Improved Actions in Lattice QCD

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May 4, 2008

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Thesis in Theoretical Physics submitted to fulfill the requirements of the the degree of Doctor of Philosophy.

Abstract

In this thesis I explore the physical effects of improved actions combined with improved operators in the framework of lattice QCD. All calculations are done in the quenched approximation, that is, when all of the dynamical fermion interactions have been suppressed by setting the determinant of the fermion matrix to a constant.

The thesis first briefly introduces lattice QCD to familiarize the reader with the basic concepts. It then describes the common numerical procedures used. It is made up of three major sections.

The first is the exploration of gauge field configurations and the study of the role of instantons in lattice QCD. In this work the Wilson gauge action and a standard 1 loop topological charge operator are used to determine the relative rates of standard cooling and smearing algorithms in pure $SU_c(3)$-color gauge theory. I consider representative gauge field configurations on $16^3 \times 32$ lattices at $\beta = 5.70$ and $24^3 \times 36$ lattices at $\beta = 6.00$. I find the relative rate of variation in the action and topological charge under various algorithms may be succinctly described in terms of simple formulae [1]. The results are in accord with recent suggestions from fat-link perturbation theory. This work is then extended to $\mathcal{O}(a^2)$-improved gauge action and $\mathcal{O}(a^2)$-improved operators [2]. In particular, an $\mathcal{O}(a^2)$-improved version of APE smearing is motivated by considerations of smeared link projection and cooling. The extent to which the established benefits of improved cooling carry over to improved smearing is critically examined. I consider representative gauge field configurations generated with an $\mathcal{O}(a^2)$-improved gauge field action on $16^3 \times 32$ lattices at $\beta = 4.38$ and $24^3 \times 36$ lattices at $\beta = 5.00$ having lattice spacings of 0.165(2) fm and 0.077(1) fm respectively. While the merits of improved algorithms are clearly displayed for the coarse lattice spacing, the fine lattice results put the various algorithms on a more equal footing and allow a quantitative calibration of the smoothing rates for the various algorithms. I find that the relative rate of variation in the action may also be described in terms of simple calibration formulae for $\mathcal{O}(a^2)$-improvement which accurately describes the relative smoothness of the gauge field configurations at a microscopic level.

In the second section the first calculation of the gluon propagator using an $\mathcal{O}(a^2)$-improved action with the corresponding $\mathcal{O}(a^2)$-improved Landau gauge fixing [3] condition is presented [4]. The gluon propagator obtained from the improved action and improved Landau gauge condition is compared with earlier unimproved results on similar physical lattice volumes of $3.2^3 \times 6.4$ fm. It is found that there is good agreement between the improved propagator calculated on a coarse lattice with lattice spacing $a = 0.35$ fm and the unimproved propagator calculated on a fine lattice with spacing $a = 0.10$ fm. This motivated us to calculate the gluon propagator on a coarse very large-volume lattice of $5.6^3 \times 11.2$ fm. The infrared behavior observed in previous studies is confirmed. The gluon propagator is enhanced at intermediate momenta and

[1] F. D. R. Bonnet, P. Fitzhenry, D. B. Leinweber, M. R. Stanford & A. G. Williams, *Phys. Rev. D* **62**, 094509 (2000) [hep-lat/0001018].

[2] F. D. R. Bonnet, D. B. Leinweber, A. G. Williams & J. M. Zanotti, *Submitted to Phys. Rev. D.* [hep-lat/0106023].

[3] F. D. R. Bonnet, P. O. Bowman, D. B. Leinweber, D. G. Richards & A. G. Williams, *Aust. J. Phys.* **52**, 939 (1999).

[4] F. D. R. Bonnet, P. O. Bowman, D. B. Leinweber & A. G. Williams, *Infrared behavior of the gluon propagator on a large volume lattice, Phys. Rev. D* **62**, 051501, (2000).

suppressed at infrared momenta. The observed infrared suppression of the Landau gauge gluon propagator is not a finite volume effect. This work is then extended to a variety of lattices with spacing ranging from $a = 0.17$ to $a = 0.4$ fm [5] to further explore finite volume and discretization effects. In this work a technique previously used for minimizing lattice artifacts, known as "tree-level correction", has also been extended. It is demonstrated that by using tree-level correction, determined by the tree-level behavior of the action being considered, it is possible to obtain scaling behavior over a very wide range of momenta and lattice spacings. This makes it possible to explore the infinite volume and continuum limits of the Landau-gauge gluon propagator.

As a final part of this thesis I present the first results for the quark propagator using an Overlap fermionic quark action [6]. I compare the results with those obtained from the standard Wilson fermion. The overlap quark action is $\mathcal{O}(a)$-improved compared with the Wilson fermion. This action realizes exact chiral symmetry on the lattice unlike the Wilson fermion and it demonstrates that the fastest way forward in this field is with improved lattice operators.

The idea of studying improved actions in lattice gauge theory was suggested to me by A/Prof. Anthony G. Williams during the *"Nonperturbative Methods in Quantum Field Theory"* workshop in early February 1998. Initially it was suggested to me that a calculation of the gluon propagator using improved action on large volumes, following a study just done with standard gauge action in Ref. [62]. The point of interest was to study the effect an improved gauge field action would have on the gluon propagator. This study would then be extended to quark actions. In the meantime when generating gauge field configurations using a computer code written in Fortran 77 (provided by Dr. Derek B. Leinweber), it occurred to me that it would be good to explore the content of these gauge field configurations. In order to do realistic calculations on large lattices we needed a gauge field configuration generator that would run on our CM5 computer and so Connection Machine Fortran (CMF) became the adopted language.

