

THE BEHAVIOUR OF STOCHASTIC RUMOURS

By

Selma BELEN

This thesis is presented for the degree of

Doctor of Philosophy of The University of Adelaide

School of Mathematical Sciences

July 2008

Contents

Abstract	xiii
Acknowledgements	xvii
1 Preliminaries	1
1.1 Structure of the thesis	1
1.2 Rumours in social life and beyond	3
1.3 The beginning of the study of epidemics	5
1.4 The mathematical theory of rumours and epidemics	6
1.5 The differences and similarities between the spread of rumours and epidemics	7
1.6 The role of deterministic models and stochastic models for some real life problems	9
1.7 Notation and terminology	10
1.8 The classical methods in rumour models	12

1.9	More general rumour models	16
2	Rumours with a general number of initial spreaders	21
2.1	Introduction	21
2.2	The model	23
2.3	Evolution of the System	26
2.4	Proportion of Ignorants Never Hearing the Rumour	30
2.5	Proportion of Total Population Never Hearing the Rumour	34
2.6	Two types of stiflers	35
3	Stochastic rumour process and transitions	45
3.1	Transitions and transition probabilities	45
3.1.1	Introduction	45
3.2	The number of transitions in [DK] and [MT] models	47
4	Impulsive control of rumours	61
4.1	Introduction	61
4.2	Impulsive control of rumours for two broadcasts	64
4.2.1	Refinement of the rumour model	64
4.3	Results for Scenario 1	69
4.4	Results for Scenario 2	76

4.5	Conclusions of two broadcasts	80
4.6	Multiple broadcasts	81
4.7	Technical Preliminaries	83
4.8	Scenario 1	84
4.9	Scenario 2	99
4.10	Comparison of Scenarios	107
5	Conclusions	109
	Appendix A	111
	Appendix B	115
	Appendix C	117
	Appendix D	125
	Appendix E	127
	Appendix F	131
	Glossary	147
	Bibliography	149

List of Figures

2.1	The graph of the equation $x = ye^y$	29
2.2	The behaviour of the function θ	33
2.3	The graphs of ϕ_1, ϕ_2 , and their derivatives as functions of α	38
2.4	The graphs of ϕ_1 and ϕ_2 depicted together.	41
2.5	The dynamical relationship between r_1 and r_2 for the case when $\alpha \rightarrow 0$. . .	43
2.6	The evolutions of r_1 and r_2 with respect to s	43
3.1	Interactions between classes and the number of outcomes	47
4.3.1	i_f vs i_b for various values of β under Scenario 1.	72
4.3.2	θ_f vs θ_b for various values of β under Scenario 1.	73
4.4.1	i_f vs i_b for various values of β under Scenario 2. The curve segment starting with \circ and ending with $+$ corresponds to the case $\beta \rightarrow 0$; the curve segment starting with \diamond and ending with \times corresponds to $\beta = 0.5$. The case $\beta \rightarrow 1$ is given by a point at the origin.	79

4.4.2 θ_f vs θ_b for various values of β under Scenario 2.	80
4.8.1 An illustration of Scenario 1 with $\alpha + \beta = 1$ and 5 broadcasts. In each simulation β is incremented by 0.2.	91
4.9.1 An illustration of Scenario 2 with $\alpha + \beta = 1$ and 5 broadcasts. In each simulation β is incremented by 0.2.	102
C.1 The Lambert- w function.	118
C.2 The graph of the equation $ye^y = x$	120
C.3 The graph of the equation $ye^y = x$	121
C.4 The behaviour of the function θ as a function of α	123
D.1 k -fold variant of the model	126

List of Tables

3.1	Possible meaningful interactions, transitions and the relative transition probabilities for different transitions at rate ρ and $r = r_1 + r_2$	55
3.2	Numerical results for extended [DK] model with respect to different sizes of population and arbitrary initial conditions.	56
3.3	Numerical results for extended [MT] model with respect to different size of population and different initial conditions.	56
3.4	Comparison actual values of [MT] and [DK] ignorants and stiflers. . . .	57
3.5	Comparison of actual values of [MT] and [DK] transitions.	58
3.6	Computational results for the extended [DK] model with respect to the population size $n := 10^8$ and $r_{1,0} = 0$ and $r_{2,0} = 0$	59
3.7	Computational results for the extended [MT] model with respect to the population size $n := 10^9$ and $r_{1,0} = 0$ and $r_{2,0} = 0$	60
E.1	Convergence for the lower bound of [DK] by DIM-1	127

