EFFECT OF SOIL VARIABILITY ON THE BEARING CAPACITY OF FOOTINGS ON MULTI-LAYERED SOIL

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APPENDIX A

Table A.1 Lower bound estimation for $c_{u1} / c_{u2} \le 1.0$.

H/B \ c_1/c_2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	**8.046**	**8.045**	**8.043**	**8.045**	**7.962**	**7.320**	**6.659**	**6.074**	**5.529**	**5.052**
0.2	**6.287**	**6.285**	**6.287**	**6.288**	**6.288**	**6.287**	**6.219**	**5.859**	**5.452**	**5.052**
0.3	**5.646**	**5.645**	**5.647**	**5.645**	**5.646**	**5.647**	**5.646**	**5.630**	**5.371**	**5.052**
0.4	**5.339**	**5.340**	**5.340**	**5.340**	**5.336**	**5.337**	**5.337**	**5.338**	**5.288**	**5.052**
0.5	**5.172**	**5.170**	**5.173**	**5.173**	**5.172**	**5.172**	**5.171**	**5.173**	**5.173**	**5.052**
0.6	**5.083**	**5.085**	**5.083**	**5.085**	**5.084**	**5.085**	**5.083**	**5.084**	**5.084**	**5.052**
0.7	**5.055**	**5.055**	**5.057**	**5.055**	**5.052**	**5.055**	**5.054**	**5.054**	**5.053**	**5.052**
0.8	**5.052**	**5.050**	**5.053**	**5.053**	**5.054**	**5.053**	**5.053**	**5.054**	**5.053**	**5.052**
0.9	**5.053**	**5.055**	**5.053**	**5.053**	**5.052**	**5.050**	**5.053**	**5.053**	**5.053**	**5.052**
1.0	**5.054**	**5.055**	**5.053**	**5.050**	**5.054**	**5.053**	**5.053**	**5.053**	**5.053**	**5.052**
1.1	**5.052**	**5.055**	**5.053**	**5.053**	**5.054**	**5.053**	**5.053**	**5.053**	**5.053**	**5.052**
1.2	**5.053**	**5.055**	**5.050**	**5.053**	**5.054**	**5.052**	**5.053**	**5.054**	**5.053**	**5.052**
1.3	**5.053**	**5.055**	**5.050**	**5.053**	**5.054**	**5.052**	**5.053**	**5.054**	**5.053**	**5.052**
1.4	**5.053**	**5.055**	**5.050**	**5.053**	**5.052**	**5.052**	**5.053**	**5.053**	**5.053**	**5.052**
1.5	**5.052**	**5.050**	**5.053**	**5.053**	**5.052**	**5.052**	**5.053**	**5.053**	**5.053**	**5.052**
1.6	**5.052**	**5.055**	**5.050**	**5.053**	**5.052**	**5.052**	**5.053**	**5.053**	**5.053**	**5.052**
1.7	**5.052**	**5.055**	**5.053**	**5.050**	**5.052**	**5.052**	**5.053**	**5.053**	**5.053**	**5.052**
1.8	**5.053**	**5.055**	**5.053**	**5.053**	**5.052**	**5.053**	**5.051**	**5.053**	**5.053**	**5.052**
1.9	**5.052**	**5.055**	**5.053**	**5.053**	**5.052**	**5.053**	**5.053**	**5.054**	**5.053**	**5.052**
2.0	**5.053**	**5.055**	**5.053**	**5.053**	**5.052**	**5.053**	**5.053**	**5.053**	**5.053**	**5.052**
2.1	**5.053**	**5.055**	**5.053**	**5.053**	**5.052**	**5.053**	**5.053**	**5.054**	**5.053**	**5.052**
2.2	**5.053**	**5.055**	**5.053**	**5.053**	**5.052**	**5.053**	**5.053**	**5.053**	**5.053**	**5.052**
2.3	**5.053**	**5.055**	**5.053**	**5.053**	**5.052**	**5.052**	**5.053**	**5.053**	**5.053**	**5.052**
2.4	**5.053**	**5.055**	**5.053**	**5.053**	**5.054**	**5.053**	**5.053**	**5.053**	**5.053**	**5.052**
2.5	**5.053**	**5.055**	**5.053**	**5.053**	**5.052**	**5.053**	**5.053**	**5.053**	**5.053**	**5.052**
2.6	**5.053**	**5.050**	**5.053**	**5.050**	**5.052**	**5.053**	**5.053**	**5.053**	**5.053**	**5.052**
2.7	**5.053**	**5.055**	**5.053**	**5.053**	**5.052**	**5.053**	**5.053**	**5.054**	**5.053**	**5.052**
2.8	**5.053**	**5.050**	**5.053**	**5.053**	**5.052**	**5.053**	**5.053**	**5.054**	**5.053**	**5.052**
2.9	**5.053**	**5.055**	**5.053**	**5.053**	**5.052**	**5.053**	**5.053**	**5.054**	**5.053**	**5.052**
3.0	**5.053**	**5.055**	**5.053**	**5.053**	**5.052**	**5.053**	**5.053**	**5.054**	**5.053**	**5.052**

Table A.2 Upper bound estimation for c_{u1} / c_{u2} < 1.0.

H/B \ c_1/c_2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	**12.697**	**12.697**	**12.618**	**10.822**	**9.256**	**8.037**	**7.088**	**6.323**	**5.706**	**5.204**
0.2	**7.349**	**7.349**	**7.349**	**7.349**	**7.349**	**7.200**	**6.666**	**6.130**	**5.630**	**5.204**
0.3	**6.187**	**6.187**	**6.187**	**6.187**	**6.187**	**6.187**	**6.187**	**5.930**	**5.554**	**5.204**
0.4	**5.697**	**5.697**	**5.697**	**5.697**	**5.697**	**5.697**	**5.697**	**5.695**	**5.476**	**5.204**
0.5	**5.442**	**5.443**	**5.442**	**5.442**	**5.442**	**5.442**	**5.442**	**5.442**	**5.393**	**5.204**
0.6	**5.294**	**5.294**	**5.294**	**5.294**	**5.294**	**5.294**	**5.294**	**5.294**	**5.294**	**5.204**
0.7	**5.226**	**5.226**	**5.226**	**5.226**	**5.226**	**5.226**	**5.226**	**5.226**	**5.226**	**5.204**
0.8	**5.205**	**5.205**	**5.205**	**5.205**	**5.205**	**5.205**	**5.205**	**5.205**	**5.205**	**5.204**
0.9	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
1.0	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
1.1	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
1.2	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
1.3	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
1.4	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
1.5	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
1.6	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
1.7	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
1.8	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
1.9	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
2.0	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
2.1	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
2.2	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
2.3	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
2.4	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
2.5	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
2.6	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
2.7	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
2.8	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
2.9	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**
3.0	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**	**5.204**

Table A.3 Lower bound estimation for 10.0 ≥ c_{u1} / c_{u2} ≥ 1.0.

H/B \ c_1/c_2	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0.1	5.052	2.766	1.967	1.552	1.293	1.111	0.975	0.871	0.788	0.722
0.2	5.052	3.000	2.230	1.812	1.544	1.353	1.210	1.101	1.013	0.941
0.3	5.052	3.232	2.480	2.059	1.780	1.585	1.439	1.321	1.226	1.147
0.4	5.052	3.447	2.723	2.303	2.014	1.808	1.653	1.531	1.430	1.347
0.5	5.052	3.655	2.955	2.535	2.246	2.035	1.873	1.745	1.640	1.553
0.6	5.052	3.858	3.180	2.765	2.472	2.260	2.094	1.963	1.856	1.768
0.7	5.052	4.052	3.403	2.988	2.698	2.485	2.323	2.193	2.088	2.001
0.8	5.052	4.241	3.620	3.213	2.926	2.718	2.560	2.435	2.334	2.249
0.9	5.052	4.426	3.833	3.435	3.160	2.958	2.806	2.684	2.584	2.503
1.0	5.052	4.597	4.040	3.658	3.396	3.203	3.054	2.938	2.843	2.763
1.1	5.052	4.756	4.237	3.878	3.630	3.448	3.310	3.198	3.107	3.031
1.2	5.052	4.898	4.417	4.085	3.862	3.693	3.561	3.459	3.372	3.299
1.3	5.052	5.030	4.583	4.278	4.074	3.923	3.807	3.714	3.632	3.568
1.4	5.052	5.055	4.737	4.455	4.268	4.130	4.027	3.944	3.874	3.818
1.5	5.052	5.050	4.880	4.618	4.442	4.317	4.221	4.145	4.084	4.034
1.6	5.052	5.050	5.013	4.768	4.604	4.487	4.394	4.325	4.269	4.221
1.7	5.052	5.055	5.050	4.913	4.756	4.645	4.559	4.491	4.440	4.394
1.8	5.052	5.055	5.053	5.043	4.900	4.795	4.713	4.650	4.597	4.553
1.9	5.052	5.055	5.053	5.053	5.034	4.935	4.859	4.798	4.748	4.706
2.0	5.052	5.055	5.053	5.053	5.052	5.053	4.996	4.936	4.890	4.850
2.1	5.052	5.055	5.053	5.053	5.052	5.052	5.051	5.054	5.023	4.986
2.2	5.052	5.055	5.053	5.053	5.052	5.053	5.053	5.054	5.052	5.053
2.3	5.052	5.055	5.053	5.053	5.054	5.053	5.053	5.054	5.052	5.052
2.4	5.052	5.055	5.053	5.053	5.054	5.053	5.053	5.054	5.053	5.053
2.5	5.052	5.055	5.053	5.053	5.054	5.053	5.053	5.054	5.053	5.052
2.6	5.052	5.055	5.053	5.053	5.052	5.053	5.053	5.054	5.053	5.053
2.7	5.052	5.055	5.053	5.053	5.054	5.053	5.053	5.054	5.053	5.053
2.8	5.052	5.055	5.053	5.053	5.052	5.053	5.053	5.054	5.053	5.052
2.9	5.052	5.055	5.053	5.053	5.052	5.052	5.053	5.054	5.052	5.052
3.0	5.052	5.055	5.053	5.053	5.052	5.053	5.053	5.054	5.052	5.052

Table A.4 Upper bound estimation for $10.0 \geq c_{u1} / c_{u2} \geq 1.0$.

H/B \ c_1/c_2	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0.1	5.204	2.877	2.060	1.636	1.372	1.189	1.053	0.950	0.868	0.799
0.2	5.204	3.112	2.328	1.903	1.627	1.433	1.288	1.174	1.082	1.005
0.3	5.204	3.342	2.581	2.151	1.866	1.664	1.512	1.391	1.293	1.211
0.4	5.204	3.558	2.820	2.390	2.101	1.893	1.732	1.603	1.497	1.409
0.5	5.204	3.767	3.052	2.624	2.329	2.115	1.950	1.816	1.706	1.614
0.6	5.204	3.967	3.278	2.851	2.555	2.337	2.170	2.038	1.929	1.837
0.7	5.204	4.162	3.498	3.076	2.780	2.566	2.402	2.271	2.164	2.075
0.8	5.204	4.352	3.714	3.300	3.013	2.803	2.644	2.517	2.412	2.325
0.9	5.204	4.535	3.925	3.524	3.248	3.047	2.892	2.770	2.671	2.587
1.0	5.204	4.707	4.132	3.748	3.485	3.292	3.145	3.027	2.932	2.853
1.1	5.204	4.866	4.329	3.969	3.722	3.539	3.398	3.287	3.197	3.122
1.2	5.204	5.012	4.516	4.183	3.954	3.784	3.653	3.547	3.461	3.391
1.3	5.204	5.145	4.688	4.384	4.175	4.021	3.900	3.803	3.724	3.657
1.4	5.204	5.204	4.847	4.568	4.379	4.240	4.132	4.045	3.974	3.913
1.5	5.204	5.204	4.995	4.736	4.562	4.435	4.338	4.261	4.197	4.144
1.6	5.204	5.204	5.133	4.893	4.730	4.611	4.521	4.449	4.390	4.342
1.7	5.204	5.204	5.204	5.039	4.886	4.774	4.688	4.620	4.565	4.518
1.8	5.204	5.204	5.204	5.175	5.032	4.927	4.845	4.780	4.727	4.683
1.9	5.204	5.204	5.204	5.204	5.170	5.070	4.993	4.931	4.880	4.838
2.0	5.204	5.204	5.204	5.204	5.204	5.199	5.132	5.073	5.024	4.984
2.1	5.204	5.204	5.204	5.204	5.204	5.204	5.204	5.200	5.161	5.122
2.2	5.204	5.204	5.204	5.204	5.204	5.204	5.204	5.200	5.204	5.204
2.3	5.204	5.204	5.204	5.204	5.204	5.204	5.204	5.200	5.204	5.204
2.4	5.204	5.204	5.204	5.204	5.204	5.204	5.204	5.200	5.204	5.204
2.5	5.204	5.204	5.204	5.204	5.204	5.204	5.204	5.200	5.204	5.204
2.6	5.204	5.204	5.204	5.204	5.204	5.204	5.204	5.200	5.204	5.204
2.7	5.204	5.204	5.204	5.204	5.204	5.204	5.204	5.200	5.204	5.204
2.8	5.204	5.204	5.204	5.204	5.204	5.204	5.204	5.200	5.204	5.204
2.9	5.204	5.204	5.204	5.204	5.204	5.204	5.204	5.200	5.204	5.204
3.0	5.204	5.204	5.204	5.204	5.204	5.204	5.204	5.200	5.204	5.204

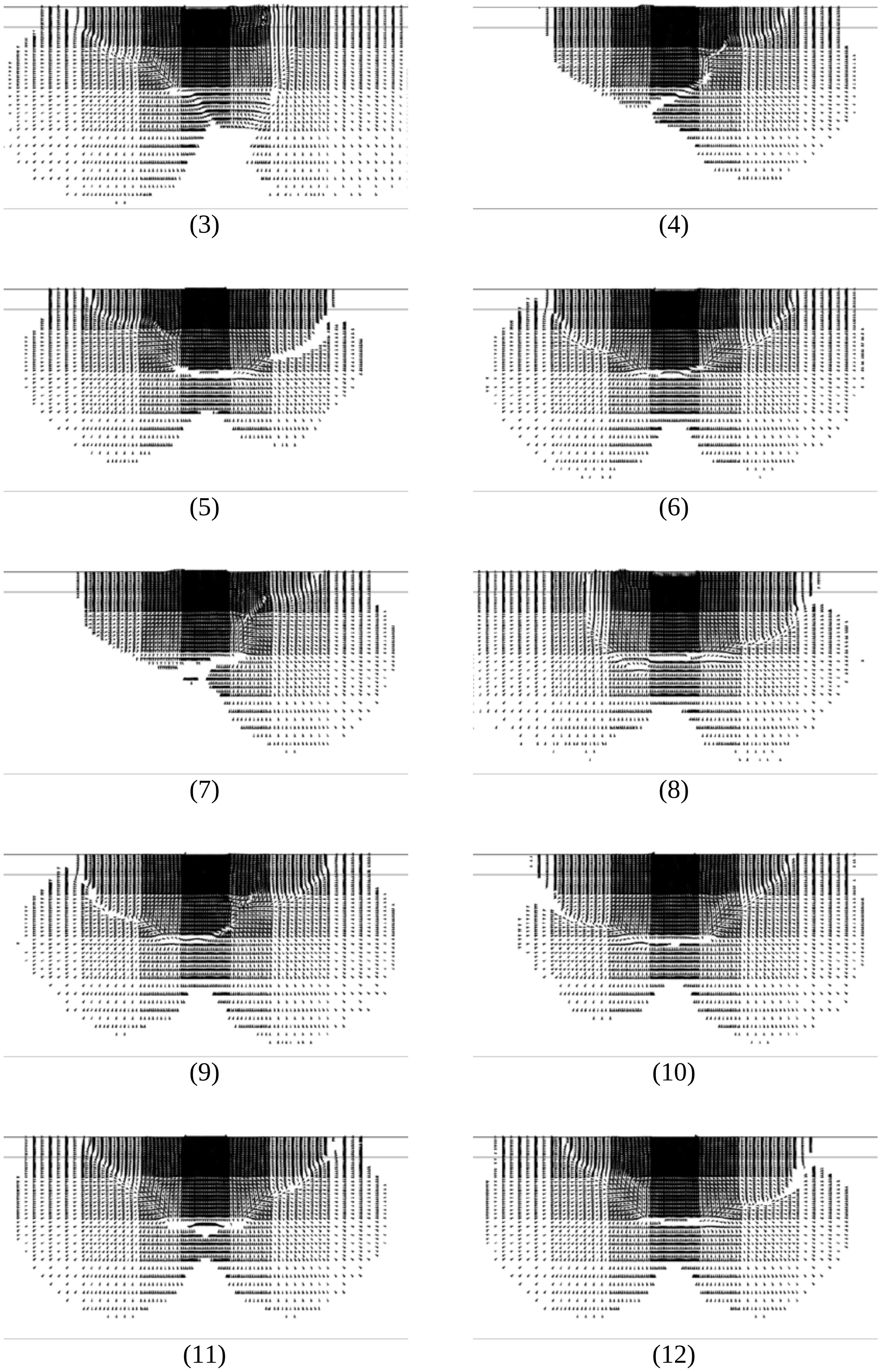
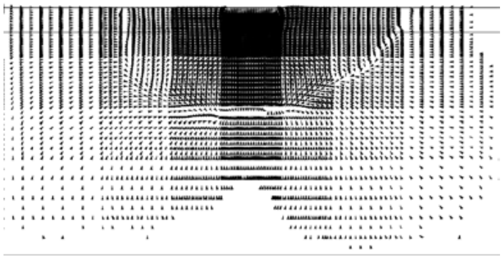


Figure A.1 Displacement vectors at near failure (two-layered spatially variable purely cohesive material).



