

# **Downlink Resource Allocation for Orthogonal Frequency Division Multiple Access Systems**

by

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# Abstract

Wireless spectral efficiency is increasingly important due to the rapid growth of demand for high data rate wideband wireless services. The design of a multi-carrier system, such as an orthogonal frequency division multiple access (OFDMA) system, enables high system capacity suited for these wideband wireless services. This system capacity can be further optimised with a resource allocation scheme by exploiting the characteristics of the wireless fading channels. The fundamental idea of a resource allocation scheme is to efficiently distribute the available wireless resources, such as the sub-carriers and transmission power, among all admitted users in the system. In this thesis, we present the findings of the investigation into the impact of several resource allocation schemes in an OFDMA environment.

We show that in an OFDMA environment without the consideration of sub-carrier assignment, the sub-optimal power allocation closed-form solution can be derived via a constrained optimisation with the duality theorem. With a perfect feedback of channel condition, the proposed low-complexity algorithm that utilises the closed-form solution can maximise the sum capacity to approach near-optimal capacity.

We derive the sub-optimal sub-carrier and power allocation closed-form solution via a similar constrained optimisation process. With an imperfect or outdated feedback of channel condition, the adaptive sub-carrier and power allocation scheme not only fails to improve but also further deteriorates the system throughput. We present and discuss the formation of the finite-state Markov channel. We show that by using the dynamics of the Markov channel, the channel quality can be reliably predicted in advance. We analyse via simulation the spectral efficiency achieved by this channel prediction scheme on an OFDMA system.

## Abstract

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We address the importance of fairness in resource allocation from a game-theoretic perspective. With different utility and preference functions that best describe the gain in users' throughput as more sub-carriers are allocated to the individual user, we formulate the resource allocation problem into cooperative and non-cooperative games. We study via simulation the effectiveness and fairness of the cooperative and non-cooperative resource allocation schemes on an OFDMA system.

Finally, we draw conclusions on our research work and outline the future research topics in connection with our current studies.

# Statement of Originality

This work contains no material that has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of the thesis, when deposited in the University Library, being available for loan and photocopying.

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# List of Abbreviations

ADC	Analog-to-Digital Converter.
ADSL	Asymmetric Digital Subscriber Line.
AWGN	Additive White Gaussian Noise.
BD	Birth-Death.
BER	Bit-error Rate.
CDMA	Code Division Multiple Access.
CPA	Constant Power Allocation.
CSI	Channel State Information.
DAB	Digital Audio Broadcasting.
DAC	Digital-to-Analog Converter.
DFT	Discrete Fourier Transform.
DVB	Digital Video Broadcasting.
DSL	Digital Subscriber Line.
FDMA	Frequency Division Multiple Access.
FPA	Fixed Power Allocation.
FSMC	Finite-state Markov Channel.
GSM	Global System for Mobile Communications.
IDFT	Inverse Discrete Fourier Transform.
ISI	Inter-symbol Interference.
KKT	Karush-Kuhn-Tucker.
MC-CDMA	Multi-carrier Code Division Multiple Access.
MC/DS-CDMA	Multi-carrier Direct Sequence Code Division Multiple Access.
MQAM	$M$ -ary Quadrature Amplitude Modulation.
MT-CDMA	Multi-tone Code Division Multiple Access.
NBD	Non-Birth-Death.

## List of Abbreviations

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NBS	Nash Bargaining Solution.
NE	Nash Equilibrium (Equilibria).
OFDM	Orthogonal Frequency Division Multiplexing.
OFDMA	Orthogonal Frequency Division Multiple Access.
OPA	Optimal Power Allocation.
P/S	Parallel-to-Serial.
PoA	Price of Anarchy.
QBD	Quasi-Birth-Death.
QoS	Quality of Service.
RBS	Raiffa-Kalai-Smorodinsky Bargaining Solution.
RRM	Radio resource management.
SINR	Signal-to-Interference-plus-Noise Ratio.
SNR	Signal-to-Noise Ratio.
S/P	Serial-to-Parallel.
SPA	Sub-optimal Power Allocation.
TDD	Time Division Duplex.
TDMA	Time Division Multiple Access.
UPA	User-prioritised Power Allocation.
WLAN	Wireless Local Area Network.

