Numerical investigations of the performance and effectiveness of thermoacoustic couples

Luke Zoontjens

School of Mechanical Engineering
The University of Adelaide
South Australia 5005

Printed February 8, 2008

Supervisors:
Carl Q. Howard
Anthony C. Zander
Ben S. Cazzolato

Abstract

Thermoacoustics is a field of study which includes devices purpose-built to exploit the phenomenal interaction between heat and sound. Thermoacoustics has been demonstrated as an effective technology which can potentially serve a variety of purposes such as cryogenics, cost-effective domestic refrigeration or electricity generation, without adverse environmental impact or commercial drawbacks such as expensive construction or maintenance costs or high part counts.

The mechanisms by which thermoacoustic devices operate at low amplitudes have been identified and effective design tools and methods are available, but the precise heat and mass transfer which occurs deep inside the core of thermoacoustic devices at high amplitudes cannot at present be precisely determined experimentally, and to date have been estimated using only relatively simple or one-dimensional computational domains. It is expected that thermoacoustic devices will need to operate at relatively high pressure amplitudes for commercial and practical applications, to achieve power densities similar to competing technologies. Clearly, advancement of these models and the methods used to investigate them will enable a better understanding of the precise heat and mass transfer that occurs within such devices.

Previous numerical studies have modelled a 'thermoacoustic couple' which consists of a single or several plates (often modelled with zero thickness) and channels within an oscillatory pressure field. In this thesis several improvements

to the ‘thermoacoustic couple’ modelspace are introduced and modelled, and compared with published results. Using the commercial CFD software *Fluent*, a two-dimensional, segregated and second-order implicit numerical model was developed which solves equations for continuity of mass, momentum and energy. These equations were computed using second-order and double-precision discretisation of time, flow variables and energy. A computational domain is presented which is capable of modelling plates of zero or non-zero thickness, is ‘self-resonant’ and able to capture the entrance and exit effects at the stack plate edges. Studies are presented in which the acoustic pressure amplitude, the thickness of the plate (‘blockage ratio’) and the shape of the plate are varied to determine their influence upon the rate of effective heat transfer, flow structure and overall efficiency.

The modelling of thermoacoustic couples with finite thickness presented in this thesis demonstrates that the finite thickness produces new results which show significant disturbances to the flow field and changes to the expected rate and distribution of heat flux along the stack plate. Results indicate that the thickness of the plate, t_s, strongly controls the generation of vortices outside the stack region and perturbs the flow structure and heat flux distribution at the extremities of the plate. Increases in t_s are also shown to improve the integral of the total heat transfer rate but at the expense of increased entropy generation.

Another contribution of this thesis is the study of the effect that leading and trailing edge shapes of stack plates have on the performance of a thermoacoustic couple. In practice, typical parallel or rectangular section stack plates do not have perfectly square edges. The existing literature considers only rectangular or zero-thickness (1-D) plates. Hence a study was performed to evaluate the potential for gains in performance from the use of non-rectangular cross sections, such as rounded, aerofoil or bulbous shaped edges. Consideration of various types of stack plate edges show that performance improvements can be made from certain treatments to the stack plate tips or if possible, stack plate profiles.

This thesis also considers the influence of thermophysical properties and phenomena associated with practical thermoacoustic devices to investigate the applicability of the numerical model to experimental outcomes. Comparisons made between results obtained using the numerical model, linear numerical formulations and experimental results suggest that the numerical model allows comparative study of various thermoacoustic systems for design purposes but is not yet of sufficient scope to fully characterise a realistic system and predict absolute levels of performance. However, the presented method of modelling thermoacoustic couples yields increased insight and detail of flow regimes and heat transportation over previous studies.

Declaration

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis being made available in the University of Adelaide Library.

The author acknowledges that copyright of published works contained within this thesis as listed below resides with the copyright holder(s) of those works.

Luke Zoontjens

Acknowledgements

The author would like to express his deep thanks and gratitude to his advisors Dr. Carl Howard, Dr. Anthony Zander, and Dr. Ben Cazzolato, whose collective enthusiasm, pragmatism and criticism have been invaluable. Without this generous support the task would have been truly arduous. The author also wishes to acknowledge Dr. Con Doolan and Dr. Bassam Dally for their valuable suggestions and discussion.