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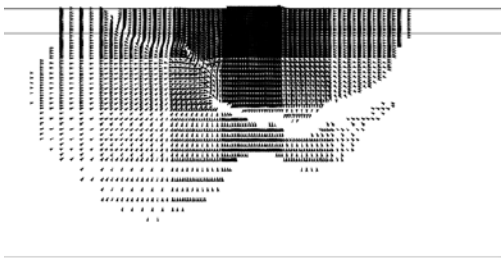

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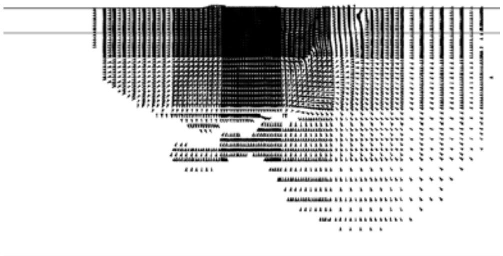

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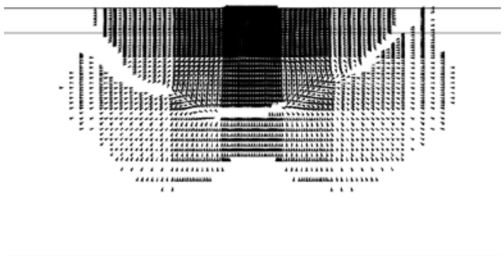

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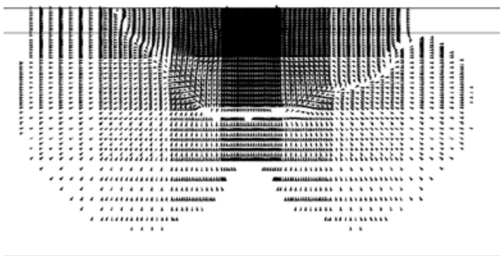

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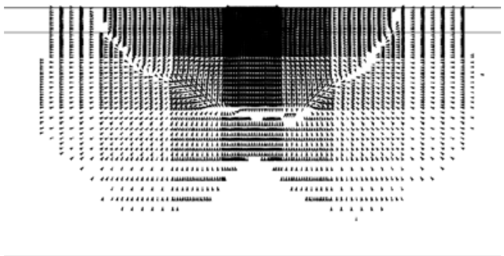

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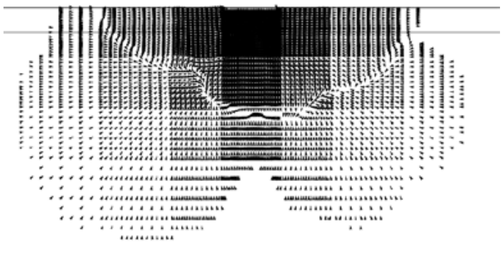

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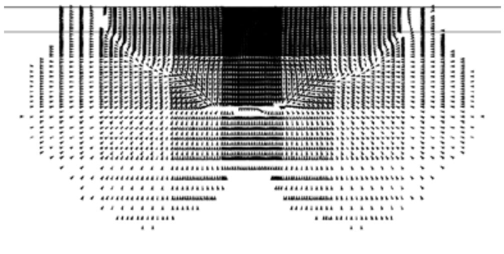

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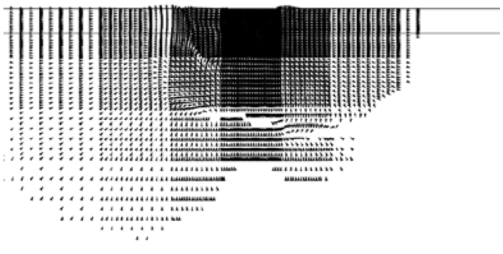

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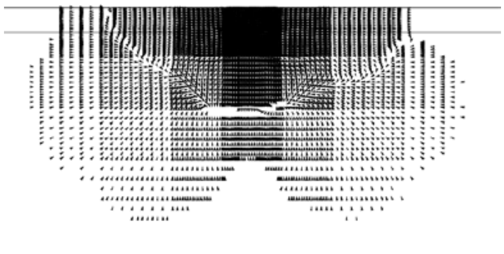
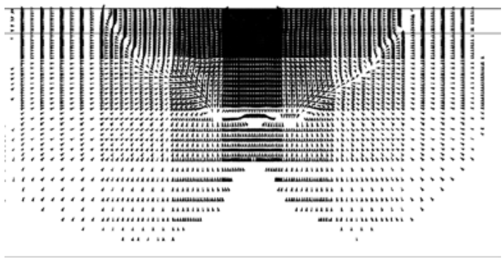


(22)

Figure A.1 Displacement vectors at near failure (two-layered spatially variable purely cohesive material). ***(Continued)***



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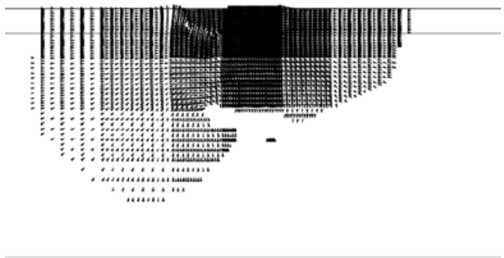

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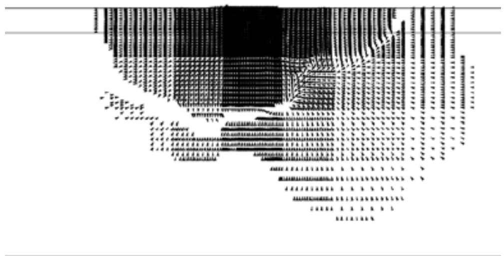


(25)

Figure A.1 Displacement vectors at near failure (two-layered spatially variable purely cohesive material). ***(Continued)***

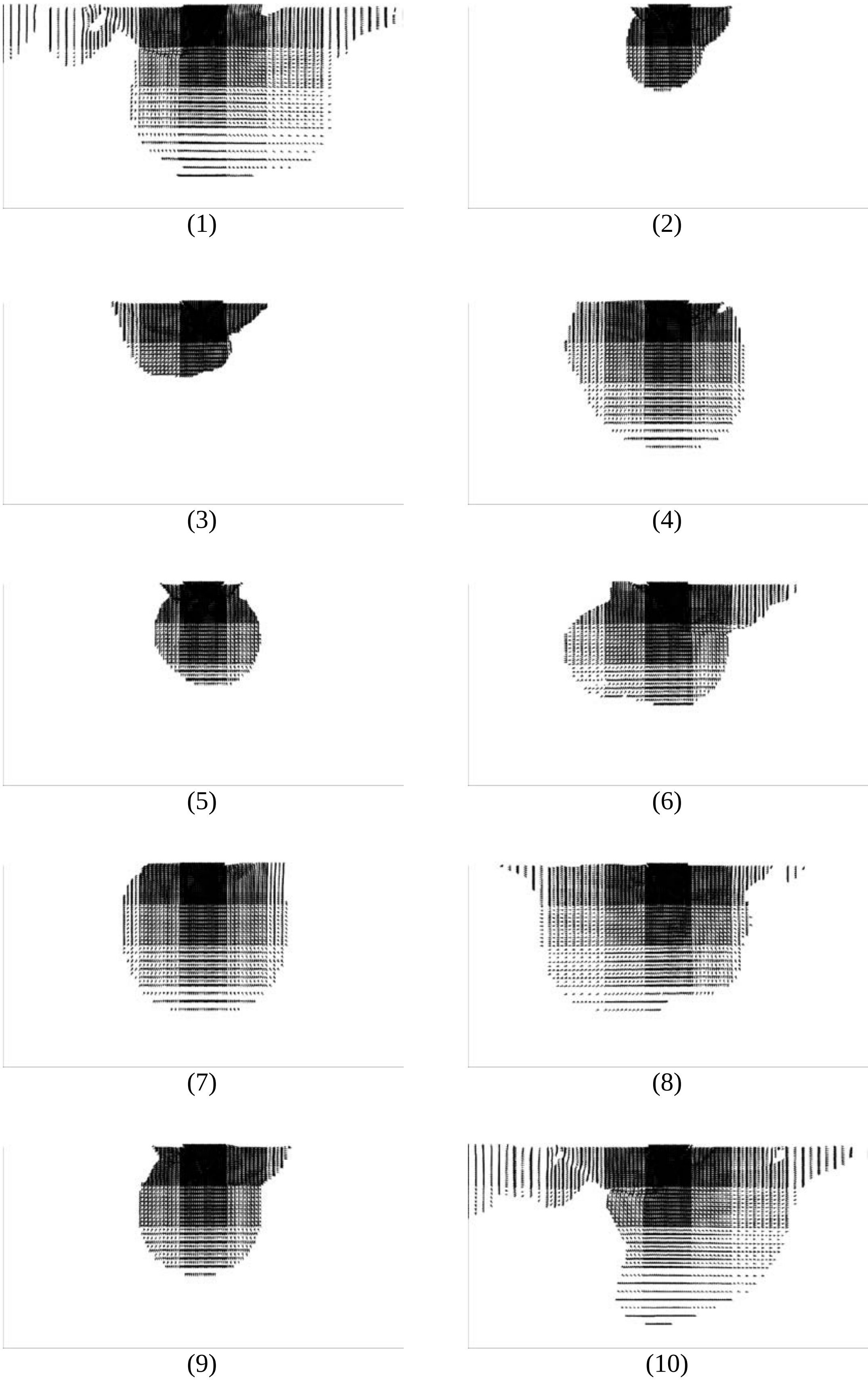


Figure A.2 Displacement vectors at near failure (single-layered spatially variable cohesive-frictional material).

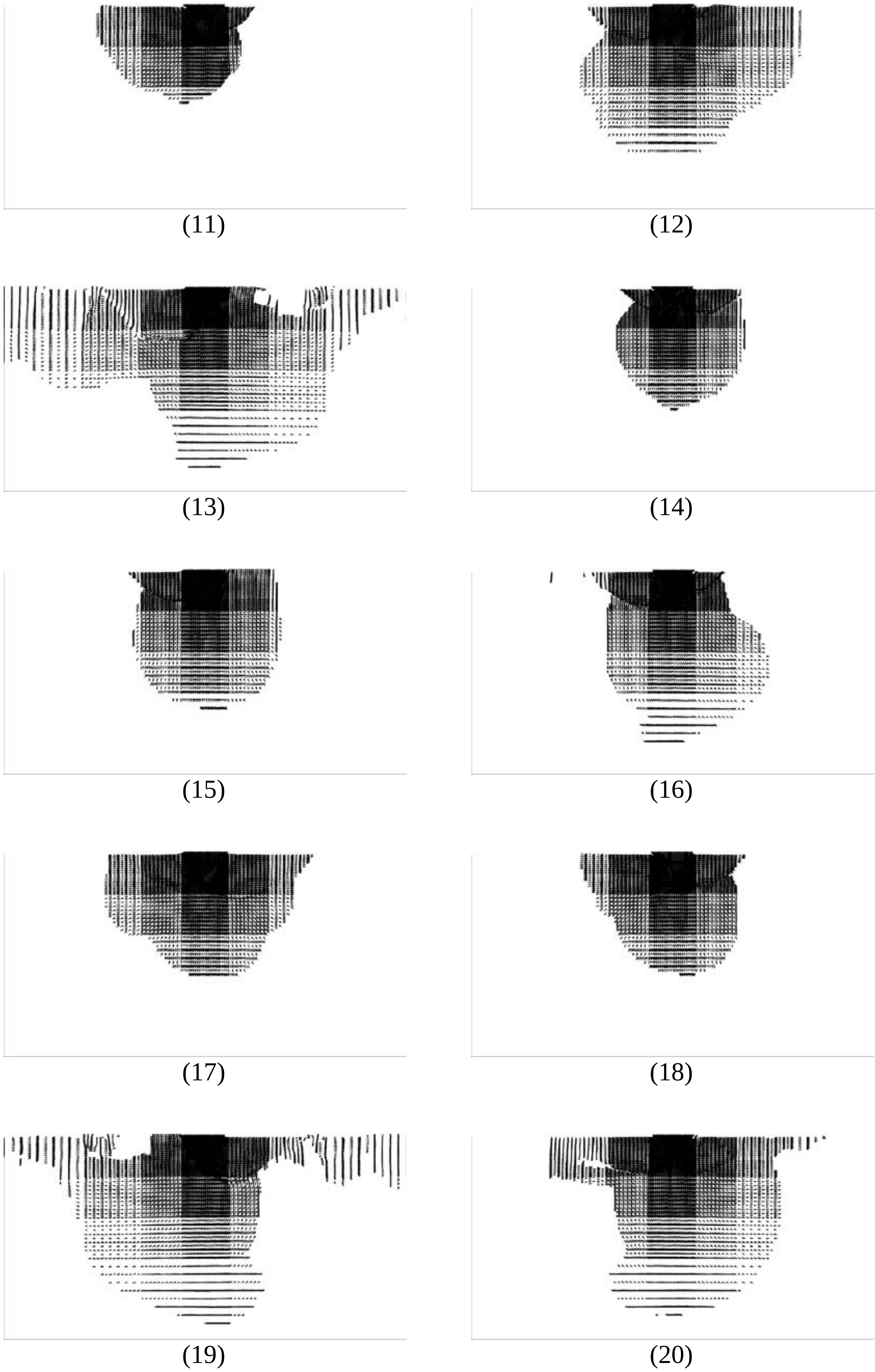
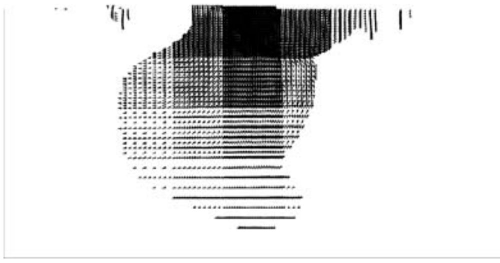


Figure A.2 Displacement vectors at near failure (single-layered spatially variable cohesive-frictional material). ***(Continued)***



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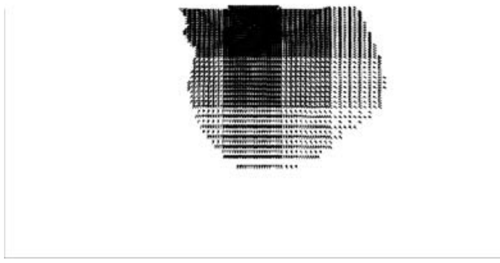

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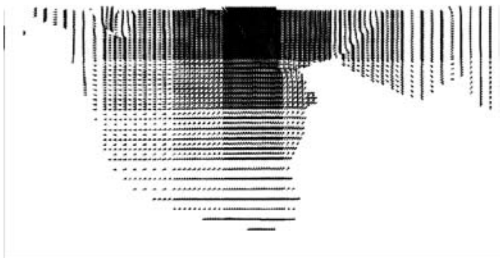

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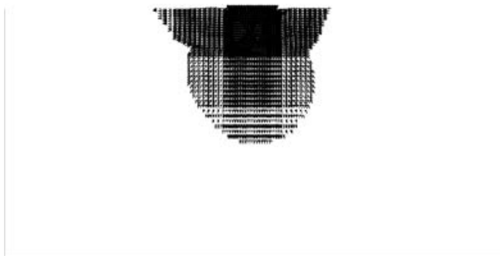

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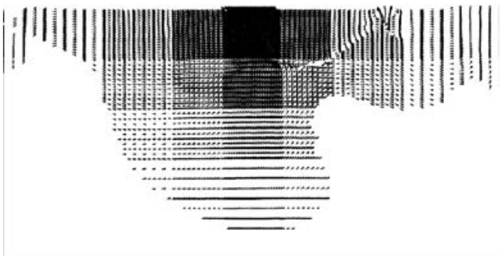

(25)

Figure A.2 Displacement vectors at near failure (single-layered spatially variable cohesive-frictional material). ***(Continued)***

APPENDIX B

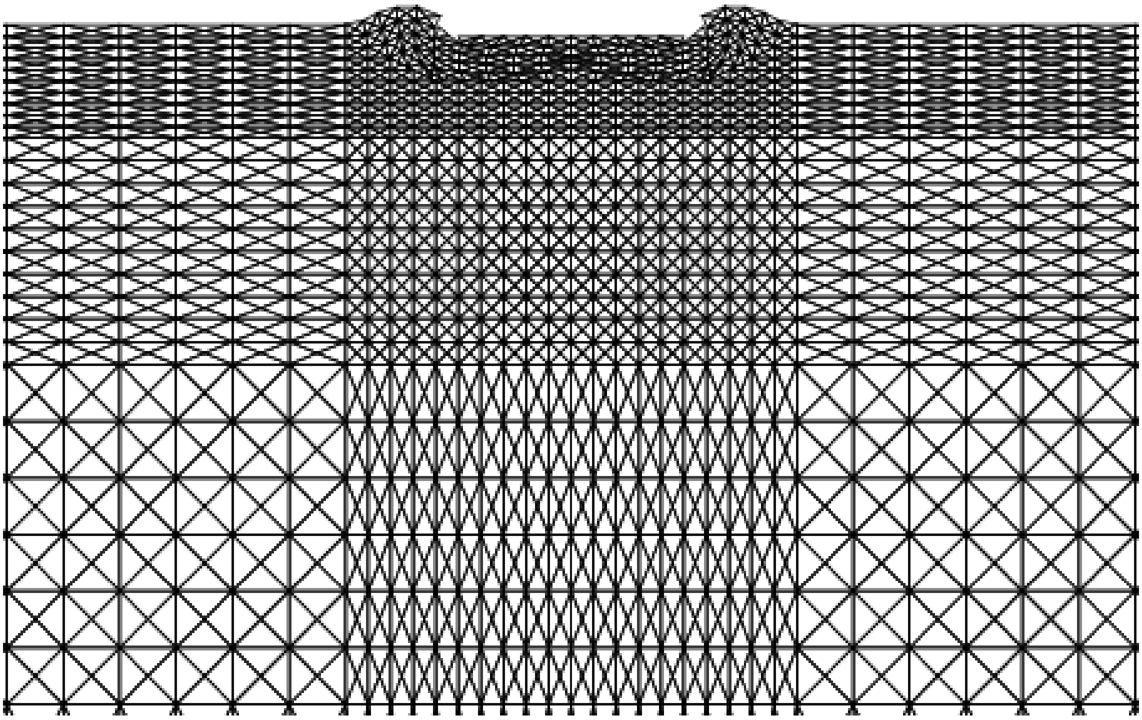

Figure B.1 Upper bound failure mechanism for COHESIVE_0.025_0.25 case (i.e. c_{u1}/c_{u2} = 0.025, H/B = 0.25).

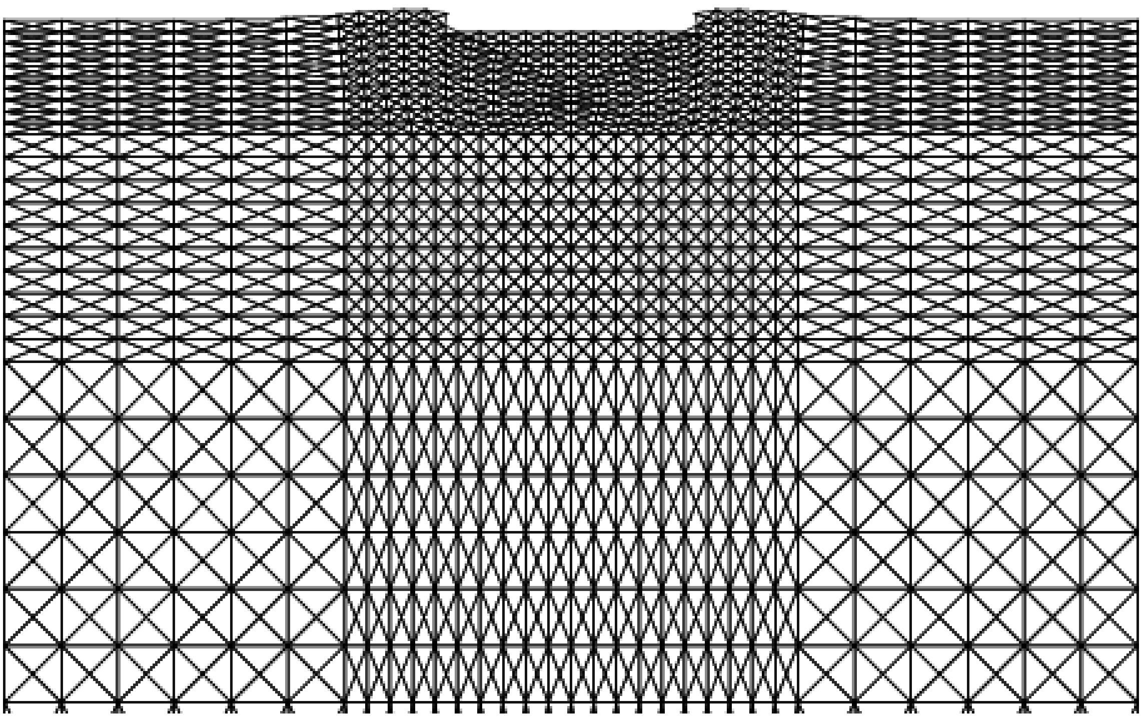

Figure B.2 Upper bound failure mechanism for COHESIVE_0.025_0.50 case (i.e. c_{u1}/c_{u2} = 0.025, H/B = 0.5).

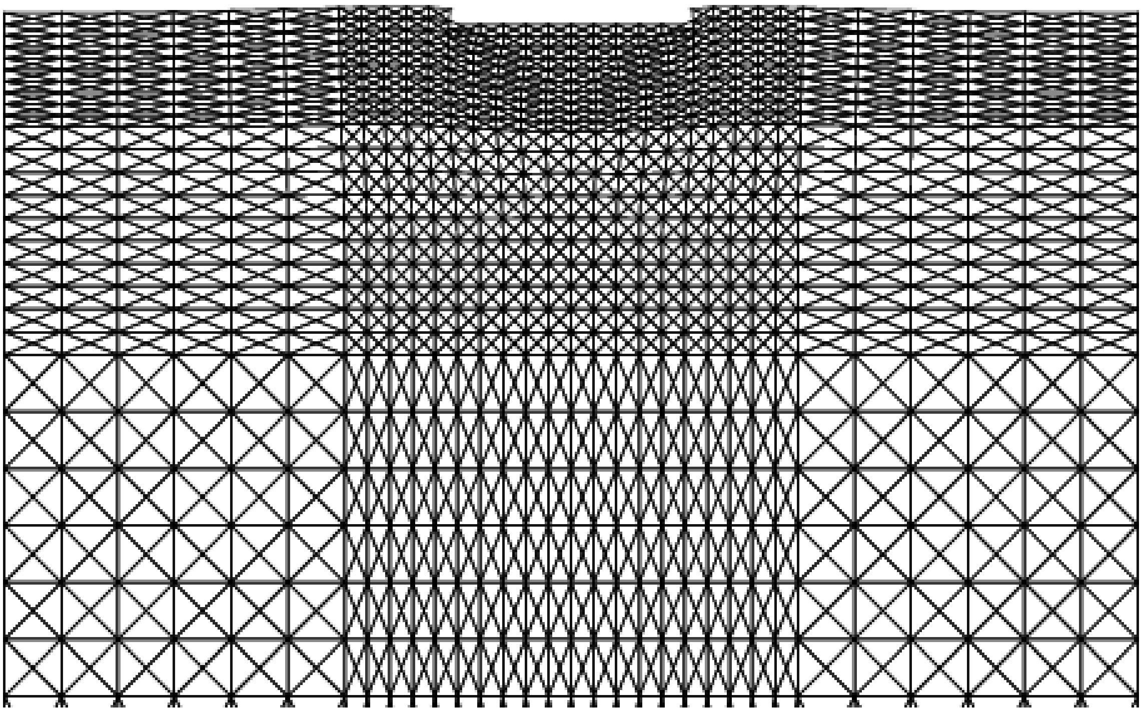

Figure B.3 Upper bound failure mechanism for COHESIVE_0.025_1.00 case (i.e. c_{u1}/c_{u2} = 0.025, H/B = 1.0).

