Investigation of External Acoustic Loadings on a Launch Vehicle Fairing During Lift-off

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Dissertation Submitted for the Award of the Degree of Doctor of Philosophy on the 19th May, 2008. Qualified on the 26th August, 2008.

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Abstract

During the lift-off of a launch vehicle, the acoustic pressure fluctuations caused by the engine exhaust gases produce high noise levels inside the cavity of the fairing and can damage the payload inside the fairing. Hence reducing the noise transmitted into the payload bay is an important area of research. Work presented in this thesis investigates the external acoustic pressure excitations on the fairing of a launch vehicle during the lift-off acoustic environment. In particular, it investigates the external sound pressure levels in the low frequency range from 50Hz to 400Hz, on the fairing during the lift-off of a launch vehicle.

This study establishes theoretical and numerical models for the prediction of external sound pressure loading on composite structures representing launch vehicles, such as a large composite cylinder referred to as a Boeing cylinder and a Representative Small Launch Vehicle Fairing (RSLVF). To predict the external sound pressure loading, various incident wave conditions were investigated, including incident plane waves, oblique plane waves and oblique plane waves with random phases that strike the circumference of the composite structures.

For the theoretical model, both the incident and scattered sound pressure fields due to incident plane waves; perpendicular to an idealised long cylinder were investigated. The results show that the scattered sound pressure field plays a major role in determining the total circumferential sound pressure field at the surface of the cylinder and cannot be ignored for the launch case.

The theoretical model was developed further for a point source, line source and oblique incident waves, and modified to determine the incident, scattered and total sound pressure fields away from the cylinder. The approach developed overcomes some limitations of previous analytical derivations.

An experiment was undertaken to determine the sound pressure patterns at the surface of a cylinder at various frequencies due to a point source positioned at a finite distance from the cylinder surface. The experimental work confirmed the accuracy of the theoretical model for a point source at a finite distance from the cylinder.

The Boundary Element Method (BEM), approach was used for the numerical investigation of the acoustic loadings. The numerical analysis was developed for various acoustic loading conditions and verified with the theoretical results, which showed that the numerical and theoretical models agree well. Both models were extended to a Boeing composite cylinder and an RSLVF for various acoustic loading conditions.

The complex acoustic environment generated during the lift-off of a launch vehicle was investigated and used as a basis for the acoustic loading on an RSLVF. To predict the acoustic excitations on an RSLVF, two different source allocation techniques were investigated, which considered acoustic sources along the rocket engine exhaust flow. The investigations were conducted both numerically and analytically. Both results agree well and show that it is possible to predict the acoustic loads on the fairing numerically and analytically.

Statement of Originality

To the best of my knowledge and belief, all the material presented in this thesis, except where otherwise referenced, is my own original work, and has not been published previously for the award of any other degree or diploma in any university. If accepted for the award of the degree of Doctor of Philosophy, I consent that this thesis be made available for loan and photocopying.

Mir Md. Maruf Morshed

Date:

Acknowledgement

I would like to acknowledge the efforts of all the people who have made a contribution toward this thesis. Many thanks to my parents, Mir Md. Morshed Ali and Shahana Morshed, who encouraged my mathematical and scientific interests from an early age, made many sacrifices to ensure I received a good education and to their financial support for this study. I would also like to thank my brother Mir Md. Mahmud Morshed and his wife Amy Louise Morshed, for their valuable support while finishing.

I am especially grateful to Professor Colin H. Hansen and Associate Professor Anthony C. Zander, for their supervision and support throughout the entire research. Without their direction and guidance, this thesis would most likely never have been completed. Thanks also to Karen Adams for her contribution to the writing corrections made to this thesis. I would also like to thank Dr Rick Morgans, Dr Carl Q. Howard and Dr Xun Li, for all of their discussions and advice. I am indebted to Silvio De Ieso, Norio Itsumi, Philip Schmidt, Joel Walker and Richard Pateman, for all their help with the design and construction of the electronics and experimental apparatus.

Finally, I would like to express my gratitude to Patricia Anderson, International Student Centre Manager, who handled my visa status during the stay in Australia.

This dissertation is dedicated to all of my family members, especially to my sister Maisha Tasnim and to brother's son who is my little nephew Mir Md. Zebediah Morshed.

