mean squared charge radii for positively charged baryons. 82
5.19 Per quark-sector (left) and corresponding charged vector meson (right) magnetic moments. 83
5.20 Charged vector meson magnetic moments. 83
5.21 g factor for ρ meson. 84
5.22 Neutral K^*-meson magnetic moment. 84
5.23 Environment-dependence for light-quark contribution to vector meson magnetic moment. 85
5.24 Per quark-sector quadrupole form-factors. 85
5.25 Vector meson quadrupole form factors for ρ^+ and K^{*+}. 86
5.26 Environment-dependence for light-quark contribution to vector meson quadrupole form-factor. 87
5.27 Quadrupole form-factor for neutral K^* meson. 87

List of Tables

3.1 J^{PC} quantum numbers and their associated meson interpolating fields. . 15
3.2 κ values, and corresponding pion masses (and uncertainties) in GeV. . . . 16
3.3 Pion ground-state mass fits from a 3×3 correlation matrix analysis. $t_{\rm start}$ and $t_{\rm end}$ denote the limits of the fit-window. Ma is the mass, in lattice units. σ is the uncertainty. $\chi^2/{\rm d.o.f}$ is the χ^2 per degree of freedom of the fit. i_κ labels the κ value as per Table 3.2. 21
3.4 Pion excited-state mass fit. Column labels are as for Table 3.3. 21
3.5 a_0 scalar meson mass vs decay channel mass. 21
3.6 1^{-+} Exotic Meson mass m (GeV) vs square of pion mass m_π^2 (GeV^2). . . 33
3.7 Strangeness ± 1 1^- Meson mass m (GeV) vs square of pion mass m_π^2(GeV^2). 33

4.1 Effect of gauge-field smearing on χ_4 hybrid pion mass determination, $t = [8, 13]$ 44
4.2 Effect of gauge-field smearing on 1^{-+} Exotic meson mass determination, $t = [5, 7]$ 53
4.3 Effect of gauge-field smearing on 1^{-+} Exotic meson mass determination, $t = [6, 8]$ 53

A.1 κ values, and corresponding pion masses (and uncertainties) in GeV. . . . 94
A.2 a_0 scalar meson mass fits. Column headings are in order, the $kappa$ number, the lower and upper bounds of the fit window, the mass, error and χ^2 from our analysis. 95
A.3 As in Table A.2, but for the $K_0^\star$. 95
A.4 As in Table A.2 but for conventional π meson operator $\bar{q}\gamma_5 q$. 95
A.5 As in Table A.2 but for conventional K meson operator $\bar{s}\gamma_5 q$. 96
A.6 As in Table A.2 but for axial-vector pion interpolator $\bar{q}\gamma_5\gamma_4 q$. 96
A.7 As in Table A.2 but for axial-vector K interpolator $\bar{s}\gamma_5\gamma_4 q$ 96
A.8 As in Table A.2 but for hybrid pion interpolating field $i\bar{q}^a\gamma_j B_j^{ab} q^b$. 97
A.9 As in Table A.2 but for hybrid K interpolating field $i\bar{s}^a\gamma_j B_j^{ab} q^b$. 97
A.10 As in Table A.2 but for hybrid pion interpolating field $i\bar{q}^a\gamma_j\gamma_4 B_j^{ab} q^b$. . . . 98
A.11 As in Table A.2 but for hybrid K interpolating field $i\bar{s}^a\gamma_j\gamma_4 B_j^{ab} q^b$. 98
A.12 As in Table A.2 but for conventional ρ-meson interpolating field $\bar{q}\gamma_j q$ for equal (left) and unequal (right) input quark masses. 98
A.13 As in Table A.2 but for conventional K^*-meson interpolating field $\bar{s}\gamma_j q$. . 99
A.14 As in Table A.2 but for conventional ρ-meson interpolating field $\bar{q}\gamma_j\gamma_4 q$. . 99

A.15 As in Table A.2 but for conventional K^*-meson interpolating field $\bar{q}\gamma_j\gamma_4 q$. 99
A.16 As in Table A.2 but for Hybrid ρ-meson interpolator $\bar{q}E_j q$. Error bars are larger than signal for lightest quark mass, so this line is omitted 100
A.17 As in Table A.2 but for Hybrid K^*-meson interpolator $\bar{q}E_j q$. Error bars are larger than signal for 3 lightest quark masses. 100
A.18 As in Table A.2 but for Hybrid ρ-meson interpolator $i\bar{q}^a\gamma_5 B_j^{ab} q^b$. 100
A.19 As in Table A.2 but for Hybrid K^*-meson interpolator $i\bar{q}^a\gamma_5 B_j^{ab} q^b$. 101
A.20 As in Table A.2 but for Hybrid ρ-meson interpolator $i\bar{q}^a\gamma_4\gamma_5 B_j^{ab} q^b$. 101
A.21 As in Table A.2 but for Hybrid K-meson interpolator $i\bar{q}^a\gamma_4\gamma_5 B_j^{ab} q^b$. . . . 101
A.22 As in Table A.2 but for pseudovector interpolating field $\bar{q}\gamma_5\gamma_4\gamma_j q$ with equal quark-antiquark masses. 102
A.23 As in Table A.2 but for pseudovector interpolating field $\bar{q}\gamma_5\gamma_4\gamma_j q$ with unequal quark-antiquark masses. 102
A.24 As in Table A.2 but for axial-vector interpolating field $\bar{q}\gamma_5\gamma_i q$ for equal quark-antiquark masses. No appropriate fit window exists for the two lightest quark-masses. 103
A.25 As in Table A.2 but for axial-vector interpolating field $\bar{q}\gamma_5\gamma_i q$ for unequal quark-antiquark masses. No appropriate fit window exists for the two lightest quark-masses. 103

C.1 Exotic meson Effective masses from the $20^3\times 40$ lattice for χ_2 with $n_{\rm src} = 35$. Results for the heaviest four quark masses are depicted. 106
C.2 Exotic meson Effective masses from the $20^3\times 40$ lattice for χ_2 with $n_{\rm src} = 35$. Results for the heaviest four quark masses are depicted. 107
C.3 Exotic meson Effective masses from the $20^3\times 40$ lattice for χ_3 with $n_{\rm src} = 35$. Results for the heaviest four quark masses are depicted. 108
C.4 Exotic meson Effective masses from the $20^3\times 40$ lattice for χ_3 with $n_{\rm src} = 35$. Results for the heaviest four quark masses are depicted. 109

F.1 Rho meson mass data 114
F.2 Pion mass data 115
F.3 Strange quark contribution to K-meson form-factor. 115
F.4 Strange quark contribution to K^*-meson charge form-factor. 115
F.5 Up quark contribution to K-meson form-factor. 116
F.6 Up quark contribution to K^*-meson charge form-factor. 116
F.7 Up quark contribution to π-meson charge form-factor. 116
F.8 Up quark contribution to ρ-meson Charge form-factor. 117
F.9 Strange quark contribution to K^* magnetic form-factor. 117
F.10 Up quark contribution to K^* magnetic form-factor. 117
F.11 Up quark contribution to ρ magnetic form-factor. 118
F.12 Strange quark contribution to K^* quadrupole form-factor. 118
F.13 Up quark contribution to K^* quadrupole form-factor. 118
F.14 Up quark contribution to ρ quadrupole form-factor. 119
F.15 Q^2 values for pion (lattice units) 119

F.16 Q^2 values for K (lattice units) 119
F.17 Q^2 values for ρ (lattice units) 120
F.18 Q^2 values for K^* (lattice units) 120