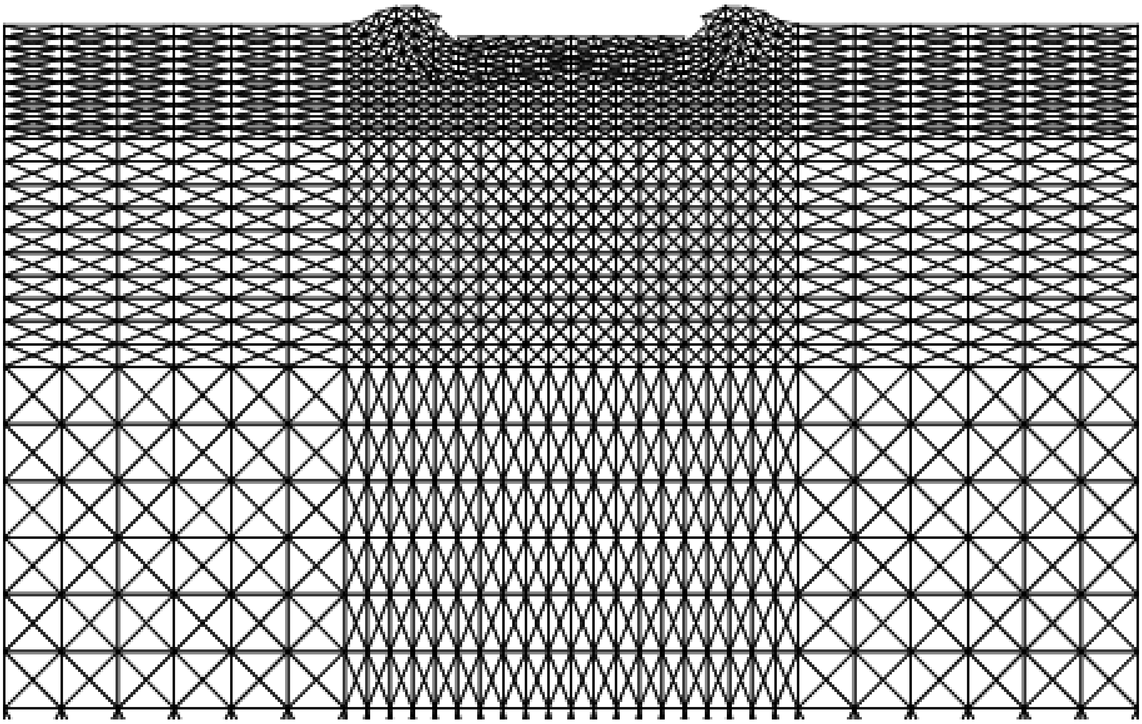

Figure B.4 Upper bound failure mechanism for COHESIVE_0.05_0.25 case (i.e. c_{u1}/c_{u2} = 0.05, H/B = 0.25).

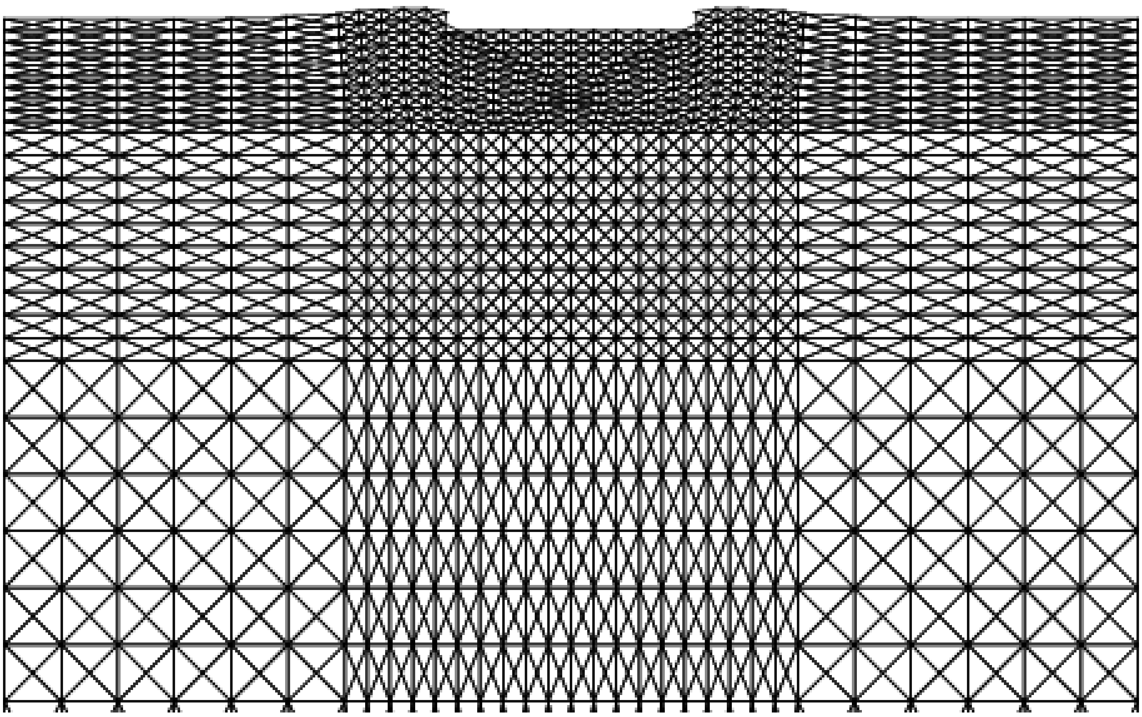

Figure B.5 Upper bound failure mechanism for COHESIVE_0.05_0.50 case (i.e. c_{u1}/c_{u2} = 0.05, H/B = 0.5).

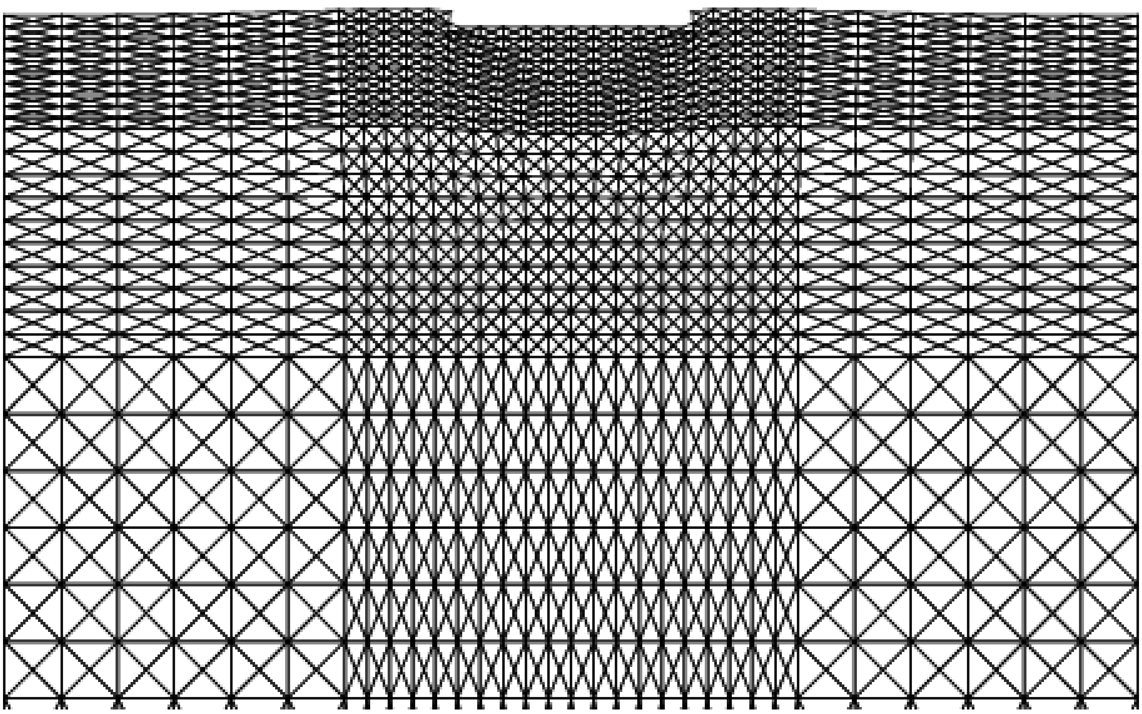

Figure B.6 Upper bound failure mechanism for COHESIVE_0.05_1.0 case (i.e. c_{u1}/c_{u2} = 0.05, H/B = 1.0).

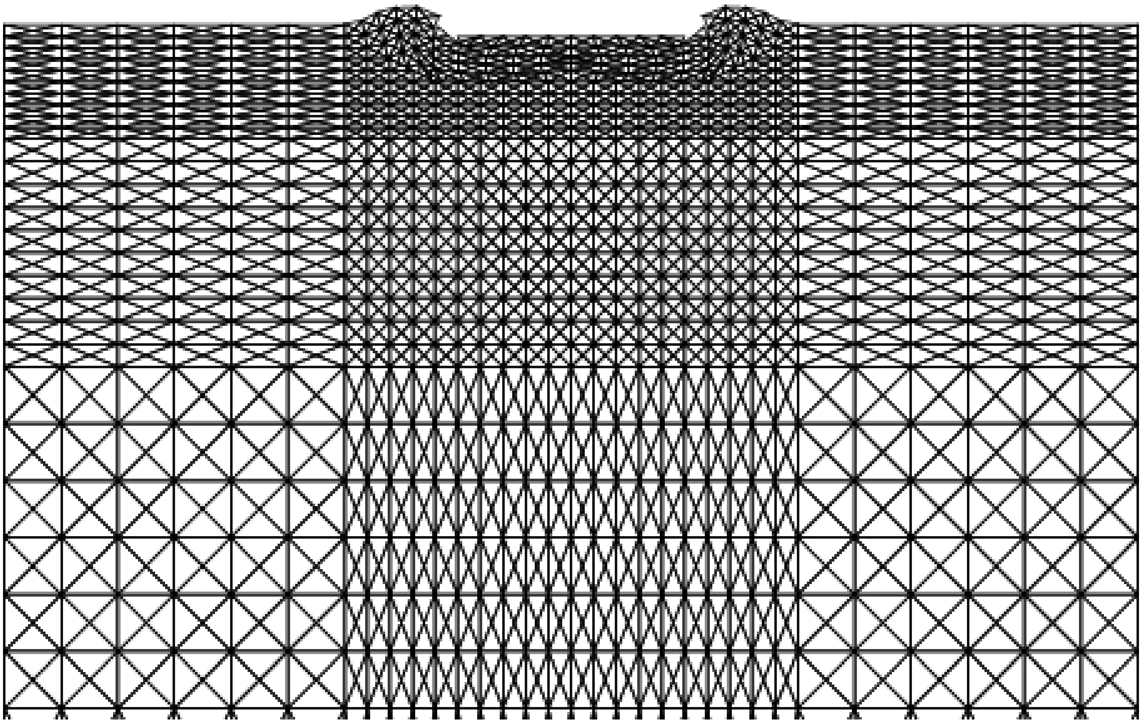

Figure B.7 Upper bound failure mechanism for COHESIVE_0.1_0.25 case (i.e. c_{u1}/c_{u2} = 0.1, H/B = 0.25).

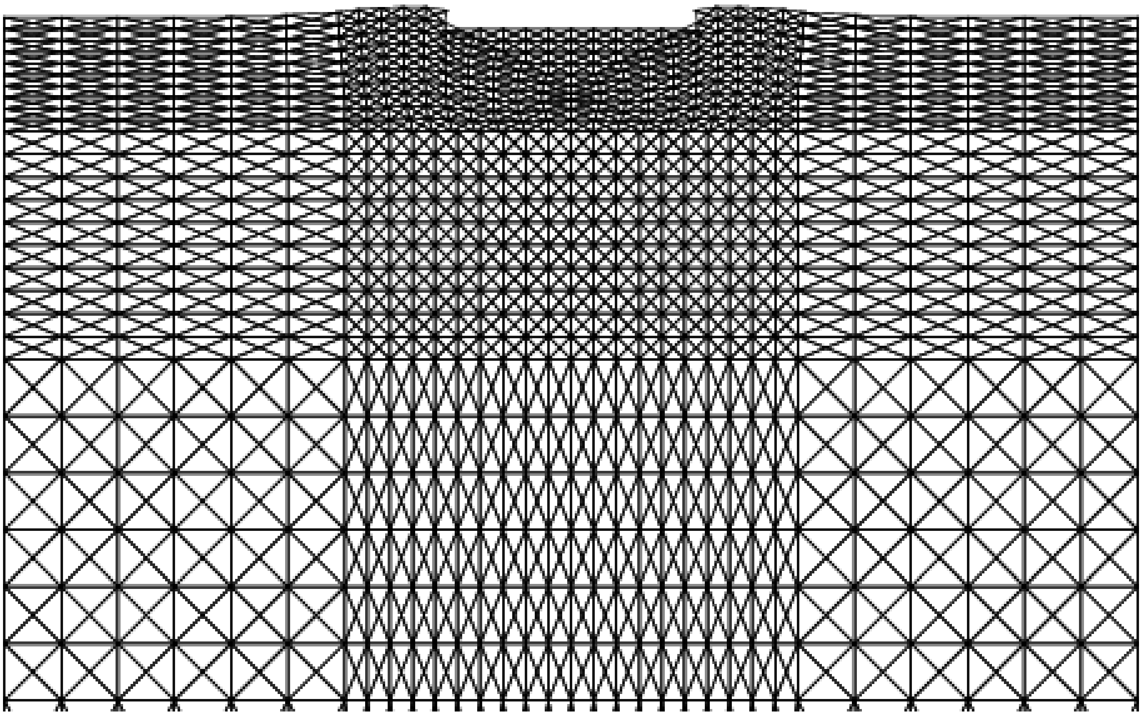

Figure B.8 Upper bound failure mechanism for COHESIVE_0.1_0.5 case (i.e. c_{u1}/c_{u2} = 0.1, H/B = 0.5).

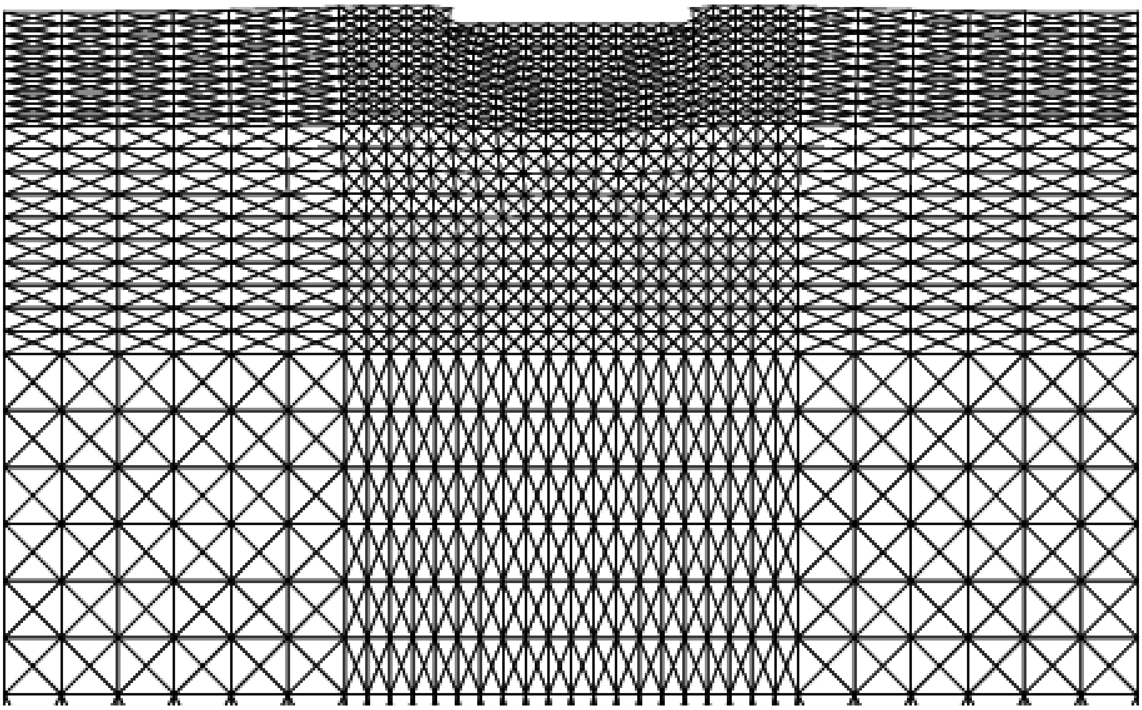

Figure B.9 Upper bound failure mechanism for COHESIVE_0.1_1.0 case (i.e. c_{u1}/c_{u2} = 0.1, H/B = 1.0).

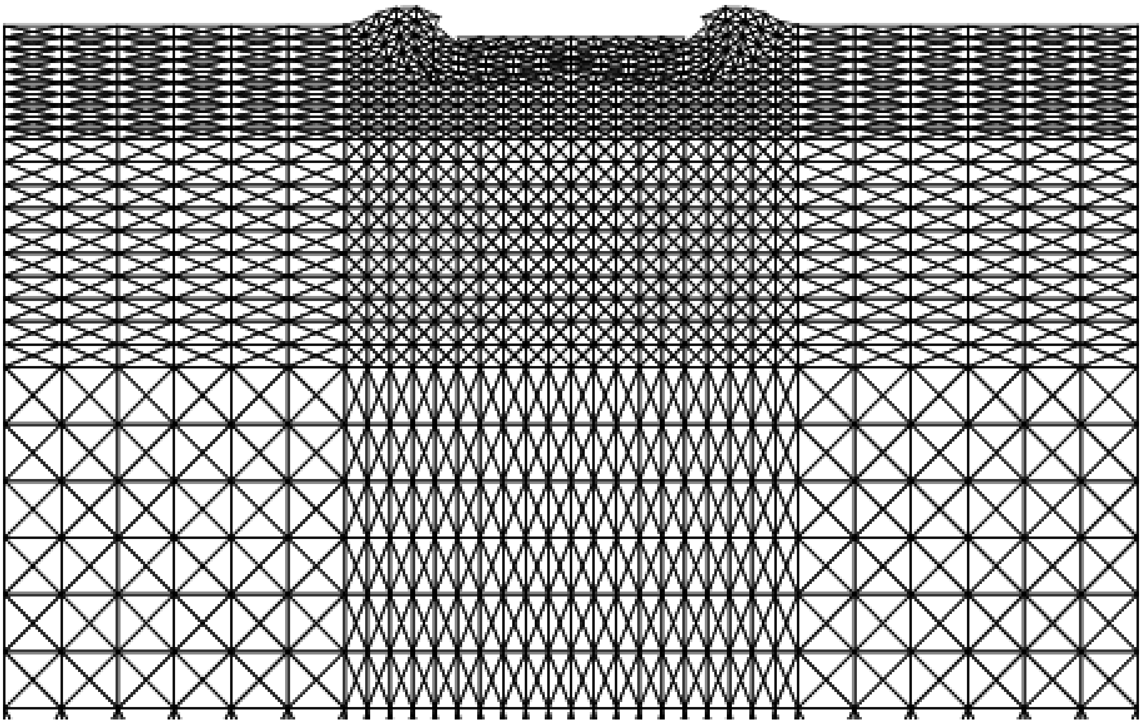

Figure B.10 Upper bound failure mechanism for COHESIVE_0.25_0.25 case (i.e. c_{u1}/c_{u2} = 0.25, H/B = 0.25).