E.2	Convergence for the lower bound of [DK] by DIM-2	128
E.3	Convergence for upper bound of [DK] by DIM-1	128
E.4	Convergence for upper bound of [DK] by DIM-2	129
E.5	Convergence for upper bound by Newton's method	129
E.6	Convergence for lower bound by Newton's method	129
F.1	The [MT] model for different population sizes n , with one initial spreader	132
F.2	i_T/i_0 results for the extended [MT] model	133
F.3	The extended [MT] model with the population $n:=10^9$	134
F.4	Final results for the extended [MT] model with $0 < \beta < 1$ and population size $n = 10^5$	136
F.5	Final results for the extended [MT] model with $0 < \beta < 1$ and population size 10^7 (n).	137
F.6	Final results for the extended [MT] model with $0 < \beta < 1$ and $n = 10^8$	138
F.7	The classical [DK] model with first type and second type of stiflers and one initial spreaders	139
F.8	i_T/i_0 for the extended [DK] model with $0 < \beta < 0.99$	140
F.9	i_T/i_0 for the extended [DK] model with $0.999 < \beta < 0.99999$	140
F.10	The extended [DK] model with $n = 10^6$	142
F.11	The extended [DK] model with $n = 10^7$	143

F.12 The extended [DK] model with $n = 10^8$ 144

F.13 The extended [DK] model with $n = 10^9$ 145

Abstract

This thesis presents results concerning the limiting behaviour of stochastic rumour processes.

The first result involves our published analysis of the evolution for the general initial conditions of the (common) deterministic limiting version of the classical Daley-Kendall and Maki-Thompson stochastic rumour models, [14].

The second result being also part of the general analysis in [14] involves a new approach to stiflers in the rumour process. This approach aims at distinguishing two main types of stiflers. The analytical and stochastic numerical results of two types of stiflers in [14] are presented in this thesis.

The third result is that the formulae to find the total number of transitions of a stochastic rumour process with a general case of the Daley-Kendall and Maki-Thompson classical models are developed and presented here, as already presented in [16].

The fourth result is that the problem is taken into account as an optimal control

problem and an impulsive control element is introduced to minimize the number of final ignorants in the stochastic rumour process by repeating the process. Our published results are presented in this thesis as appeared in [15] and [86].

Numerical results produced by our algorithm developed for the extended [MT] model and [DK] model are demonstrated by tables in all details of numerical values in the appendices.

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being made available in all forms of media, now or hereafter known.

Signed..... Date.....

”Sevgi sabırlıdır, sevgi şevkatlidir.

Sevgi kıskanmaz, övünmez, böbürlenmez.

Sevgi kaba davranmaz, kendi çıkarımı

aramaz. Sevgi haksızlığa sevinmez,

gerçek olanla sevinir. Sevgi

her şeye katlanır, her şeye inanır,

her şeyi umut eder, her şeye dayanır.”

Acknowledgements

I would like to thank Charles E. M. Pearce, my supervisor, for his many productive suggestions, guidance and support during the years of my graduate research works. Especially, I am grateful to him very much, for his suggestions and final work on my thesis during our long meetings in the ICNAAM (International Conference of Numerical Analysis and Applied Mathematics) held in September 2007 in Corfu in Greece.

I would like also to express my thanks to Timothy Langtry for his reading of the manuscript of my thesis and his many useful and valuable suggestions during his co-supervision for about some months while I was visiting UTS for about seven months in 2002 as a doctorate student.

I am also thankful Yalçın Kaya who expressed his interest in my work and shared with me his knowledge of control of the dynamical systems and provided many useful references, encouragement and support through the years of my research.

I would like also to thank to Liz Cousins for her friendly encouragement in the first year of my graduate study and to Peter Gill for his attention and friendly encourage-

ment on my study.

Of course, I am grateful to my parents for their warmest support and love. Without them this work wouldn't have come into this stage.

Finally, I wish to thank the following: Yvonne, Tim, Claire and Ito (for their friendships); *and* my sister (because she asked me to).

Selma Belen

September, Greece 2007