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9.24 Numerically calculated sound pressure levels on the circumferential nodes at a height of z = 2.17m from the bottom face of the RSLVF at various one-third-octave centre frequencies from 125Hz to 200Hz, using the non-unique source allocation method. The data used for the numerical calculation for each point source are given in Tables 9.3, 9.4, 9.5 and 9.6. [$x_1 = 15D_e$, $x_2 = 5D_e$ and reference pressure 20μPa] 238

9.25 Numerically calculated sound pressure levels on the circumferential nodes at a height of z = 2.17m from the bottom face of the RSLVF at various one-third-octave centre frequencies from 250Hz to 400Hz, using the non-unique source allocation method. The data used for the numerical calculation for each point source are given in Tables 9.3, 9.4, 9.5 and 9.6. [$x_1 = 15D_e$, $x_2 = 5D_e$ and reference pressure 20μPa] 238

A1 Characteristic behavior of *Bessel functions of the first kind* of order 0, and 1 with respect to the argument z. 251

A2 Characteristic behavior of *Bessel functions of the second kind* of order 0 and 1 with respect to the argument z. 252

Glossary of Symbols

$\boldsymbol{a}$	Radius of a cylinder
a	Observation point
$\boldsymbol{a_e}$	Speed of sound in the exhaust flow
A_m	Coefficients [defined by equations (3.16)]
b	Distance between two point sources [see Figure (5.8)]; also, frequency band (Ch.9)
c	Speed of sound
$C(q)$	Solid angle from volume V [defined by equations (8.7)]
d	Distance between the nearest source and the cylinder surface [see Figure (5.8)]
D_e	Nozzle exit diameter
E_m^i	Amplitudes of the incident waves [see equation (3.22)]
E_m^o	Amplitudes of the outgoing waves [see equations (3.22), (3.26) and (3.27)]
f	Frequency of sound
F	Driving force in the medium (Ch.3); also, thrust of the rocket engine (Ch.9)
$G(a/r'), G(r_p, r_q), G(R)$	Acoustic Green's function [see equations (5.1), (5.2) and (8.1)]
H_m	Bessel function of the 3rd kind or Hankel function
H_{m+1}	Hankel Function of $(m+1)$th order
H_{m-1}	Hankel Function of $(m-1)$th order
$H_m^1(z)$	Hankel function for incoming waves [defined by equation (A.5)]
$H_m^2(z)$	Hankel function for outgoing waves [defined by equation (A.6)]
H'_m	Derivatives of the Hankel function of mth order
H'_0	Derivatives of the Hankel function of 0th order
H_1	Derivatives of the Hankel function of first order
$I^s(r, k, \phi_i)$	Scattered sound intensity as a function of radial distance r, wavelength k and azimuthal angle ϕ_i [defined by equation (4.4)]
J_m	Bessel function of the first kind of mth order
J'_m	Derivatives of the mth order Bessel function
J_{m+1}	Bessel function of $(m+1)$th order
J_{m-1}	Bessel function of $(m-1)$th order
J'_0	Derivatives of the 0th order Bessel function
J_1	Bessel function of first order

k	Wave number
L	Length of a cylinder
L_{p,b,ϕ_i}	Sound pressure level at a frequency band b of interest [defined by equation (9.8)]
L_{p,OA,ϕ_i}	Overall sound pressure level [defined by equation (9.9)]
L_{p,seg,b,ϕ_i}	Sound pressure level corresponding to each segment at a frequency band b of interest [defined by equation (9.14)]
$L_{p,\text{OA},seg,b,\phi_i}$	Overall sound pressure level over the entire segments of the exhaust flow at a frequency band b of interest [defined by equation (9.15)]
L_w	Overall acoustic power level [defined by equation (9.2)]
$L_{w,b}$	Acoustic power level at a frequency band b of interest [defined by equation (9.5)]
$L_{w,seg}$	Overall acoustic power level of exhaust flow segments [defined by equation (9.10)]
$L_{w,seg,b}$	Acoustic power level of exhaust flow segments at a frequency band b of interest [defined by equation (9.11)]
n	Number of the point sources (Chs.5, 7); also, number of nozzles (Ch.9)
N	Total number of point sources (Ch.5); also, number of surface elements (Ch.8)
$N_i(\xi)$	Element shape functions [see equation (8.11)]
N_m	Bessel function of the 2nd kind or Neumann function
N'_m	Derivatives of the mth order Neumann function
N_{m+1}	Neumann function of the $(m+1)$th order
N_{m-1}	Neumann function of the $(m-1)$th order
N'_0	Derivatives of the 0th order Neumann function
N_1	Neumann function of first order
$p(a,\phi_i)$	Resultant pressure as a function of cylinder radius a and azimuthal angle ϕ_i [defined by equations (3.21) and (3.24)]
p_a	Resultant sound pressure at the surface of a cylinder [defined by equation (3.26)]
p^i	Incident wave pressure [see equation (3.7)]
P	Acoustic pressure
$P(p), P(r_p)$	Acoustic pressure at a point p (Ch.8)
$P(p),\ P_i(p)$	Constant nodal pressure on a point p; also, constant nodal pressure for ith elements [see equations (8.9) and (8.10)]
P_o	Equilibrium pressure in the medium
$P_{s\infty}(r,\phi)$	Scattered sound pressure as a function of radial distance r and azimuthal angle ϕ [defined by equation (3.20)]
P^i	Incident wave pressure [defined by equation (7.1)]
$P^i(a,k,\phi_i)$	Incident wave pressure as a function of cylinder radius a, wave number k and azimuthal angle ϕ_i [defined by equation (3.10)]