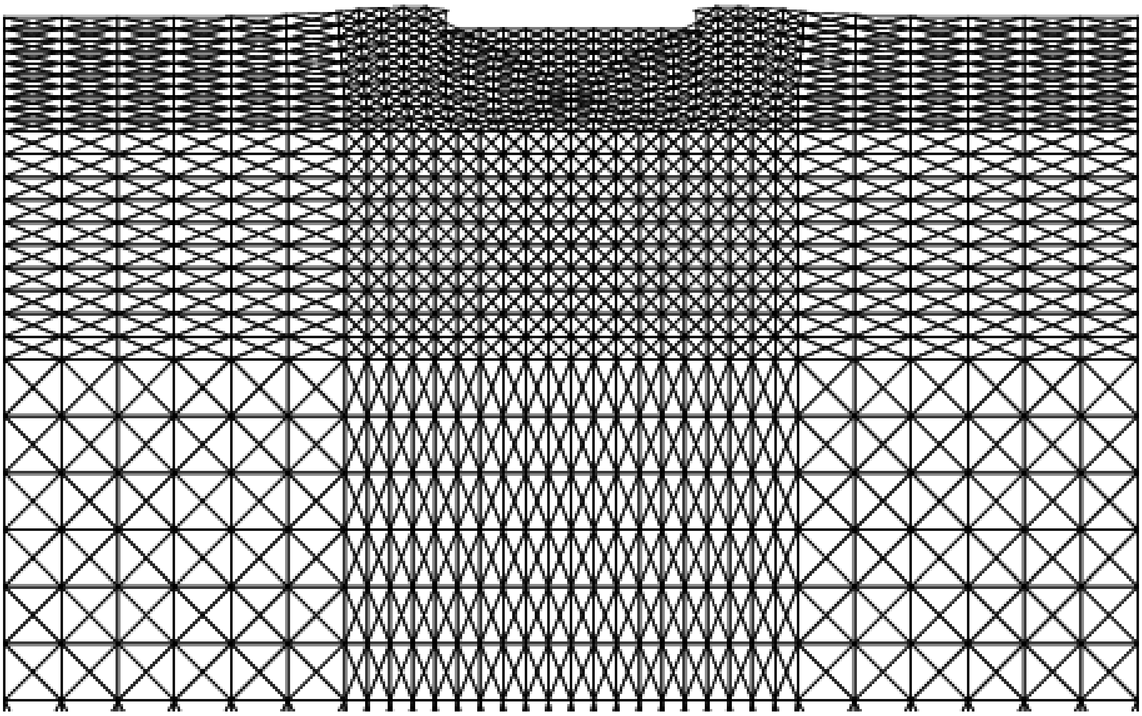

Figure B.11 Upper bound failure mechanism for COHESIVE_0.25_0.5 case (i.e. $c_{u1}/c_{u2} = 0.25$, $H/B = 0.5$).

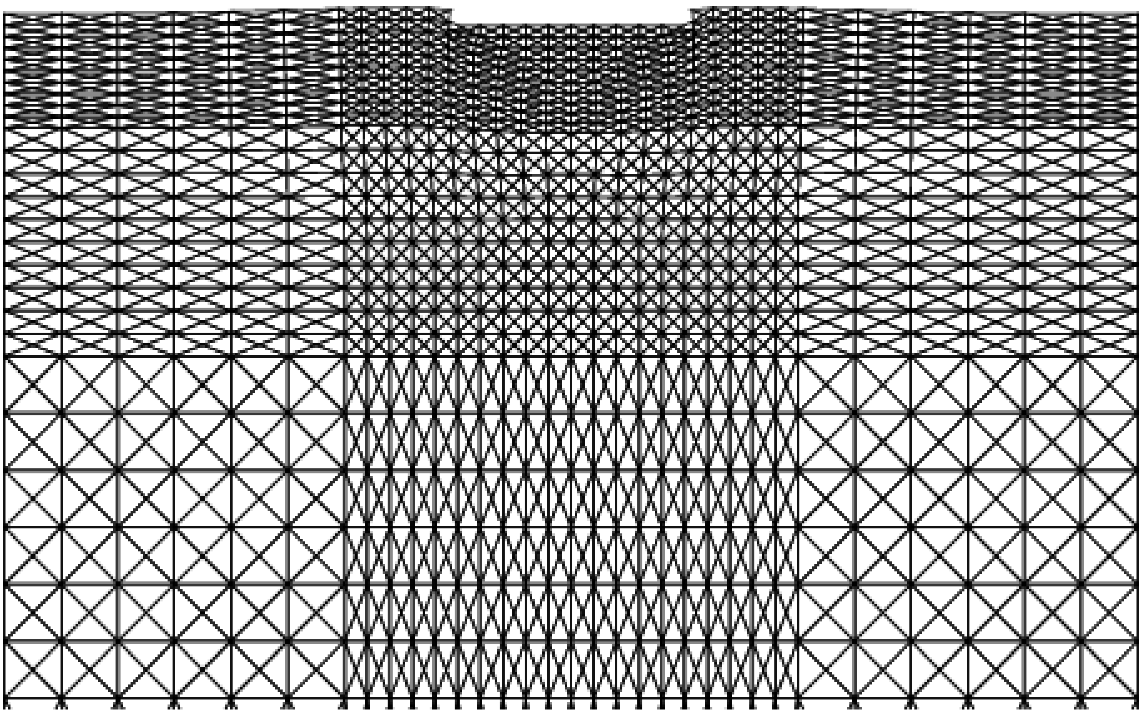

Figure B.12 Upper bound failure mechanism for COHESIVE_0.25_1.0 case (i.e. $c_{u1}/c_{u2} = 0.25$, $H/B = 1.0$).

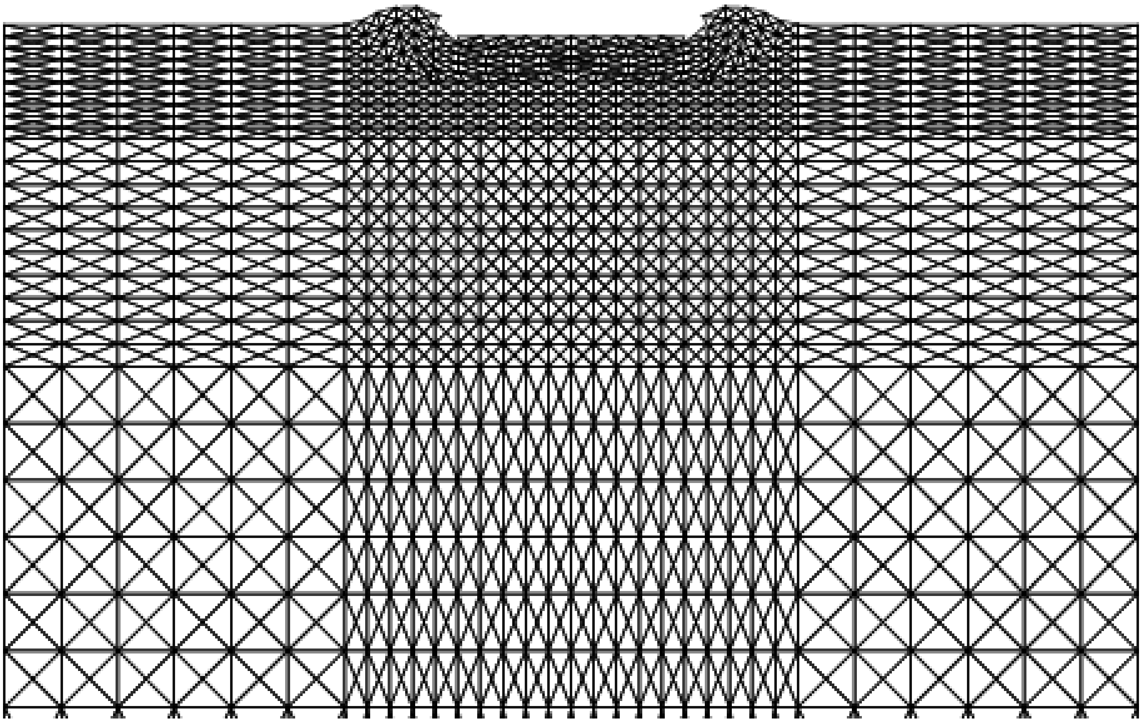

Figure B.13 Upper bound failure mechanism for COHESIVE_0.333_0.25 case (i.e. c_{u1}/c_{u2} = 0.333, H/B = 0.25).

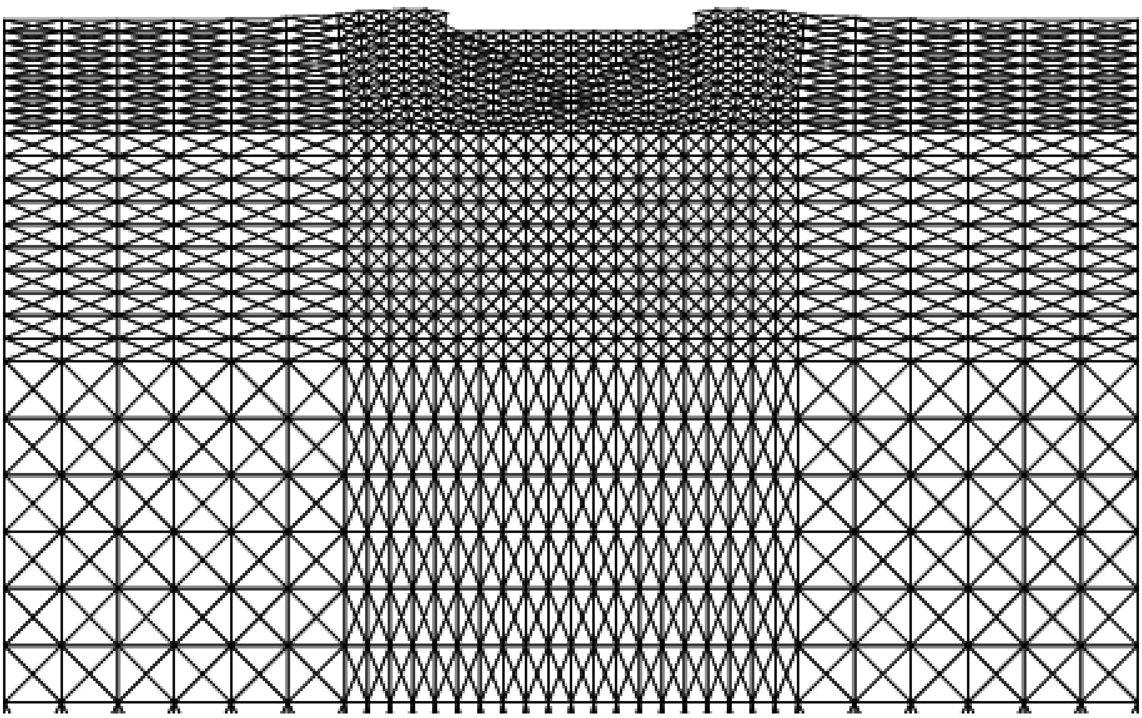

Figure B.14 Upper bound failure mechanism for COHESIVE_0.333_0.5 case (i.e. c_{u1}/c_{u2} = 0.333, H/B = 0.5).

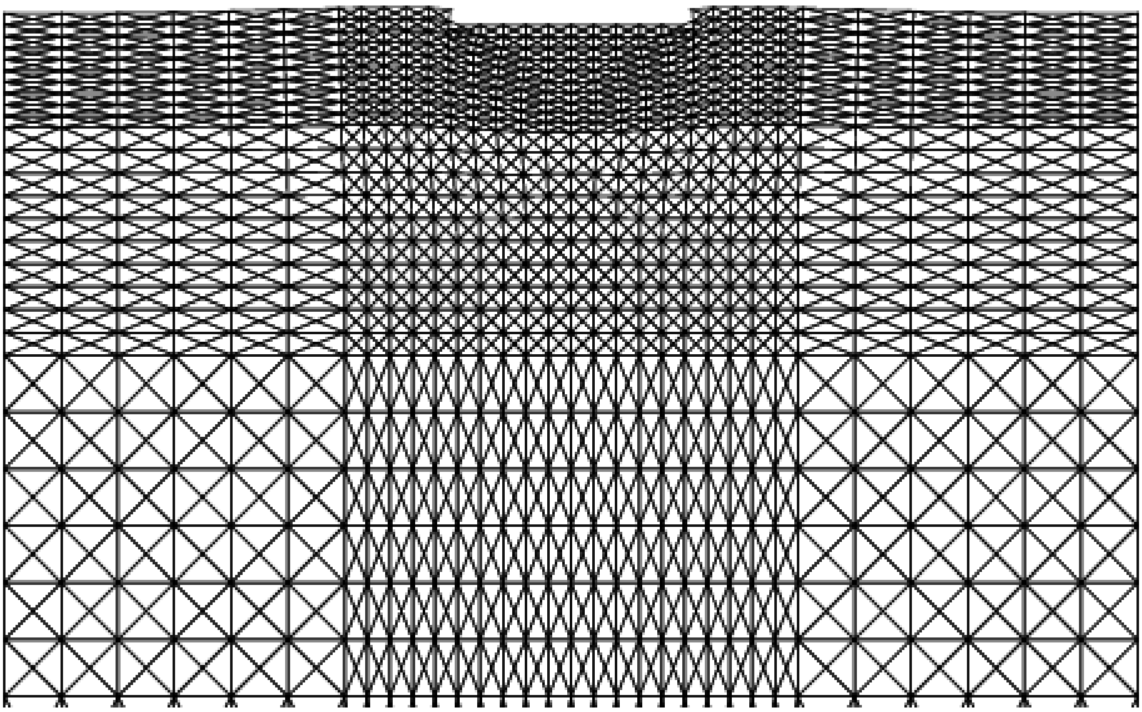

Figure B.15 Upper bound failure mechanism for COHESIVE_0.333_1.0 case (i.e. c_{u1}/c_{u2} = 0.33, H/B = 1.0).

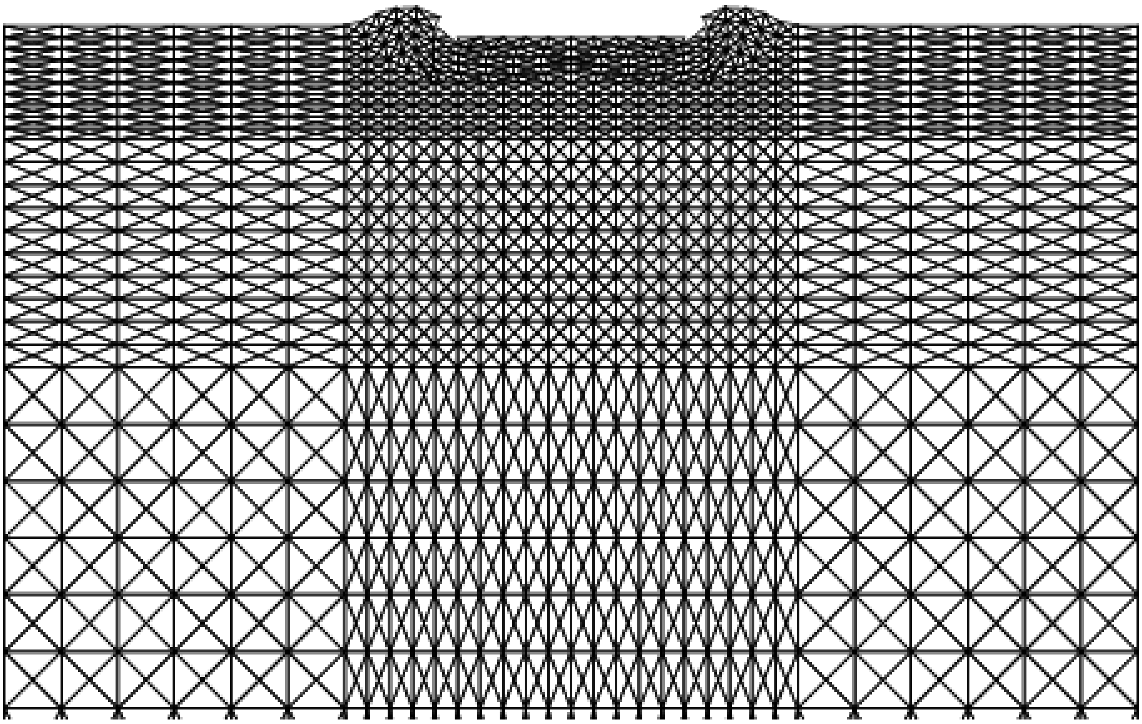

Figure B.16 Upper bound failure mechanism for COHESIVE_0.5_0.25 case (i.e. c_{u1}/c_{u2} = 0.5, H/B = 0.25).

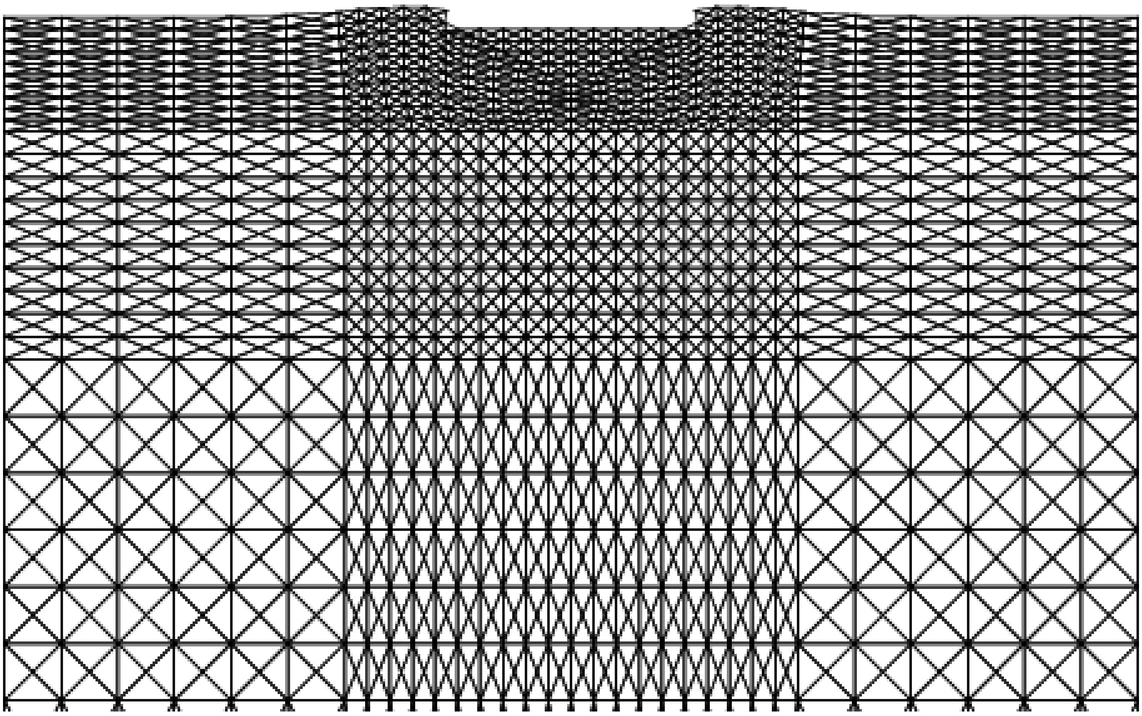

Figure B.17 Upper bound failure mechanism for COHESIVE_0.5_0.5 case (i.e. c_{u1}/c_{u2} = 0.5, H/B = 0.5).

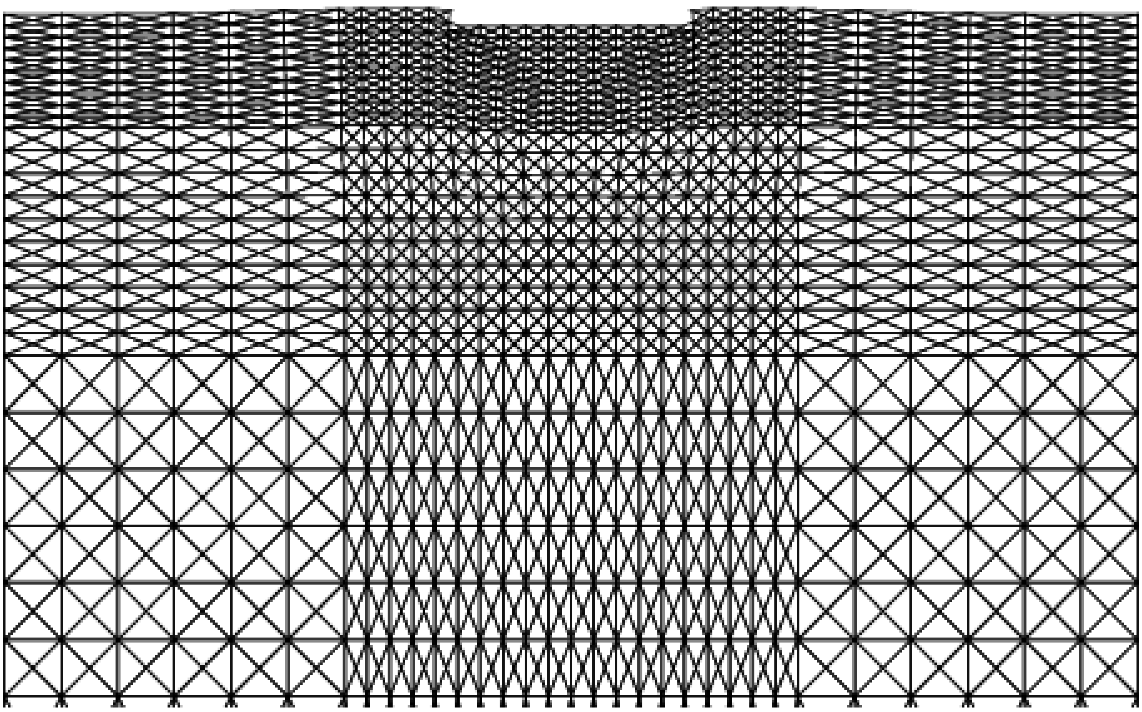

Figure B.18 Upper bound failure mechanism for COHESIVE_0.5_1.0 case (i.e. c_{u1}/c_{u2} = 0.5, H/B = 1.0).

