

Static-light matrix elements on a dynamical anisotropic lattice

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The static-light matrix element needed to determine f_B is studied on an anisotropic lattice with $N_f = 2$. The improvement in precision due to stout links and all-to-all propagators is investigated.

1. INTRODUCTION

The determination of heavy-light decay constants and form factors relevant to CKM phenomenology is a priority for lattice QCD. However, to make a meaningful contribution the statistical and systematic uncertainties in these calculations must be reduced to a few percent for simulations carried out in the full theory. This is extremely costly, and new methods to improve precision while keeping computational costs modest are important. We report preliminary results from our investigations of heavy-light physics on anisotropic lattices using stout links and all-to-all propagators.

2. SIMULATION DETAILS

The gauge action used in the simulations described here is Symanzik-improved with a negative adjoint term. It has been used previously in a study of glueballs [1] and in an exploratory study of quenched heavy-quark physics on anisotropic lattices [2]. The fermion action is designed specifically for large anisotropies when the temporal lattice spacing a_t is much finer than the spatical lattice spacing, a_s . It incorporates the usual Wilson term in the temporal direction and a Hamber–Wu term in the spatial directions to remove doublers. The action has been discussed in detail in Refs. [2,3] and is written

$$S' = \bar{\psi}' M_r \psi' - \frac{r a_t}{2} \bar{\psi}' \left(D_t^2 - \frac{g}{2} \epsilon_i E_i \right) \psi' + s a_s^3 \bar{\psi}' \sum_i D_i^4 \psi' \quad (1)$$

where the prime indicates that fields have been rotated, $M_r = \mu_r \gamma_i D_i + \gamma_0 D_t + \mu_r m_0$, $\mu_r = (1 + 1/2 a_t m_0)$ and the spatial derivative is improved with a two-hop term. The usual Wilson parameter is $r = 1$ and the Wilson-like parameter $s = 1/8$. To reduce the coupling to UV gluon modes we use stout links [5]. The preservation of differentiability with respect to the link variables of this smearing technique means that the hybrid Monte Carlo algorithm can be straightforwardly applied. Quantum fluctuations renormalise the anisotropy $\xi = a_s/a_t$, and the parameters ξ_g and ξ_q in the gauge and quark actions respectively must be tuned such that the measured anisotropy takes its required value. In the quenched theory the gluon and quark actions can be separately tuned. However, in unquenched simulations this is no longer true and the quarks and gluons must have a common anisotropy to recover the correct continuum limit of the theory. The non-perturbative tuning of this action is under investigation. We require that the anisotropy measured from the sideways potential agrees with that determined from the low-momentum relativistic pion dispersion relation. This simultaneous tuning requires three initial simulations to find the self-consistent point [4]. The results presented here are based on one (untuned) simulation. The results do not therefore have a physical interpretation; the point of interest is rather the precision which can be reached using anisotropic lattices and all-to-all propagators.

3. RESULTS

In this exploratory simulation we use the hybrid Monte Carlo algorithm with $N_f = 2$ and de-

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Table 1

Simulation parameters for one of three runs needed to tune the anisotropy. u_s, u_t are the tadpole-improvement coefficients in the spatial and temporal directions.

| Gauge Action | | Quark Action | |
|--------------|----------|--------------|--------|
| β | 1.51 | $a_t m_0$ | -0.057 |
| ξ_g | 8.0 | ξ_q | 6.0 |
| u_s | 0.752121 | u_s, u_t | 1.0 |

generate sea and valence quark masses. We have generated 250 gauge configurations on a $8^3 \times 48$ lattice, with $m_\pi/m_\rho \sim 0.65$. The parameter values are given in Table 1.

The combination of anisotropic lattices and stout links allows us to reach momenta, $p > 1\text{GeV}$. Figure 1 shows the light pseudoscalar meson dispersion relation obtained from point propagators and Jacobi smeared quark fields. The quark mass is close to the strange quark and the point $n^2 = 6$ corresponds to momentum (1, 1, 2) in units of $2\pi/a_s L$ or $p \sim 1.5\text{GeV}$. This result is encouraging since it implies that high-momentum light pseudoscalars can be reliably simulated on these dynamical lattices. This is relevant for studies of semileptonic decays such as $B \rightarrow \pi \ell \nu$ which have in the past been restricted to the low-momentum regime. We have not explicitly disentangled the benefits of the anisotropic lattice from the stout links but experience in the quenched case without stout links, indicates that spatial momenta higher than (2, 0, 0) are difficult to resolve, implying that stout links play an important rôle in these high-momentum simulations.