Thanks go to the administrative and technical staff of School of Mechanical Engineering, who have made my experience positive and enjoyable.

This work has been supported by the South Australian Partnership for Advanced Computing (SAPAC), and the Australian Partnership for Advanced Computing (APAC) by means of access to high-capacity Linux clusters and valued technical advice. The author is also grateful for financial support received through the Australian Research Council (ARC) and the Faculty of Engineering, Computer and Mathematical Sciences at the University of Adelaide.

$$\lim_{z\to\infty}\left[\left(\left(\bar{X}^T\right)^{-1}-\left(\bar{X}^{-1}\right)^T\right)!+\frac{1}{z}\right]^2+\sin^2(\phi)+\cos^2(\phi)=\sum_{n=0}^{\infty}\frac{\cosh(\xi)*\sqrt{1-\tanh^2\xi}}{2^n}$$

Sometimes, despite its initial appearance, the answer can be that simple.

Contents

Abbreviations

AHX	ambient heat exchanger
CFD	computational fluid dynamics
CHX	cold heat exchanger
GWP	global warming potential, Section 2.1.3, p2.1.3
HDTAR	heat driven thermoacoustic refrigerator
HX	heat exchanger
HHX	hot heat exchanger, Section 2.1.1, p7
ODP	ozone depletion potential, Section 2.1.3, p13
OPTR	orifice pulse tube refrigerator
RSM	Reynolds stress model
S-A	Spalart-Allmaras
SETAC	shipboard electronics thermoacoustic chiller, Section 2.1.4, p15
SSBLA	short-stack boundary layer approximation, Section 2.2.2, 27
TAC	thermoacoustic couple, Section 1.2, p2
TALSR	thermoacoustic life science refrigerator, Section 2.1.4, p15
TAR	thermoacoustic refrigerator, Section 2.1, p5
TASHE	thermoacoustic stirling heat-engine, Section 2.1.6, p18
TEWI	total equivalent warming potential, Section 1, p1
ts	time step, e.g. ts0700 refers to the 700th time step
TTE	turbulent transport equation

Notation

English letters

A	cross-sectional area, m^2
BR	blockage ratio
c	(acoustics) gas sound speed, m/s
c_p	heat capacity, J/kgK
c_{pk}	gas heat capacity, J/kgK
c_{ps}	solid heat capacity, J/kgK
C	Courant number, Section B.4, p200
COP	coefficient of performance
COP_r	Carnot relative coefficient of performance
COP_{tc}	Carnot efficiency of thermoacoustic stack, Section 3, p57
d	wall thickness, m, Equation (2.20), p52
DR	drive ratio
e	spatial grid sizing ratio or exponent, Section B.3, p190
ESDM	experimental standard deviation of the mean, Equation (3.3), p65
$\dot{E}_2$	time-averaged acoustic power, W, Section5.2.3.3, p120
$\dot{W}_{\text{diss}}$	rate of energy dissipation, W
f	frequency, Hz, also thermal function
f_M	Moody friction factor, Section 2.2.3, p29
f_0	first resonant frequency, Hz
f_κ	spatial average thermal function
f_υ	spatial average viscous function
g	gravitational acceleration, m/s^2
G	cost function, B.3.2, p195
h	enthalpy, J/kg, in other texts may refer to stack plate-spacing (=$2y_0$), m, Section 4.1.5, p82
$\dot{h}$	heat flux, W/m^2
$\dot{H}_2$	total power, W
j	$=\sqrt{-1}$
k	(acoustics) gas wavenumber, m^{-1}
k	(thermodynamics) gas thermal conductivity, W/mK, Equation (2.5), p12
k	(statistics) coverage factor, a.k.a. Student's t-factor, Section 3.2.1.2, p66
l	stack plate thickness (=$2t_s$), m, Section 2.1.2, p11
L	length, m
$\dot{m}$	periodic mass flow rate amplitude, kg/s, Equation (2.20), p52
M	(fluid dynamics) Mach number, Equation (4.10), p80
M	(physics) Molecular weight, kg/mol, Equation (B.14), p210
M_a	acoustic Mach number (as defined by Swift (1988)), Equation (4.8), p79
n_x	number of mesh grid intervals along edge in x direction
n_y	number of mesh grid intervals along edge in y direction