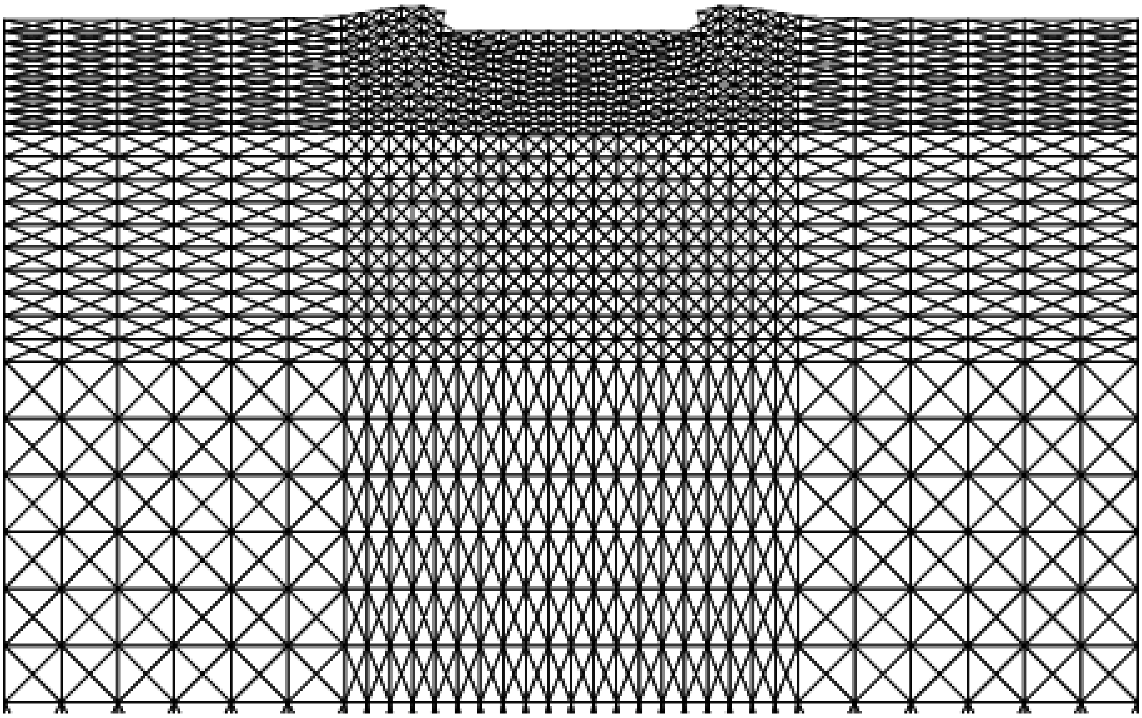

Figure B.19 Upper bound failure mechanism for COHESIVE_0.75_0.25 case (i.e. c_{u1}/c_{u2} = 0.75, H/B = 0.25).

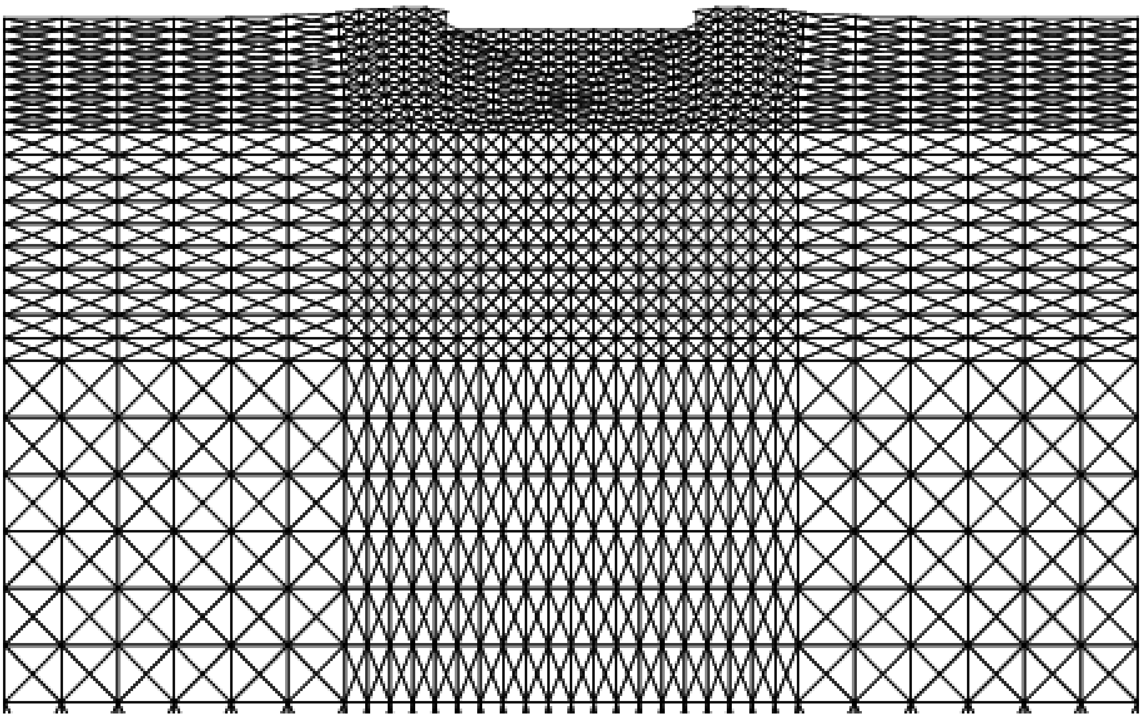

Figure B.20 Upper bound failure mechanism for COHESIVE_0.75_0.5 case (i.e. c_{u1}/c_{u2} = 0.75, H/B = 0.5).

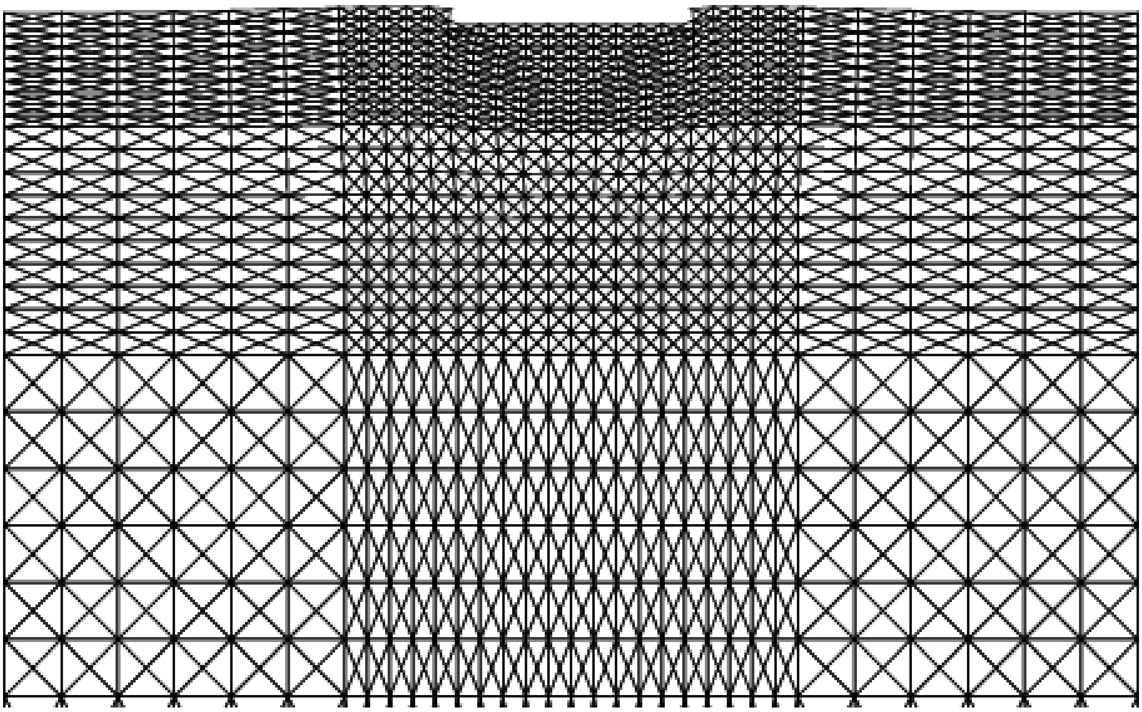

Figure B.21 Upper bound failure mechanism for COHESIVE_0.75_1.0 case (i.e. c_{u1}/c_{u2} = 0.75, H/B = 1.0).

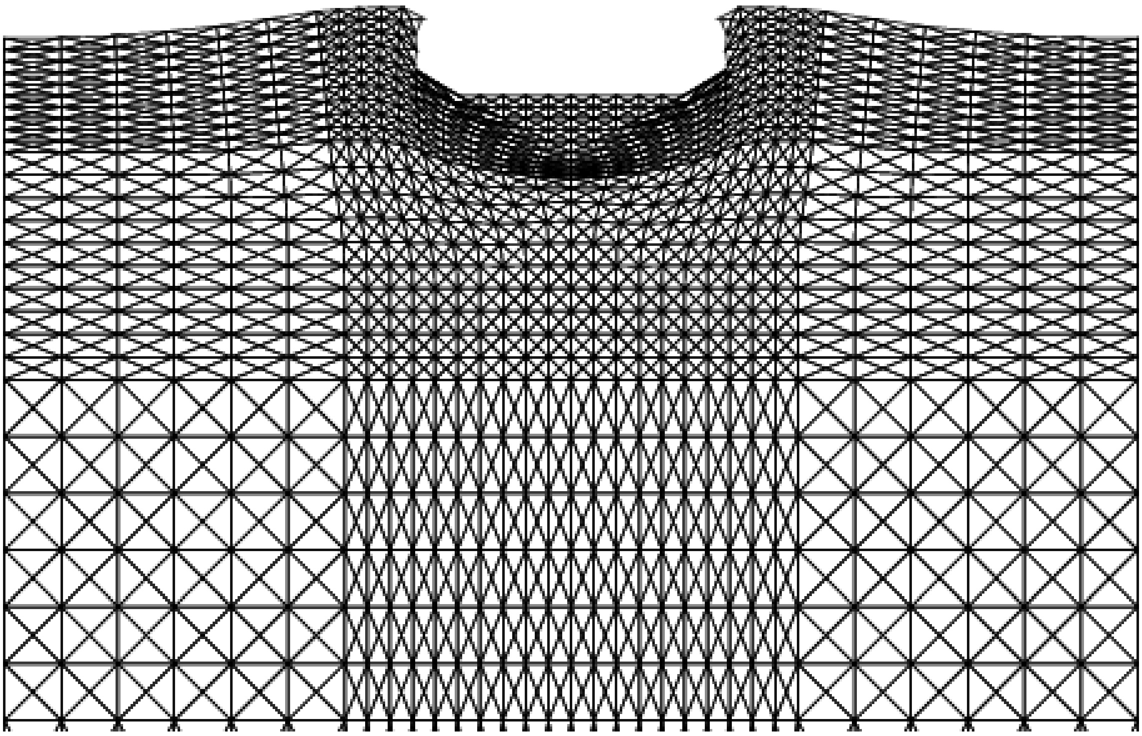

Figure B.22 Upper bound failure mechanism for single-layered deterministic homogeneous case (i.e. c_{u1}/c_{u2} = 1.0)

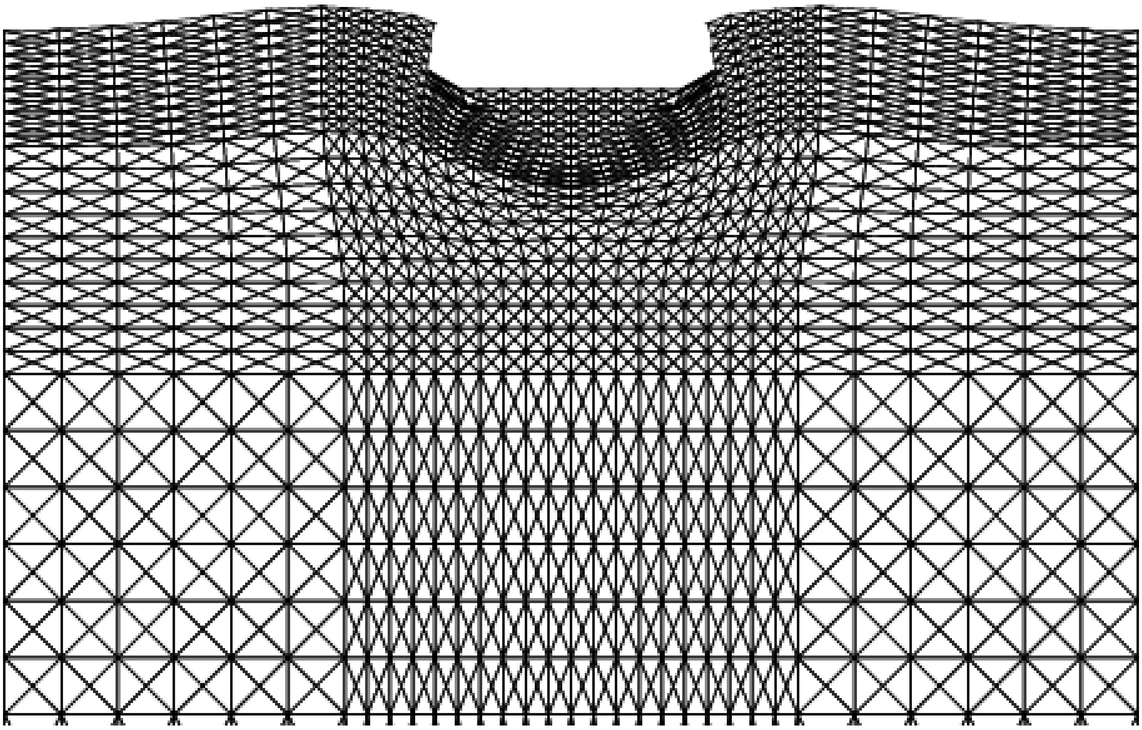

Figure B.23 Upper bound failure mechanism for COHESIVE_1.333_0.25 case (i.e. c_{u1}/c_{u2} = 1.33, H/B = 0.25).

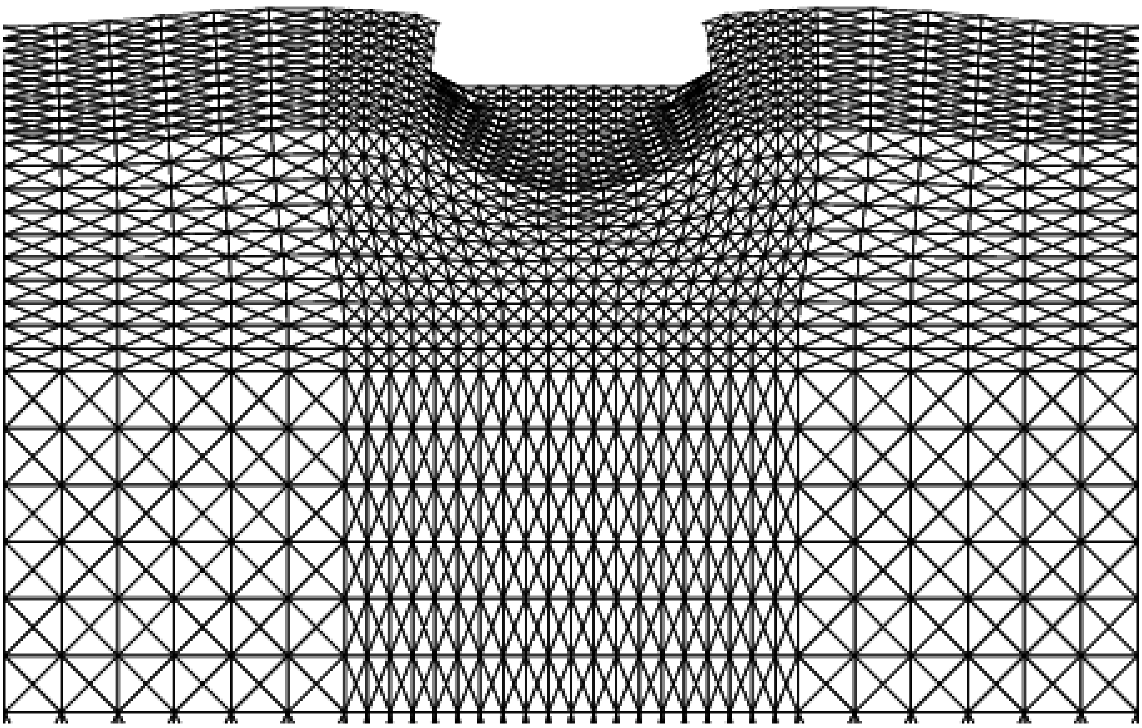

Figure B.24 Upper bound failure mechanism for COHESIVE_1.333_0.5 case (i.e. c_{u1}/c_{u2} = 1.33, H/B = 0.5).

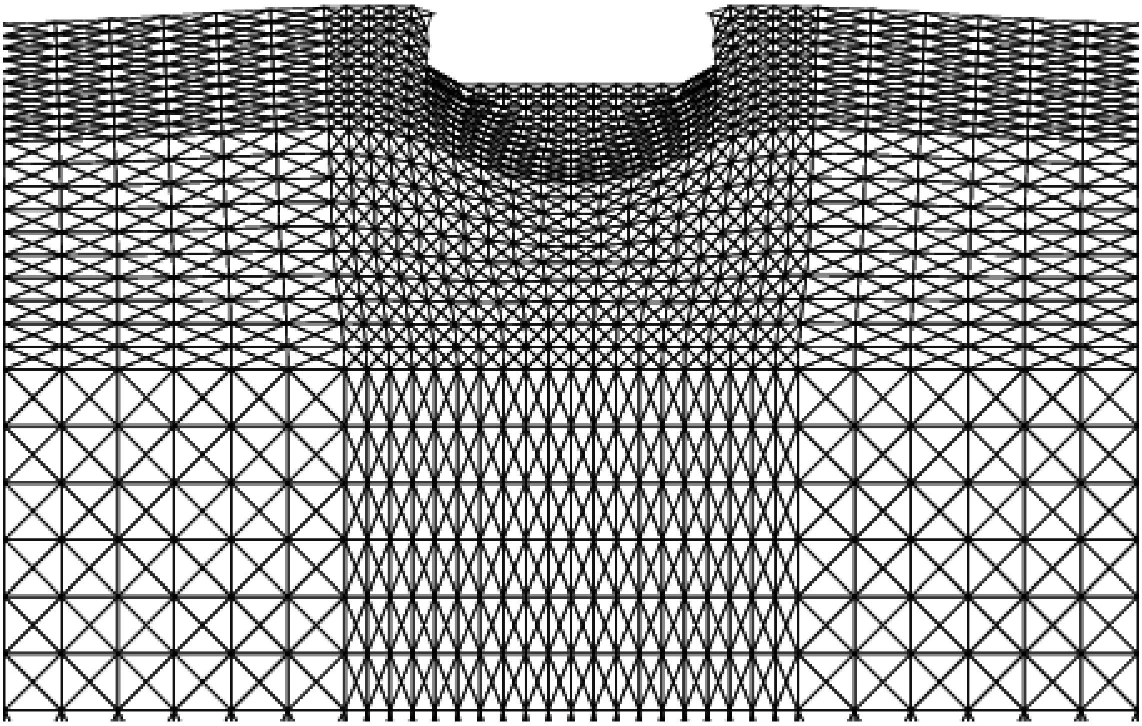

Figure B.25 Upper bound failure mechanism for COHESIVE_1.333_1.0 case (i.e. c_{u1}/c_{u2} = 1.33, H/B = 1.0).