3.1. All-to-all propagators

A new algorithm to estimate all-to-all propagators was presented at this conference [6]. With all-to-all propagators much more information can be extracted from a gauge configuration than with point sources, which is of particular relevance for expensive dynamical simulations. The algorithm discussed in Ref. [6] uses low-mode dominance corrected with a stochastic estimator which yields the exact all-to-all propagator in a finite number of quark inversions. Once again we use stout link smearing on the gauge fields and

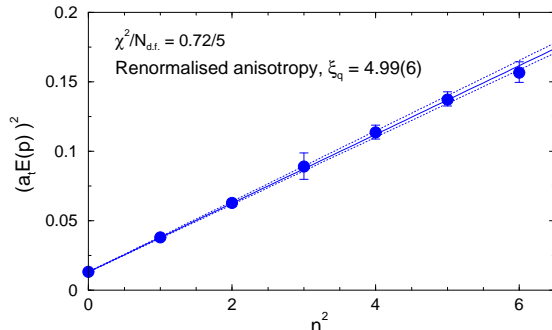


Figure 1. Dispersion relation for the light quark in the simulation; $a_t m_q = -0.057$, close to the strange quark mass.

Jacobi smearing on the (light) quarks.

3.2. Matrix elements

We now study the effect of these all-to-all propagators on the static-light correlator. Although relatively cheap to calculate, this is known to be noisy and difficult to determine with precision. In addition, using all-to-all propagators in the static approximation means that f_B can be extracted from a combination of smeared-local (SL) and smeared-smeared (SS) correlators rather than the restriction to local-local (LL) and SL correlators when point sources are used. The advantages are clearly illustrated in Figure 2. The plot shows the effective mass of a heavy-light meson determined using both point and all-to-all propagators. The heavy quark was simulated in the static approximation and the light quark is close to the strange quark mass. It is important to note that the all-to-all dataset consists of 130 configurations whereas the equivalent statistical precision using point sources requires 250 configurations.

Using point sources the effective mass and amplitude (which yields f_B) is determined from a combination of the LL and SL data in a region where they overlap. Figure 2 shows that this is true only for a few timeslices but also that the purely local data in that region are becoming very noisy, decreasing the reliability and precision with which f_B can be extracted. In contrast, the all-to-all datasets agree over a much larger number of timeslices reducing the uncertainties in the fitted

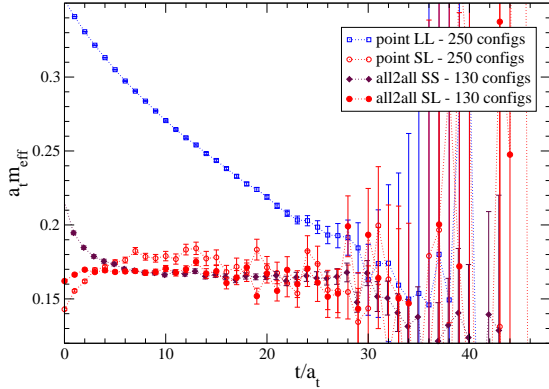


Figure 2. Effective mass of the static-light meson. The light quark is close to the strange quark mass. Open symbols are determined with point propagators and the solid points with all-to-all propagators. The all-to-all data at large t/a_t have been hidden for the sake of clarity.

mass and amplitude. Figure 3 shows a comparison of the matrix-element amplitude determined from point and all-to-all propagators. These are essentially double sliding window plots. The vertical spread represents the systematic error in the extracted value of Z_L^2 as the fit range is varied for the point LL and SL correlators and for the all-to-all SL and SS correlators respectively. The plots are on the same scale and the all-to-all data are very impressive especially since these required only half the number of configurations compared to the point sources.

4. SUMMARY

The static-light matrix element was computed using all-to-all propagators for the light quark on an anisotropic (but untuned) lattice with $N_f = 2$, using a stout-link background. The great advantage of being able to use smeared-smeared correlators was demonstrated by a direct comparison with the effective mass taken from conventional point propagators. The tuning of the improved, dynamical anisotropic action is currently being carried out, and the resulting parameters will be used for a determination of f_B . Work is also in progress to study radially excited S-waves and P-

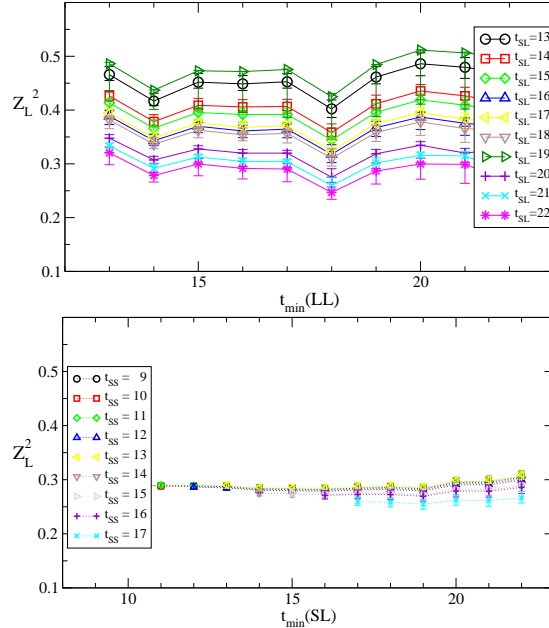


Figure 3. A comparison of the stability of the amplitude with respect to t_{\min} for point (upper panel) and all-to-all (lower panel) propagators.

waves; the preliminary results of this are highly encouraging.

5. ACKNOWLEDGEMENTS

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