English letters (continued)

N_R	Reynolds number, Section 2.17, p30
$N_{R,S}$	streaming (oscillatory) Rayleigh number, Section 4, p71
$N^c_{R,S}$	critical streaming Rayleigh number, Equation (4.11), p80
p	pressure, Pa
p_m	mean operating pressure, Pa
Q	thermal power, W
r	internal radius, m, Equation (2.20), p52
r_h	hydraulic radius (=A/Π), m
R	characteristic transverse dimension, m
R	(physics) universal gas constant, Equation (B.14), p210
R^c	(computing) residual of continuity, Equation (3.1), p62
s	entropy, J/kgK
$\dot{S}_{\text{gen}}$	rate of entropy generation per unit volume, W/m^{3}K
S	total system entropy, J/kgK
t	time, s, Equation (2.20), p52
t_s	stack plate half-thickness, m, Section 2.1.2, p11
T_{crit}	critical temperature, K, Section 2.1.1, p7
T_k	gas temperature, K
T_m	mean temperature, K
T_s	stack temperature, K
T_0	ambient temperature, K, Equation (2.20), p52
T_m	mean temperature, K, Section 2.1.1, p7
u	velocity, velocity component in x direction, m/s
U	volumetric flow rate, m^3/s
U_{95}	Expanded uncertainty of measurement at the 95% confidence limit, Section 3.2.1.2, p66
v	velocity component in y direction, m/s
V	volume, m^3
x	axial or horizontal co-ordinate, m, Equation (2.20), p52
x'	axial distance from centre of resonator, m
y	transverse or vertical co-ordinate/dimension, m
y_0	stack plate half-spacing, m, Section 4, p71
Z	impedance, Pa.s/m

Greek symbols

β	thermal expansion coefficient (Classical linear theory), Equation (2.15), p25
γ	ratio of specific heats
δ_κ	thermal penetration depth, m, Section 2.1.3, p13
$\delta_{\kappa m}$	mean thermal penetration depth, m
δ_υ	viscous penetration depth, m, Section 2.3.3, p44
Δt	computational time step, s
Δt_{CFL}	time step limit recommended using the Courant-Friedrichs-Lewy criterion, s, Equation (B.12), p201
$\Delta T_{k,hx}$	axial gas temperature difference across heat exchanger, K
Δx	computational mesh interval spacing in x direction, m
Δy	computational mesh interval spacing in y direction, m
ϵ_s	heat capacity ratio, Section 2.2.1, p23
κ	thermal diffusivity, m^2/s
λ	wavelength, m
μ	dynamic viscosity, kg/ms
ξ	(stack design) normalised stack length, Equation (2.7), p13
ξ_c	(stack design) normalised stack centre position, Equation (2.7), p13
$\lvert\xi_1\rvert$	gas displacement amplitude ($=\lvert u_1\rvert/2\pi f$), m
Π	perimeter, m
ρ	density, kg/m^3
σ	Prandtl number, Pr, Section 2.1.3, p13
σ_{ref}	reference Prandtl number, Section 2.3.4, p50
υ	kinematic viscosity, m^2/s
ϕ	(acoustics) phase angle, in radians unless noted otherwise
ϕ	(stack geometry) volumetric porosity
ω	angular frequency ($=2\pi f$), radians/s
Ω	vorticity, s^{-1}, Section 2.3.3, p44

Subscripts

a	acoustic
bl	boundary layer
crit	critical
eff	effective
h	hydraulic, e.g. hydraulic radius r_h
hx	heat exchanger
$hxsf$	heat exchanger surface
k	gas
m	mean, average
ref	reference
s	solid
t	time-averaged, e.g. $\langle \dot{h} \rangle_t$ is the time-averaged heat flux
x	x or axial direction (Figure 3.1, Section 3.1)
y	y or transverse direction
0	ambient, prevailing
1	first order, complex
κ	thermal
υ	viscous