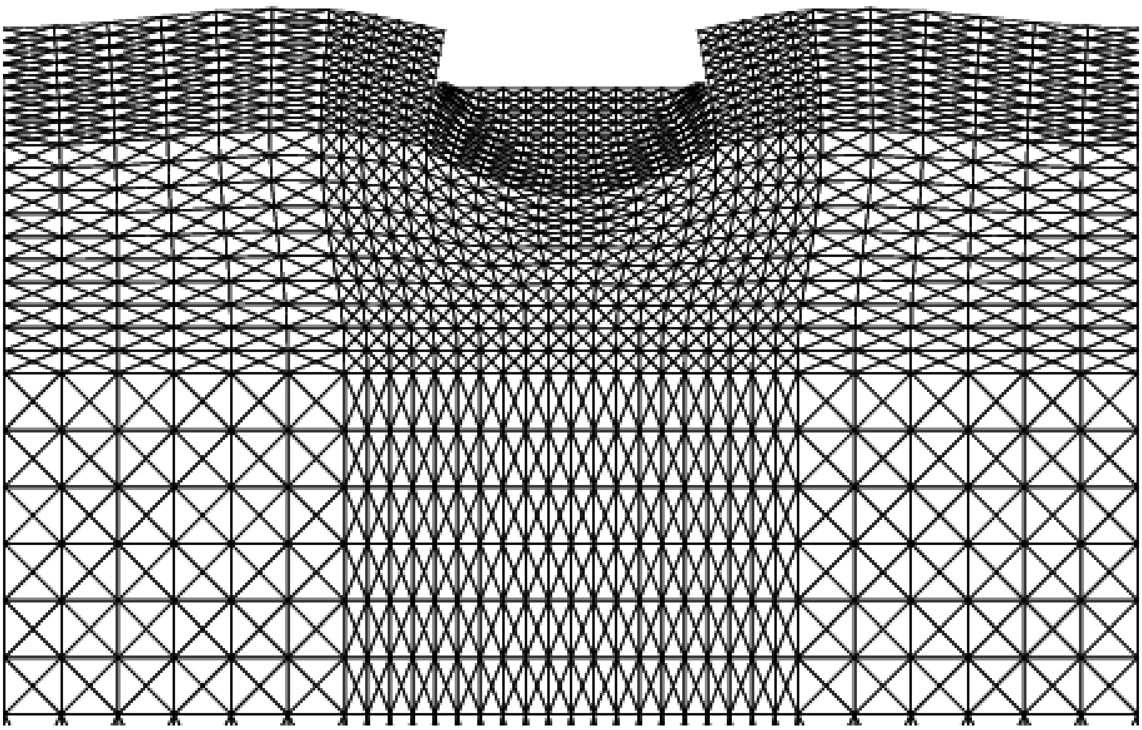

Figure B.26 Upper bound failure mechanism for COHESIVE_2.0_0.25 case (i.e. c_{u1}/c_{u2} = 2.0, H/B = 0.25).

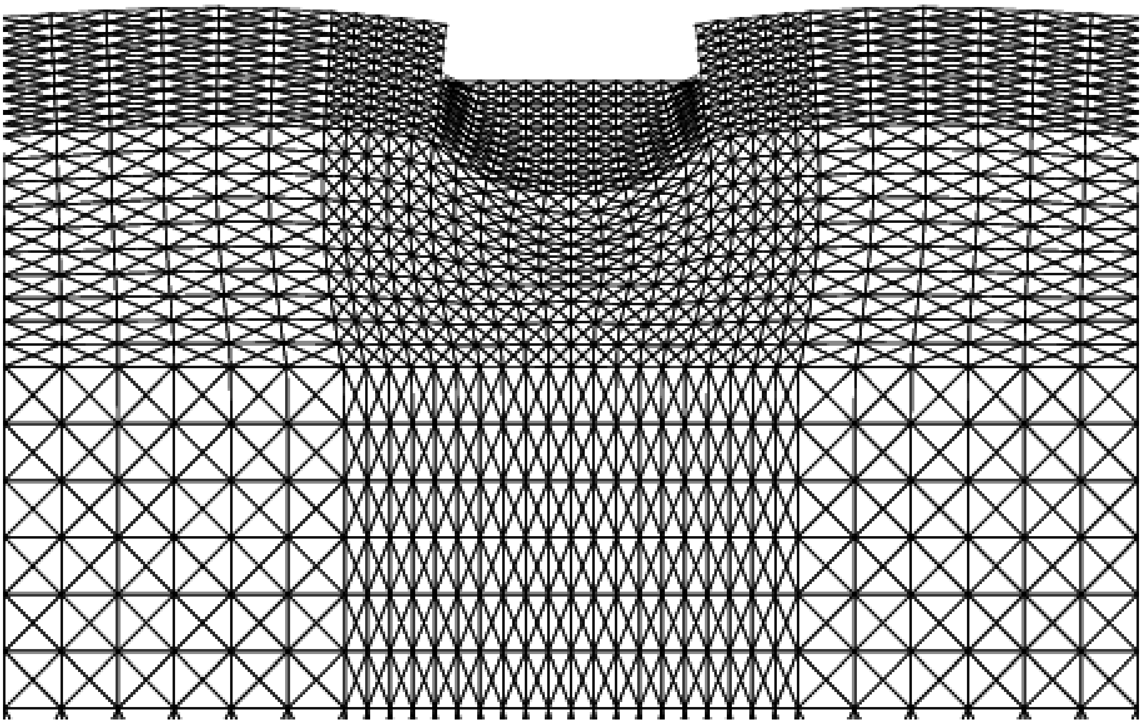

Figure B.27 Upper bound failure mechanism for COHESIVE_2.0_0.5 case (i.e. c_{u1}/c_{u2} = 2.0, H/B = 0.5).

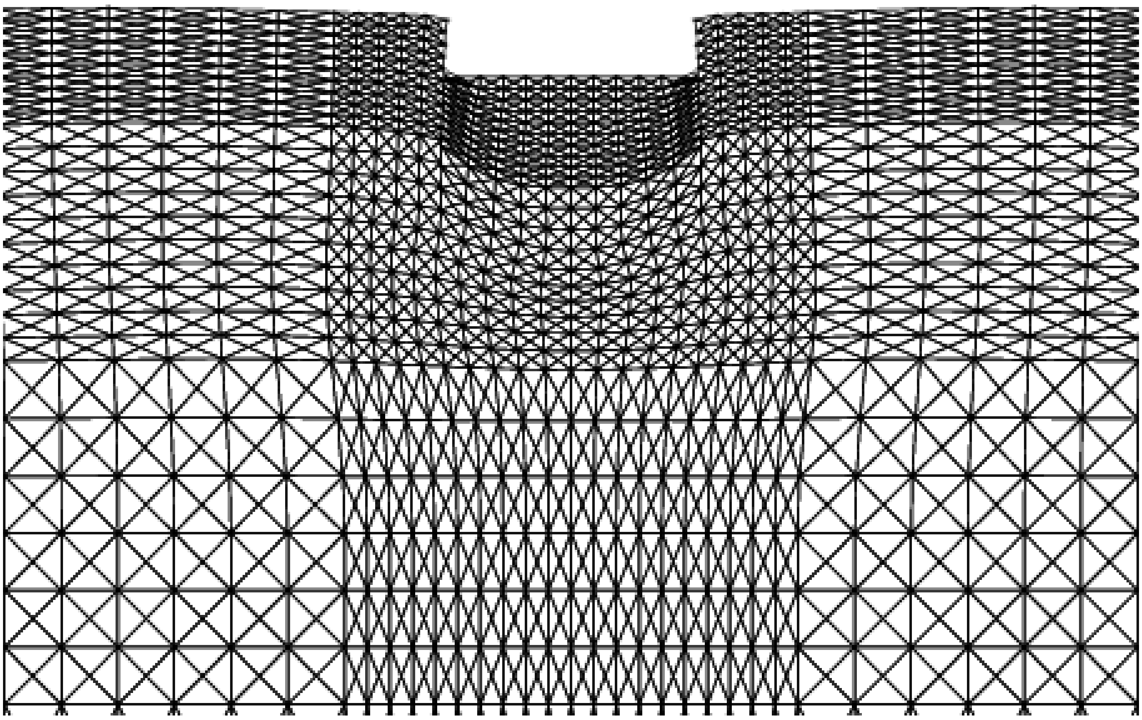

Figure B.28 Upper bound failure mechanism for COHESIVE_2.0_1.0 case (i.e. c_{u1}/c_{u2} = 2.0, H/B = 1.0).

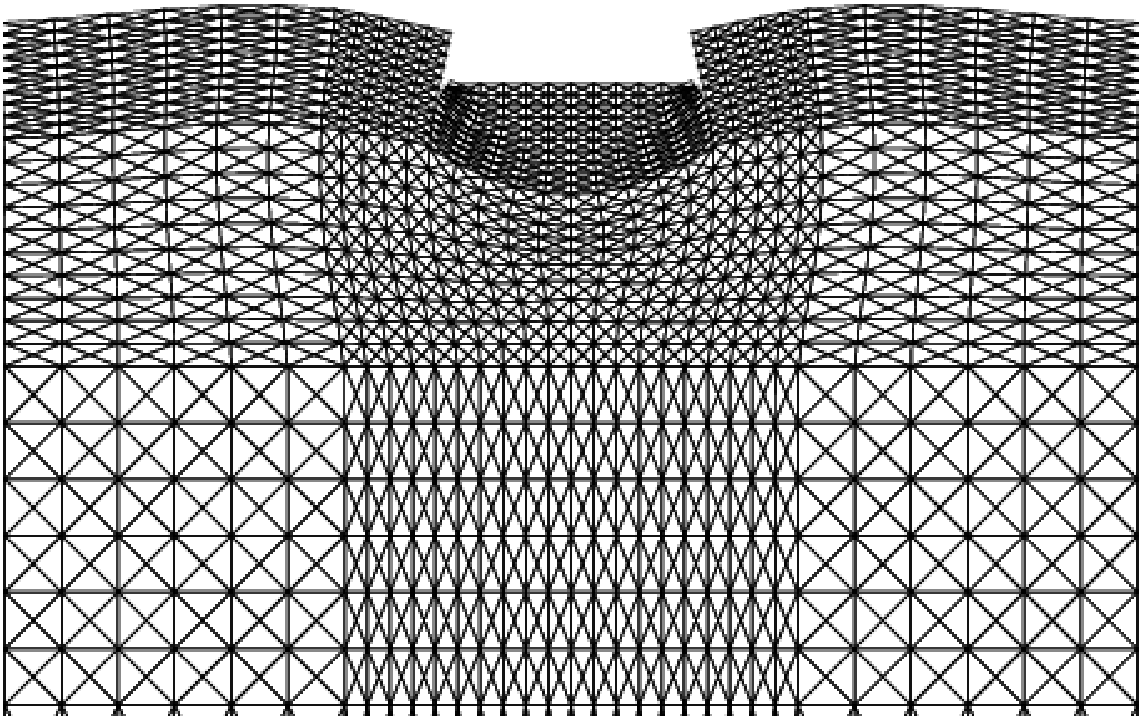

Figure B.29 Upper bound failure mechanism for COHESIVE_3.0_0.25 case (i.e. c_{u1}/c_{u2} = 3.0, H/B = 0.25).

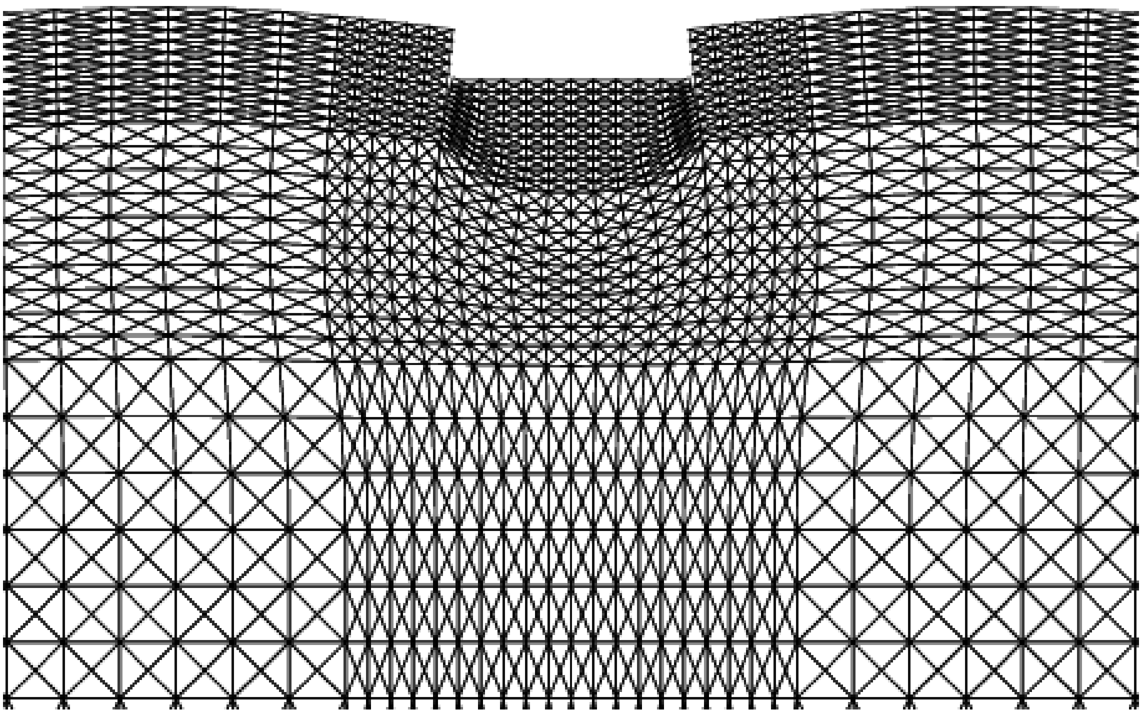

Figure B.30 Upper bound failure mechanism for COHESIVE_3.0_0.5 case (i.e. c_{u1}/c_{u2} = 3.0, H/B = 0.5).

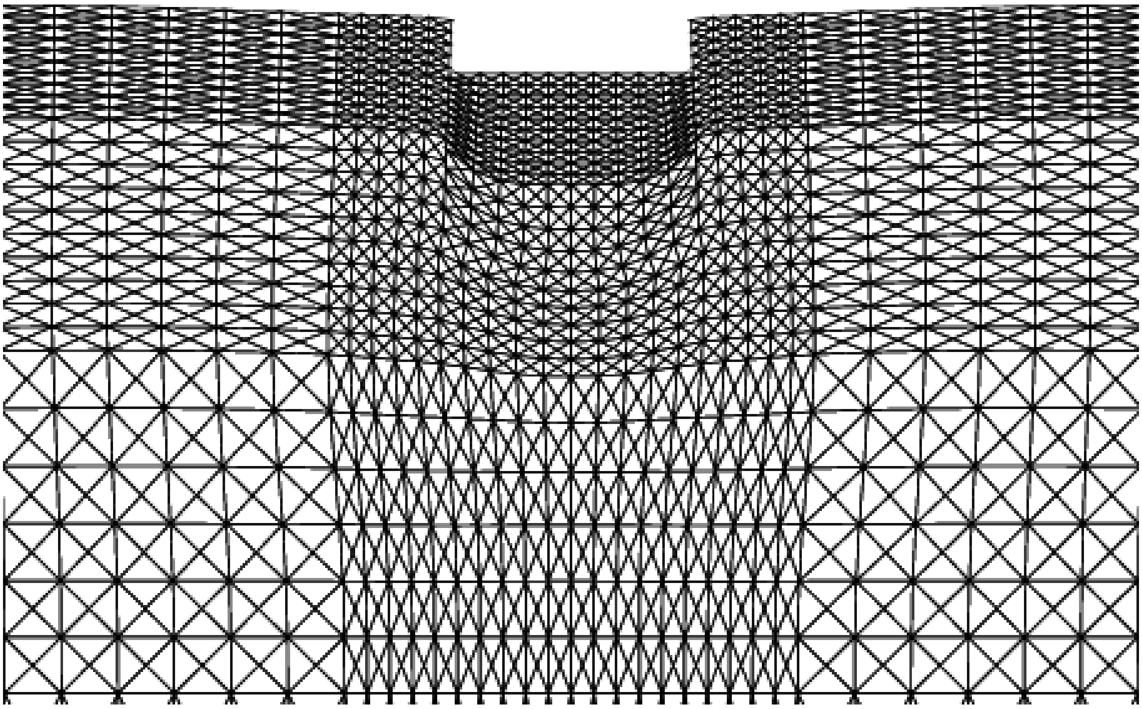

Figure B.31 Upper bound failure mechanism for COHESIVE_3.0_1.0 case (i.e. c_{u1}/c_{u2} = 3.0, H/B = 1.0).

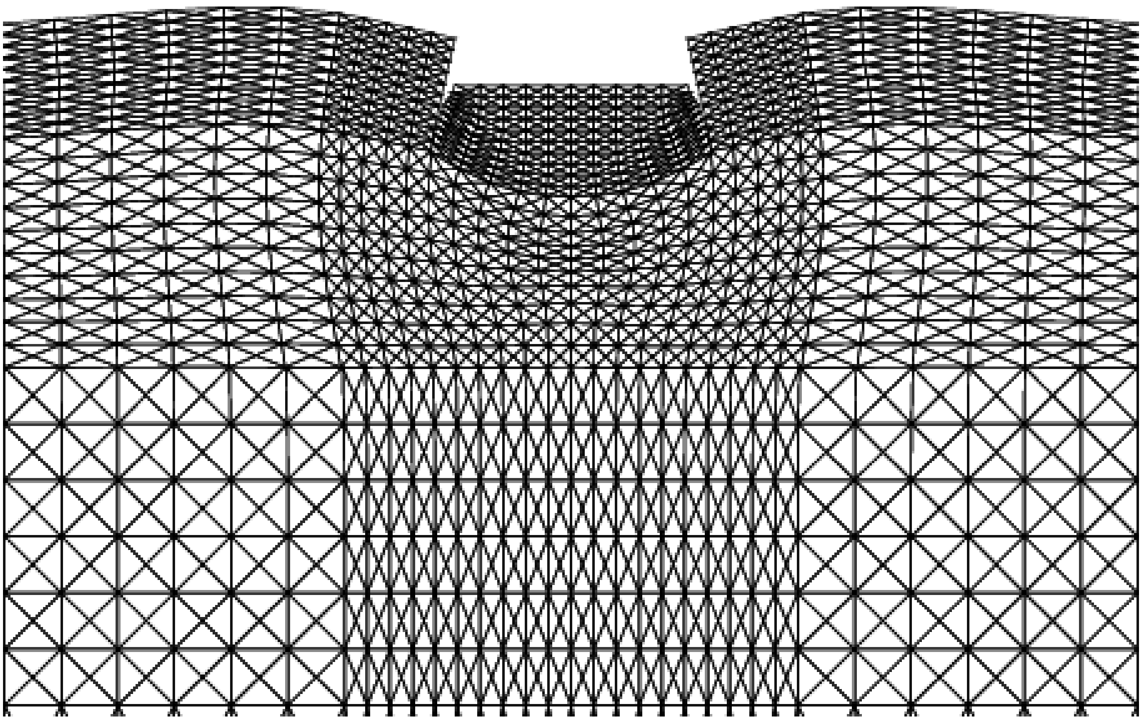

Figure B.32 Upper bound failure mechanism for COHESIVE_4.0_0.25 case (i.e. c_{u1}/c_{u2} = 4.0, H/B = 0.25).

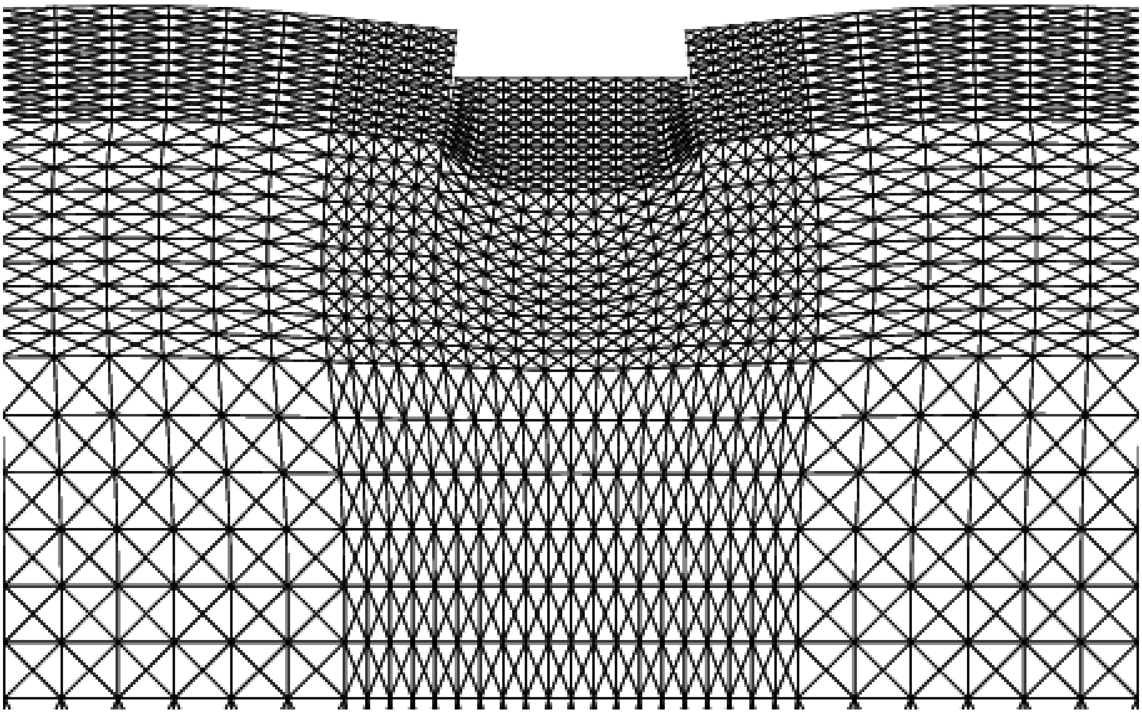

Figure B.33 Upper bound failure mechanism for COHESIVE_4.0_0.5 case (i.e. c_{u1}/c_{u2} = 4.0, H/B = 0.5).