# List of Symbols

$\mathbf{A}$	State transition matrix.
$a_{i,j}$	Transition probability of state $i$ to $j$ , for all $i, j \in \mathbb{Z}^+$ .
$\alpha_l(t)$	Time-varying amplitude of the wireless channel response at the $l^{\text{th}}$ path.
$\beta_{k,n}$	Sub-carrier assignment factor for user $k$ at sub-carrier $n$ .
$b$	Number of sub-carriers within one sub-band.
$B$	Received signal bandwidth.
$C$	Shannon's capacity.
$\chi$	Decibel shadow fading component.
$\chi_\sigma$	Decibel standard deviation of shadow fading component.
$\Delta f$	Sub-carrier spacing.
$\Delta T$	Length of cyclic prefix (also known as guard interval).
$d$	Instantaneous distance between base station and the corresponding mobile user.
$d_0$	Relative distance between base station and any arbitrary mobile user.
$\eta$	Path loss exponent.
$f_d$	Maximum Doppler frequency.
$\mathcal{F}_k$	Fairness index for user $k$ .
$\gamma_{k,n}$	Signal-to-noise-ratio for user $k$ at sub-carrier $n$ .
$h(t)$	Impulse response of the wireless fading channel.
$h_{k,n}$	Channel fading coefficient for user $k$ at sub-carrier $n$ .
$itr$	Number of iterations in an iterative algorithm.
$K$	Number of users.
$\kappa$	Decibel zero-mean Gaussian variable of zero decibel standard deviation.
$\lambda$	Lagrange multiplier.
$L_P(\cdot)$	Lagrangian of primal objective.
$L_D(\cdot)$	Lagrangian of dual objective.

## List of Symbols

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$\mathbf{L}_q$	The $q^{\text{th}}$ partition of lumpable states.
$\mu$	Lagrange multiplier.
$N$	Number of sub-carriers.
$n_{k,n}$	Additive white Gaussian noise for user $k$ at sub-carrier $n$ .
$\nu$	Lagrange multiplier.
$O(\cdot)$	Computational complexity of an iterative algorithm.
$\boldsymbol{\pi}$	Steady-state probability vector.
$\pi_i$	Steady-state probability for state $i$ , for all $i \in \mathbb{Z}$ .
$P^{\max}$	Total power budget for one transmission cycle.
$P_{k,n}$	Instantaneous transmit power for user $k$ at sub-carrier $n$ .
$PL(\cdot)$	Decibel path loss.
$R$	Transmission rate.
$\sigma_{k,n}^2$	Noise variance for user $k$ at sub-carrier $n$ .
$s_i$	State $i$ , for all $i \in \mathbb{Z}$ .
$S_t$	Markov process at time $t$ .
$\tau_{\max}$	Maximum delay spread.
$\tau_l$	Time delay of the wireless channel response at the $l^{\text{th}}$ path.
$T$	OFDMA symbol duration.
$u_k$	Utility function for user $k$ .
$w_k$	Weighting factor for user $k$ .
$x_{k,n}$	Transmitted signals for user $k$ at sub-carrier $n$ .
$y_{k,n}$	Received signals for user $k$ at sub-carrier $n$ .

# List of Publications

1. **T. K. Chee**, C.-C. Lim, J. Choi, “Sub-optimal Power Allocation for Downlink OFDMA Systems”, in *Proceedings of the IEEE 60th Vehicular Technology Conference - Fall*, vol. 3, pp. 2105-2109, September 2004.
2. **T. K. Chee**, C.-C. Lim, J. Choi, “Adaptive Power Allocation with User Prioritization for Downlink OFDMA Systems”, in *Proceedings of the IEEE 9th International Conference on Communications System*, pp. 210-214, September 2004.
3. **T. K. Chee**, C.-C. Lim, J. Choi, “A Lumpable Finite-State Markov Model for Channel Prediction and Resource Allocation in OFDMA Systems”, in *Proceedings of the IEEE 1st International Conference on Wireless Broadband and Ultra Wideband Communications*, March 2006.
4. **T. K. Chee**, C.-C. Lim, J. Choi, “Channel Prediction using Lumpable Finite-State Markov Channels in OFDMA Systems”, in *Proceedings of the IEEE 63rd Vehicular Technology Conference - Spring*, May 2006.
5. **T. K. Chee**, C.-C. Lim, J. Choi, “A Cooperative Game Theoretic Framework for Resource Allocation in OFDMA Systems”, in *Proceedings of the IEEE 10th International Conference on Communications System*, October 2006.
6. **T. K. Chee**, C.-C. Lim, J. Choi, “Sub-channel and Power Allocation with Channel Prediction Using Lumpable Finite-State Markov Model”, *submitted for journal publication*.
7. **T. K. Chee**, C.-C. Lim, J. Choi, “A Cross-Layer Resource Allocation for OFDMA Systems Using Cooperative and Non-Cooperative Game Theory”, *submitted for journal publication*.

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