I started writing the computer code to generate the gauge field configuration in the $SU_c(2)$ with the help of Dr. Derek B. Leinweber, who introduced me to the basic concepts in lattice QCD. I then extended this code to the $SU_c(3)$ gauge group. This is commonly known as the standard Wilson gauge action. After investigating with some of the optimization possibilities, I moved on to code an $\mathcal{O}(a^2)$-improved gauge action. The code uses a masking procedure for the link update. I have generalized the masking procedure for any planar gauge field action in $SU_c(N)$, Ref. [18].

From there it was very obvious that by applying a continuous repetition of some sections of code that I written, that some bigger Wilson loops could easily be included in the action and hence highly improved actions could be easily constructed. The only difficulty was to calculate the improvement coefficients.

I then moved on to study smearing algorithms. I adapted the gauge field configuration code to a cooling and a 1×2 and 2×1 improved cooling code in which we inserted higher order loop operators. This was the tool used to explore gauge field configurations and their topological structures. Once the short range quantum fluctuations are removed it is possible to see instantons. Instantons are believed to play a crucial role in the spontaneous chiral symmetry breaking mechanism. We improved

[5] F. D. R. Bonnet, P. O. Bowman, D. B. Leinweber, A. G. Williams & J. M. Zanotti, *Infinite volume and continuum limits of the landau gauge gluon propagator*, *Phys. Rev. D* **64**, 034501 (2001) [hep-lat/0101013].

[6] F. D. R. Bonnet, P. O. Bowman, D. B. Leinweber, A. G. Williams & J. Zhang, *Overlap Propagator in Landau Gauge*, *to be Submitted to Phys. Rev. D.*

the topological charge operator from the clover term to an (1×2 and 2×1) $\mathcal{O}(a^2)$–improved topological charge operator (see Appendices, Sections E.16 and E.17). This code was subsequently adapted by Sundance Bilson-Thompson so that he could insert higher order loops. I have also inserted my $\mathcal{O}(a^2)$–improved operator to construct an $\mathcal{O}(a^2)$–improved smearing algorithm. Using these tools I have calibrated the relative rates of cooling and smearing.

Another piece of work on gauge fixing, reviewed in Chapter 8, was led by Dr. Patrick O. Bowman, Ref. [63]. There I supplied the gauge field configurations and checked some of the analytical work. For the gluon propagator work I supplied all of the lattice configurations with the exception of the $32^3 \times 64$ used in Ref. [62]. The analysis was primarily carried out by Dr. Patrick O. Bowman and partly inspired by the one carried out in hep-lat/0106023. While this gluon propagator work is not being presented here as my own Ph. D. qualifying work, I am a co–author on the subsequent papers and so I have therefore decided to include a review of this work in Chapter 9.

I have also made some contribution in the construction of the Fat–link quark action (with and without the clover term) developed by James M. Zanotti. These contributions involve the code for the Reunitarization of the smeared links, Appendix E.21. Because of the code developed for the improved lattice definition of the $F_{\mu\nu}(x)$ term I have also made some contribution to the Fat–link clover quark action although I will not discuss about this work in the following thesis.

My main contribution for the overlap quark propagator study was in the analysis of the propagator data. The overlap propagators were generated by Dr. Jianbo Zhang and the research was also carried out in collaboration with A/Prof. Anthony G. Williams and Dr. Derek B. Leinweber. The quark propagators for the Wilson fermion were generated by a computer code parallelized by James M. Zanotti and originally written by Prof. Frank X. Lee.

The anisotropic lattice code has not been used in any calculations yet although it has been tested and verified. The code was extended from the isotropic improved generator code in $SU_c(3)$. After a literature search, we decided to implement the action described in Ref. [31] for the anisotropic Wilson action and in Ref. [11, 32] for the improved anisotropic case.

Apart from the work on the gauge fixing and the gluon propagator, done in collaboration with Dr. Patrick O. Bowman, and which for completeness is briefly reviewed in Chapters 8 and 9 respectively, this thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other institution and to the best of knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

Frédéric D. R. Bonnet Date: 20^{th} of September 2001.

Acknowledgement

I would like to thank my supervisors Assoc. Prof. Anthony G. Williams and Dr. Derek B. Leinweber for their help and their guidance throughout my studies. I would like to give special thanks to Assoc. Prof. Anthony G. Williams for encouraging me to take a project in Lattice Gauge Theory and for his clear vision which lead to this thesis. I would also like to thank Dr. Derek B. Leinweber for helping me with most of the aspects of this thesis.

I would like to gratefully acknowledge Dr. Patrick O. Bowman for analysing all of the gauge field configurations in the study of the gluon propagator, James M. Zanotti for calculating the various lattice spacings and allowing me to use the quark propagator code, Dr. Jianbo Zhang for generating the overlap quark propagators, Francis Vaughan and Paul Coddington of the South Australian Centre for Parallel Computing and the Distributed High-Performance Computing Group for support in some of the technical problems and for keeping the CM5 running, and to Ramona Adorjan for her hard and dedicated work on daily computer maintenance.

Special thanks to Prof. Anthony W. Thomas and Assoc. Prof. Anthony G. Williams for working so hard to keep the CSSM running as a research centre and providing a stimulating and pleasant research environment.

I would also like to thank Olivier Leitner and Francois Bissey for their friendship and for sharing some of their research interests with me.

Finally I acknowledge all of the people at the CSSM for providing such a good environment for studying physics. I think that Adelaide University is extremely fortunate to have such a world–class research centre as the CSSM.

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