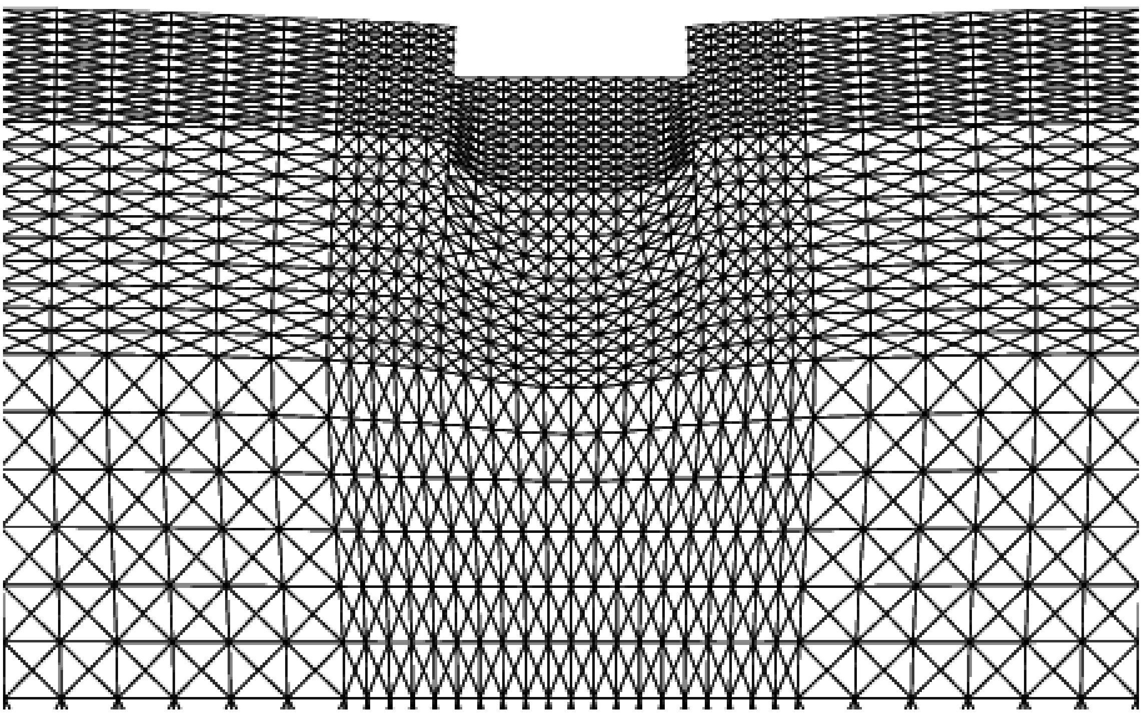

Figure B.34 Upper bound failure mechanism for COHESIVE_4.0_1.0 case (i.e. c_{u1}/c_{u2} = 4.0, H/B = 1.0).

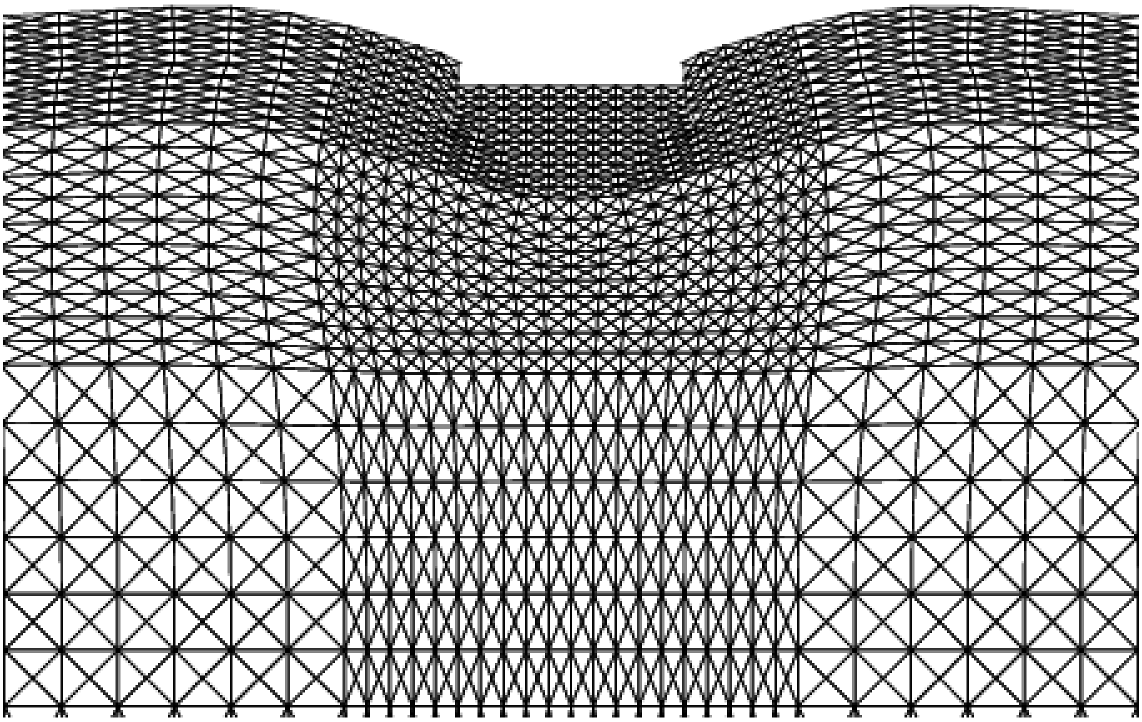

Figure B.35 Upper bound failure mechanism for COHESIVE_10.0_0.25 case (i.e. c_{u1}/c_{u2} = 10.0, H/B = 0.25).

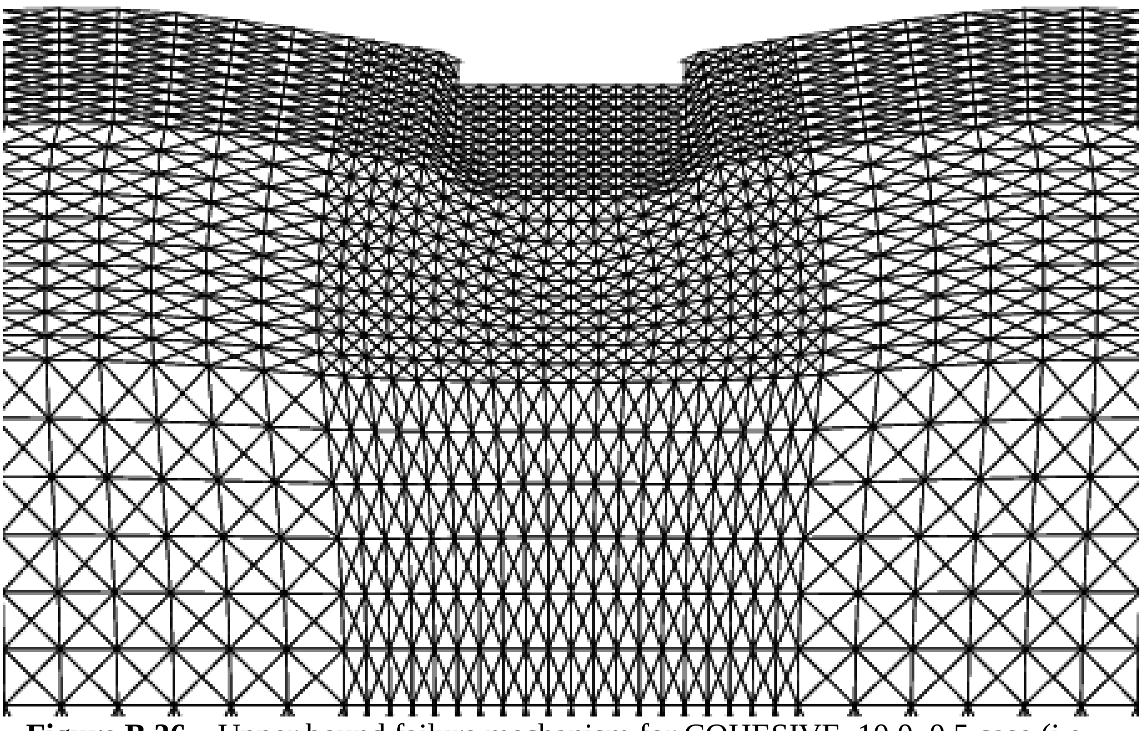

Figure B.36 Upper bound failure mechanism for COHESIVE_10.0_0.5 case (i.e. c_{u1}/c_{u2} = 10.0, H/B = 0.5).

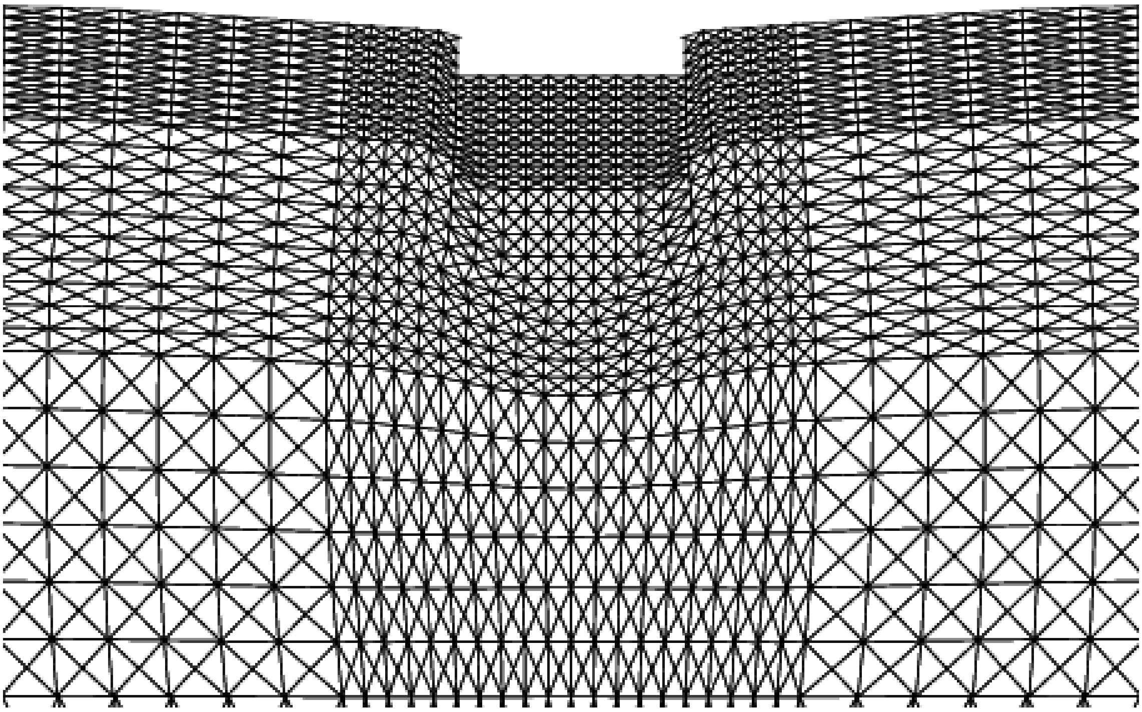

Figure B.37 Upper bound failure mechanism for COHESIVE_10.0_1.0 case (i.e. c_{u1}/c_{u2} = 10.0, H/B = 1.0).

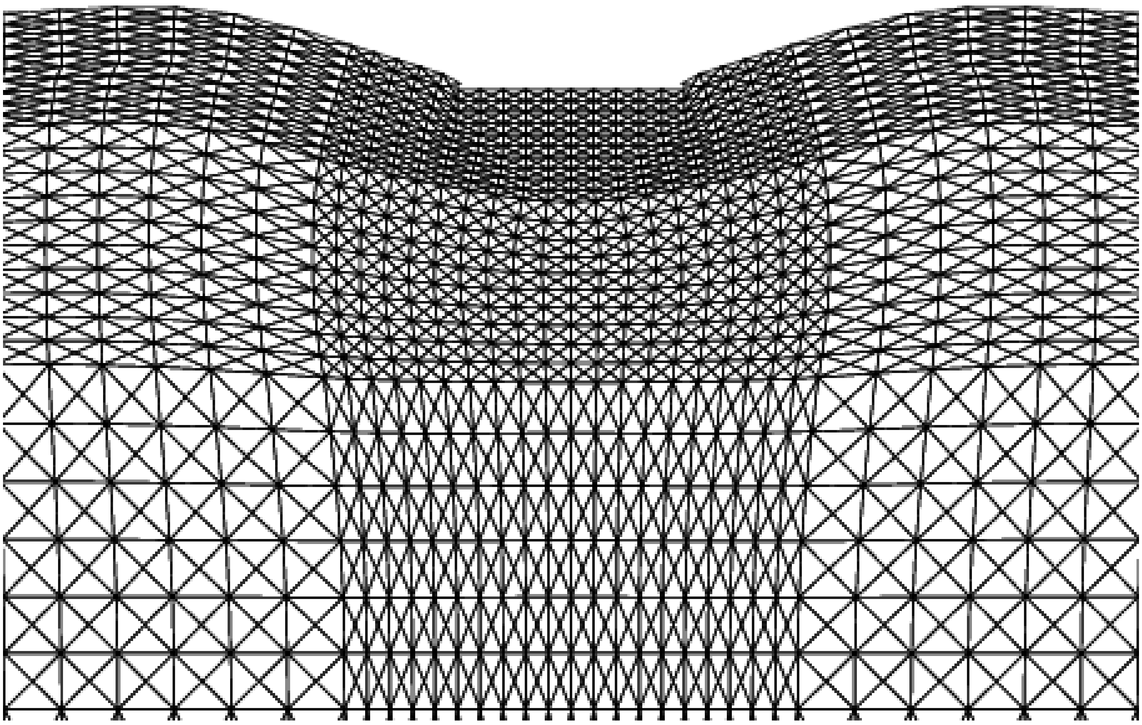

Figure B.38 Upper bound failure mechanism for COHESIVE_20.0_0.25 case (i.e. c_{u1}/c_{u2} = 20.0, H/B = 0.25).

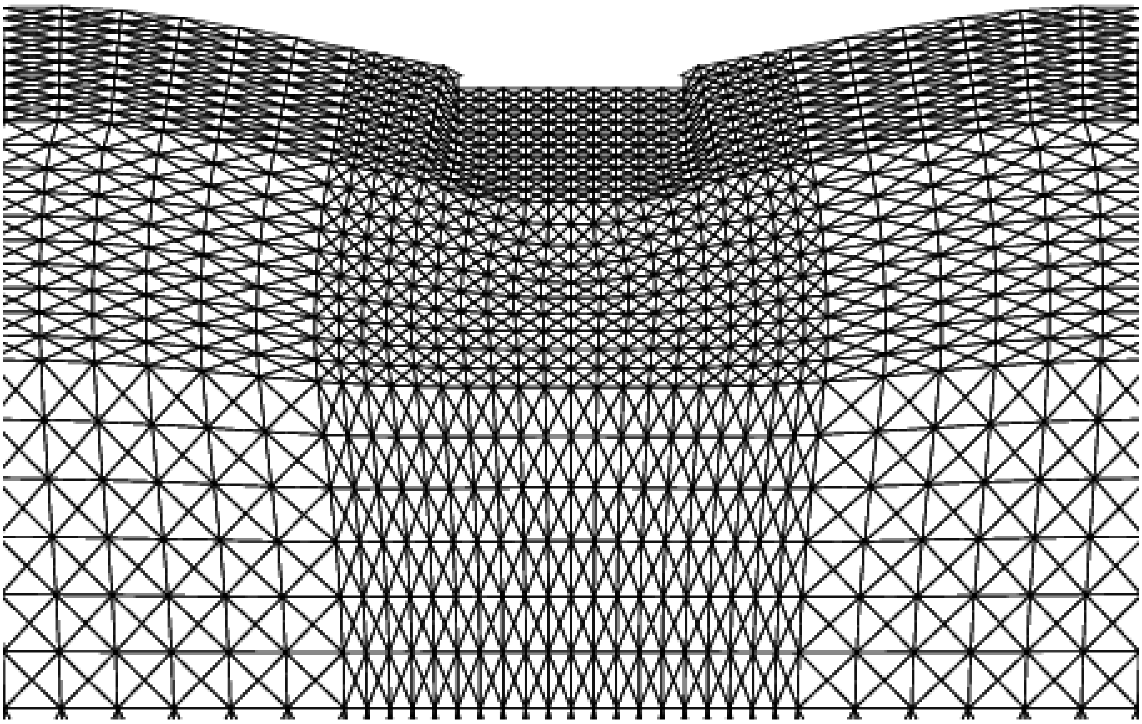

Figure B.39 Upper bound failure mechanism for COHESIVE_20.0_0.5 case (i.e. c_{u1}/c_{u2} = 20.0, H/B = 0.5).

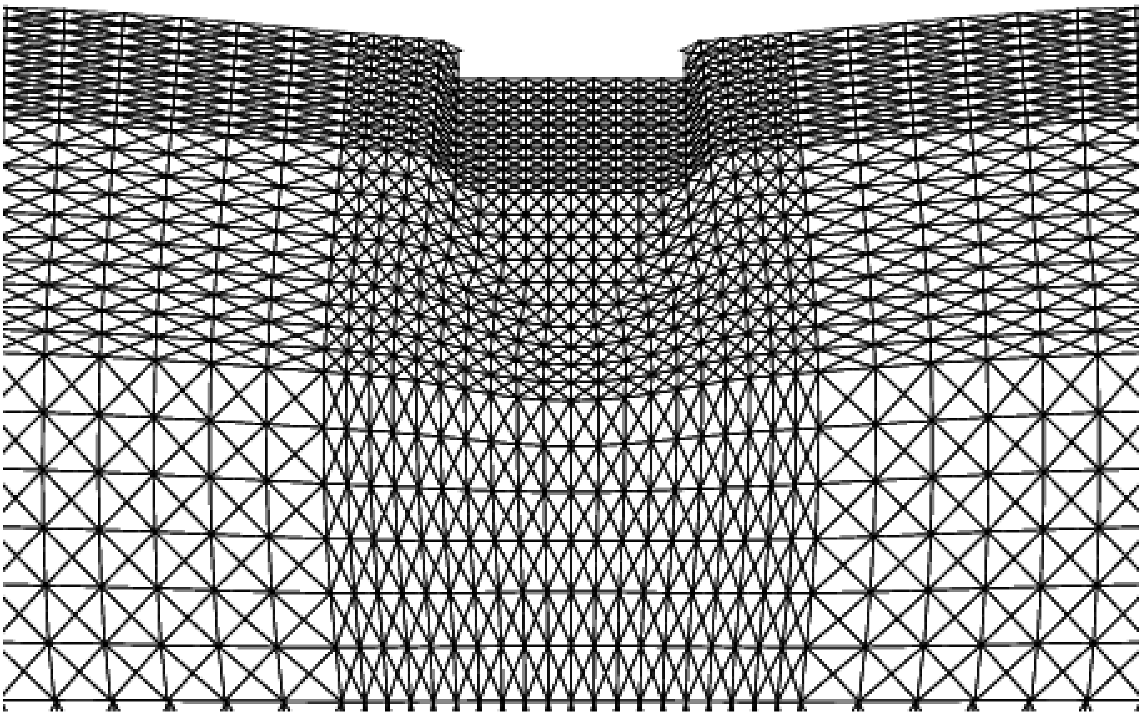

Figure B.40 Upper bound failure mechanism for COHESIVE_20.0_1.0 case (i.e. c_{u1}/c_{u2} = 20.0, H/B = 1.0).

