

# **Electromagnetic properties of baryons from lattice QCD**

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*To my family*

*“Whatever you do will be insignificant,  
but it is very important that you do it.”*

M. K. Gandhi

# Abstract

Electromagnetic properties of the octet and decuplet baryons are calculated in quenched QCD on a  $20^3 \times 40$  lattice with a lattice spacing of 0.128 fm using the fat-link irrelevant clover (FLIC) fermion action.

FLIC fermions enable simulations to be performed efficiently at quark masses as low as 300 MeV. By combining FLIC fermions with an improved conserved vector current we ensure that discretization errors occur only at  $O(a^2)$  while maintaining current conservation.

Magnetic moments, charge radii and magnetic radii are extracted from the electric and magnetic form factors for each individual quark sector. From these the corresponding baryon properties are constructed.

Our results for the octet baryons are compared with the predictions of Quenched Chiral Perturbation Theory ( $Q\chi$ PT) and experimental values where available. Results for the charge radii and magnetic moments of the octet baryons are in accord with the predictions of the  $Q\chi$ PT, suggesting that the sum of higher order terms makes only a small contribution to the chiral expansion. The regime where chiral physics dominates remains to be explored. We establish the non-analytic behavior of the charge radii and magnetic moment in the case of octet baryons. The neutron charge radius suggests that the chiral regime is still far away. We establish substantial environment sensitivity in the quark behavior in the low mass region. We establish that the  $u$  and  $d$  quarks make substantial and important contribution to the magnetic moment of the  $\Lambda$  contradicting the predictions of the Simple Quark Model.

We present the  $E0$  and  $M1$  form factors of the decuplet baryons and the charge radii and magnetic moments. We compare the decuplet baryon results with the lattice calculation of charge radii and magnetic moments of octet baryons. We establish that the environment sensitivity is far less pronounced in the case of the decuplet baryons compared to that in the octet baryons. A surprising result is that the charge radii of the decuplet baryons are generally smaller than that of the octet baryons.

Magnetic moment of the  $\Delta^+$  shows a turn over in the low quark mass region, making it smaller than the proton magnetic moment. This is consistent with the expectations of the Quenched Chiral Perturbation Theory. A similar turn over is also noticed in the magnetic moment of the  $\Sigma^{*0}$ , but not for  $\Xi^*$  where only kaon loops can appear in Quenched QCD.

We present results for the higher order moments of the decuplet baryons, i.e., the electric quadrupole moment  $E2$  and the magnetic octupole moment  $M3$ . With these results we provide the first conclusive analysis which shows that decuplet baryons are deformed. The electric quadrupole moment of the  $\Omega^-$  baryon is positive when the negative charge factor is included, and is equal to  $0.014 \pm 0.005 \text{ fm}^2$ , indicating an oblate shape.

# **Statement of originality**

This work contains no material which has been accepted for the award of any other degree or diploma in any university or tertiary institution. To the best of my knowledge and belief this contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.



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