$P^i(r,k,\phi_i)$	Incident sound pressure as a function of radial distance r, wave number k and azimuthal angle ϕ_i [defined by equation (4.1)]
P^s, p_s	Scattered sound pressure
$P^s(a,k,\phi_i)$	Scattered sound pressure as a function of cylinder radius a, wave number k and azimuthal angle ϕ_i [defined by equation (3.11)]
$P^s(r,k,\phi_i)$	Scattered pressure as a function of radial distance r, wave number k and azimuthal angle ϕ_i [defined by equation (4.3)]
$P^I(q)$	Incident acoustic pressure at point q (Ch.8)
P_f^I	Incident acoustic pressure on the field points [see equation (8.18)]
$P_a^t(a,k,\phi_i)$	Total sound pressure as a function of cylinder radius a, wave number k and azimuthal angle ϕ_i [defined by equation (3.18)]
$P_a^t(R_i,k,a,\phi_i,Q_s)$	Total sound pressure at the surface of a cylinder due to a point source [defined by equation (5.5)]
$P'(R_i)$	Spatially dependent factor [defined by equation (5.4)]
Q_s, $Q_{s,b}$, $Q_{s,seg,b}$	Source strength [see equations (5.4), (9.7) and (9.13)]
r_n	Distance between the origin of the cylinder to the nth number of point sources [defined by equation (5.6)]
r_p	Distance to integration point on the boundary from the centre of the body (Ch.8)
r_q	Distance to point q from the centre of the body (Ch.8)
r'	Source distance [see equation (5.1)]
R	Distance between a point source and observation point [see equation (5.2)]; also, oblique resultant distance between a point source and an observation point [defined by equation (7.3)]
R_i	Distances between a point source and ith number of observation points [see equations (5.3) and (7.4)]
R'	Distance between a point source and an projected observation point [see Figure (7.1); also, defined by equation (7.2)]
S	Boundary surface (Ch.8)
t	Time
$\boldsymbol{u}$	Directional particle velocity
$u(p), u_n(r_p)$	Outward normal particle velocity at a point p (Ch.8)
U_e	Fully expanded exit velocity
W_b	Acoustic power of a source at a frequency band b of interest [defined by equation (9.6)]
$W_{seg,b}$	Acoustic power of a source corresponding to a segment of exhaust flow at a frequency band b of interest [defined by equation (9.12)]
x	Distance travelled by sound waves along the x-axis; also, node position along the x coordinate (Ch.8)

x_t	Core length
y	node position along the y coordinate (Ch.8)
z	Cylinder axis
ω	Angular frequency
β	Elevation angle
γ	Specific heat ratio
γ_m	Phase angles for *m*th order
ϕ_i	Azimuthal or circumferential angle
ψ , ψ_s	Velocity potential
$\psi_m^s(a,k,\phi_i)$	Velocity potential as a function of cylinder radius a, wavelength k and azimuthal angle ϕ_i [defined by equation (4.2)]
λ	Wavelength
ρ	Density of air in the exhaust flow
ρ_o	Equilibrium density in the fluid
$\delta(a-r')$, $\delta(r_p-r_q)$	Dirac delta function [see equations (5.1) and (8.1)]
Γ	Gamma function [defined by equation (A.3)]
ε_m	Constant terms used in the equations
∇	Laplacian operator or differential operator
Δx	Length of exhaust flow slices
Δf_b	Bandwidth of the frequency band, b