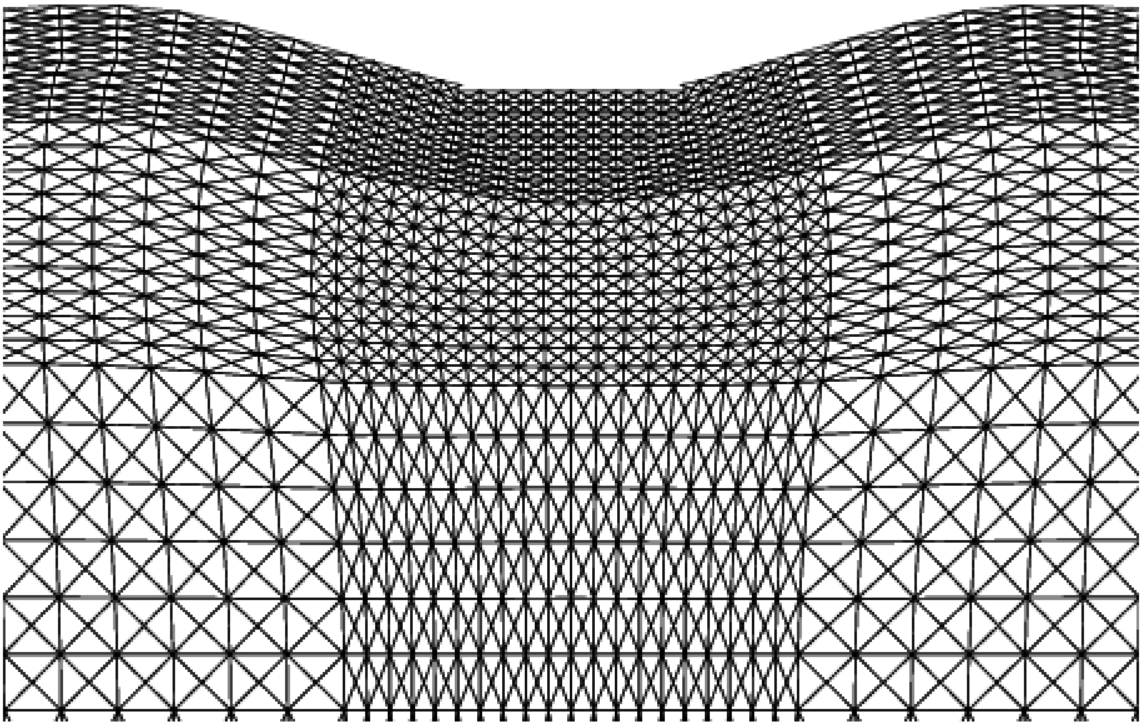

Figure B.41 Upper bound failure mechanism for COHESIVE_40.0_0.25 case (i.e. c_{u1}/c_{u2} = 40.0, H/B = 0.25).

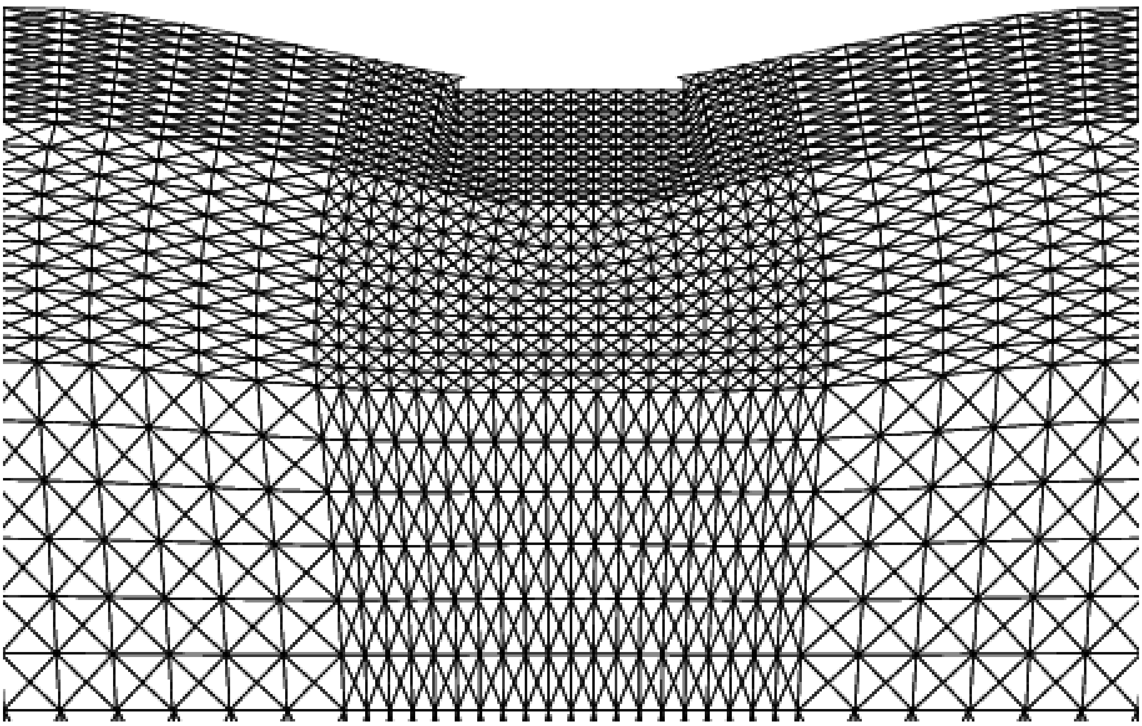

Figure B.42 Upper bound failure mechanism for COHESIVE_40.0_0.5 case (i.e. c_{u1}/c_{u2} = 40.0, H/B = 0.5).

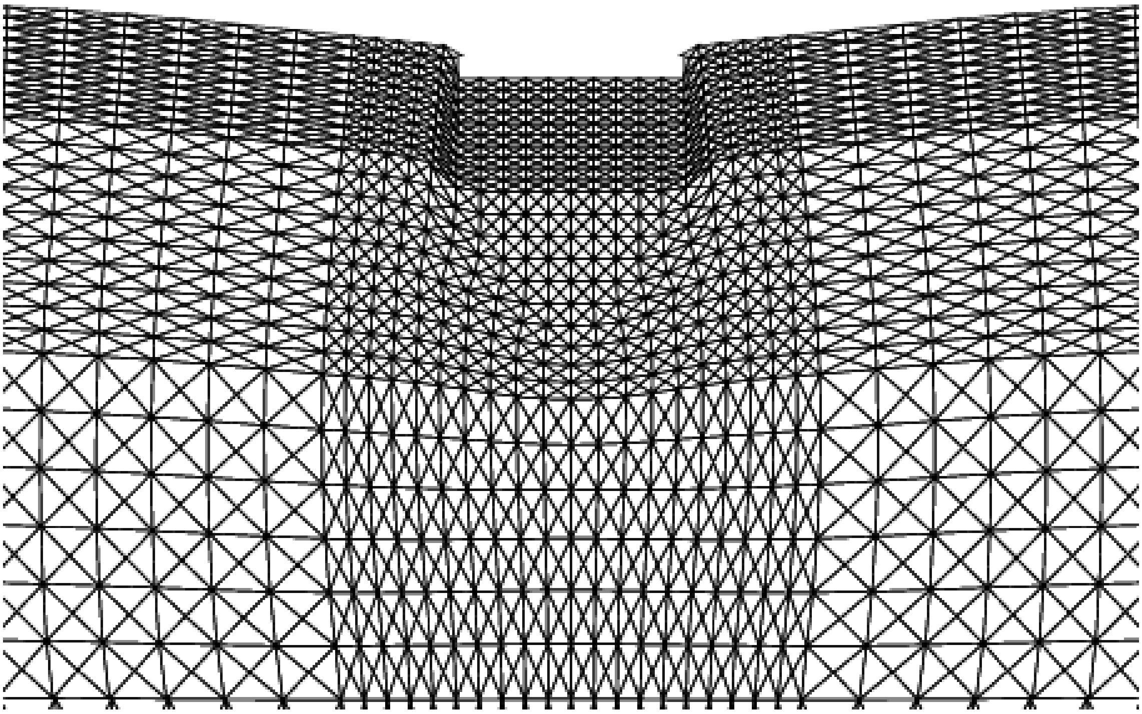

Figure B.43 Upper bound failure mechanism for COHESIVE_40.0_1.0 case (i.e. c_{u1}/c_{u2} = 40.0, H/B = 1.0).

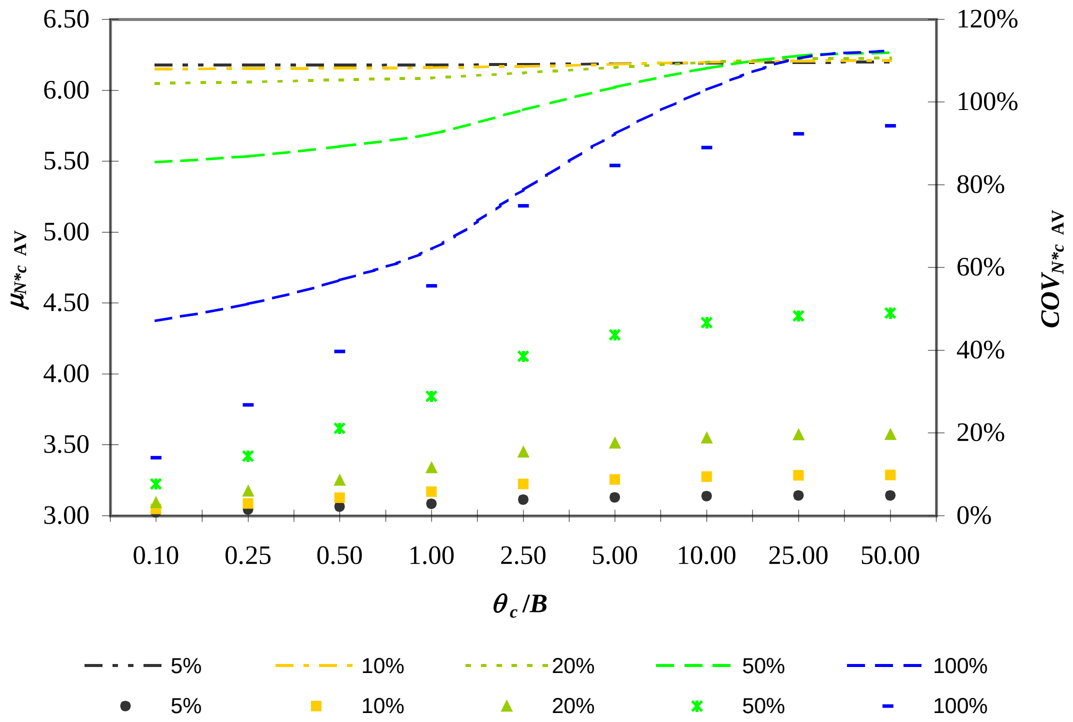

Figure B.44 The variation of $\mu_{N^*c\ AV}$ and $COV_{N^*c\ AV}$ with respect to COV_c and θ_c/B for COHESIVE_0.025_0.25 case (where μ_{c1} / μ_{c2} = 0.025 and H/B = 0.25).

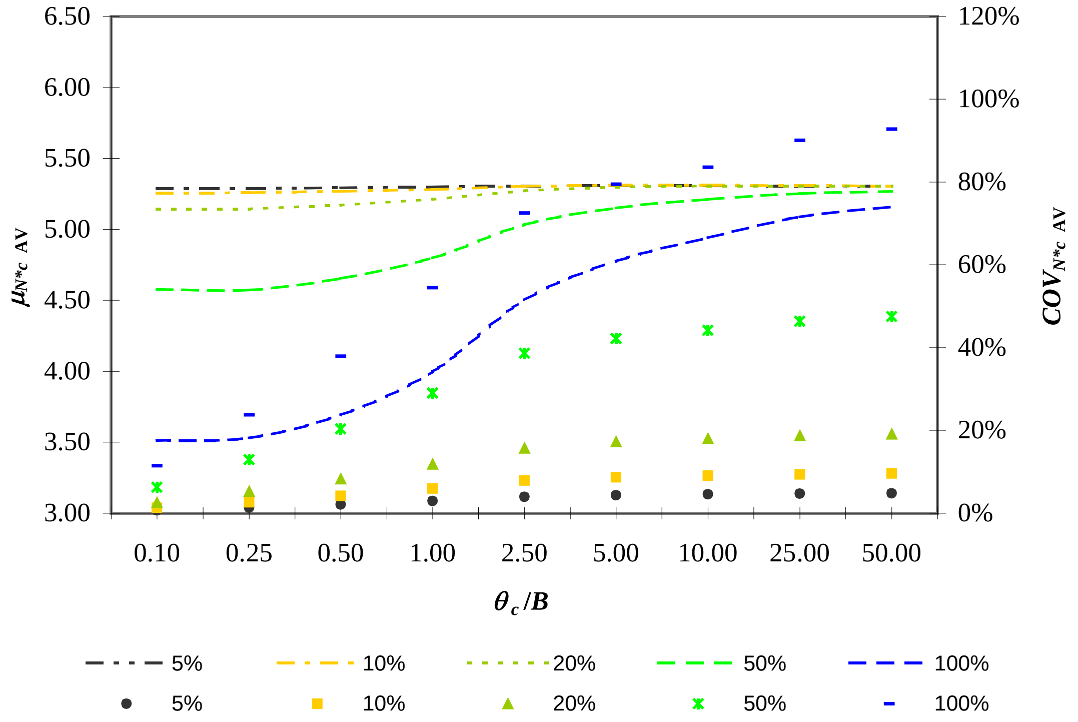

Figure B.45 The variation of $\mu_{N^{*}c\ \mathrm{AV}}$ and $COV_{N^{*}c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.025_0.5 case (where μ_{c1} / μ_{c2} = 0.025 and H/B = 0.5).

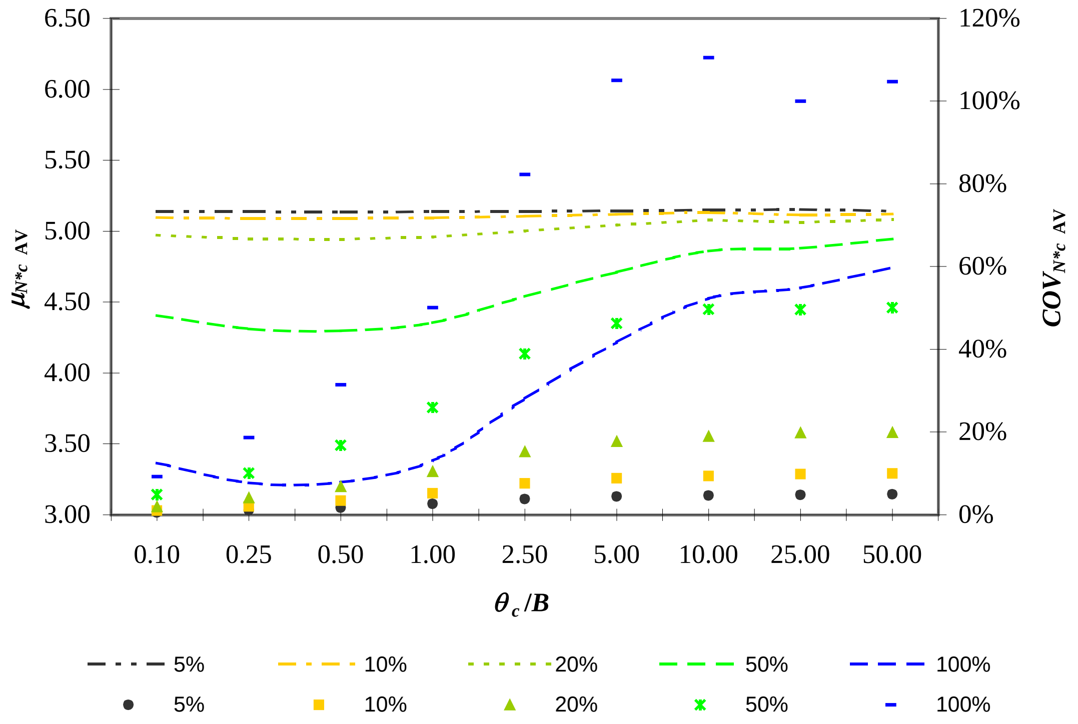

Figure B.46 The variation of $\mu_{N^{*}c\ \mathrm{AV}}$ and $COV_{N^{*}c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.025_1.0 case (where μ_{c1} / μ_{c2} = 0.025 and H/B = 1.0).

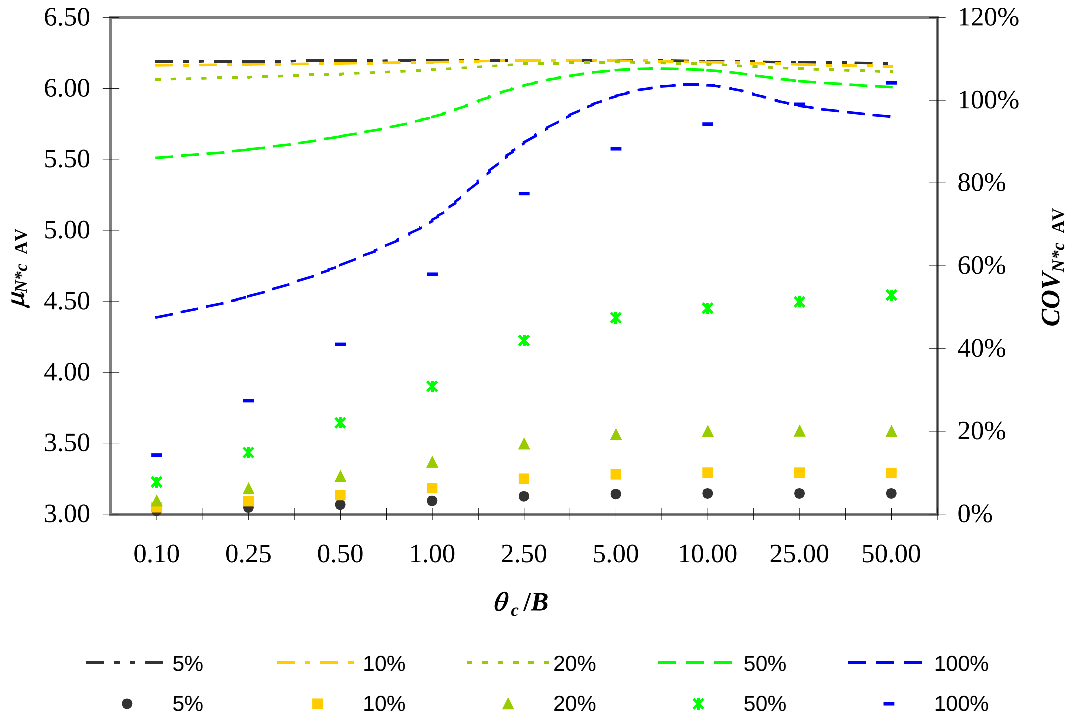

Figure B.47 The variation of $\mu_{N^*c\ \mathrm{AV}}$ and $COV_{N^*c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.05_0.25 case (where μ_{c1} / μ_{c2} = 0.05 and H/B = 0.25).

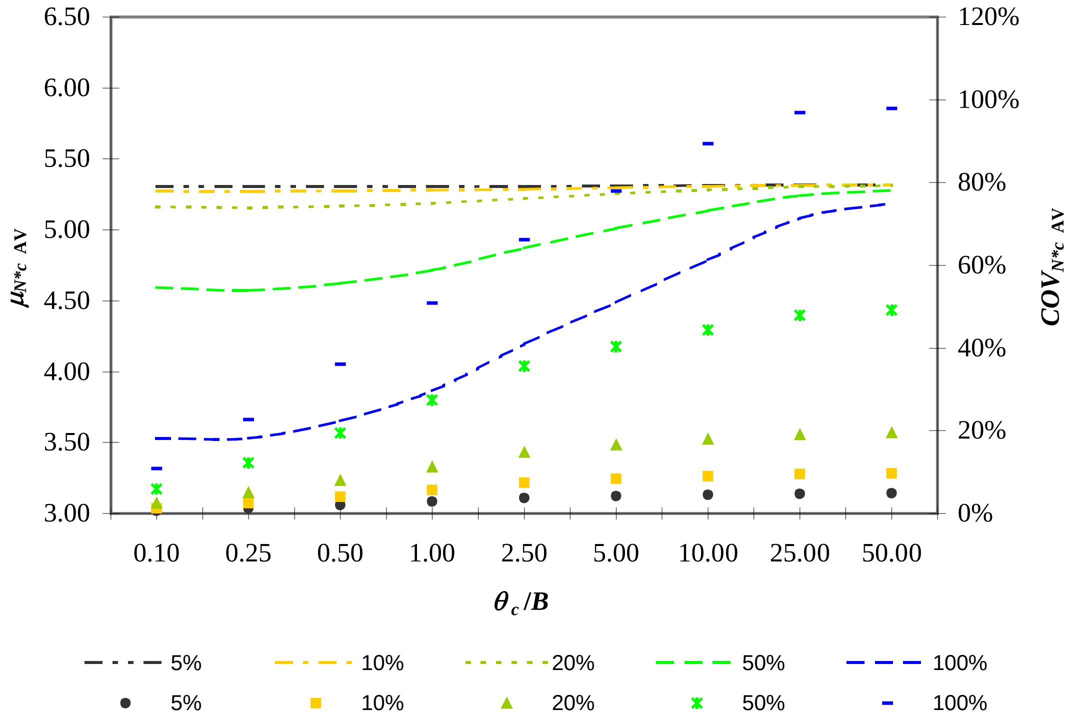

Figure B.48 The variation of $\mu_{N^*c\ \mathrm{AV}}$ and $COV_{N^*c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.05_0.5 case (where μ_{c1} / μ_{c2} = 0.05 and H/B = 0.5).

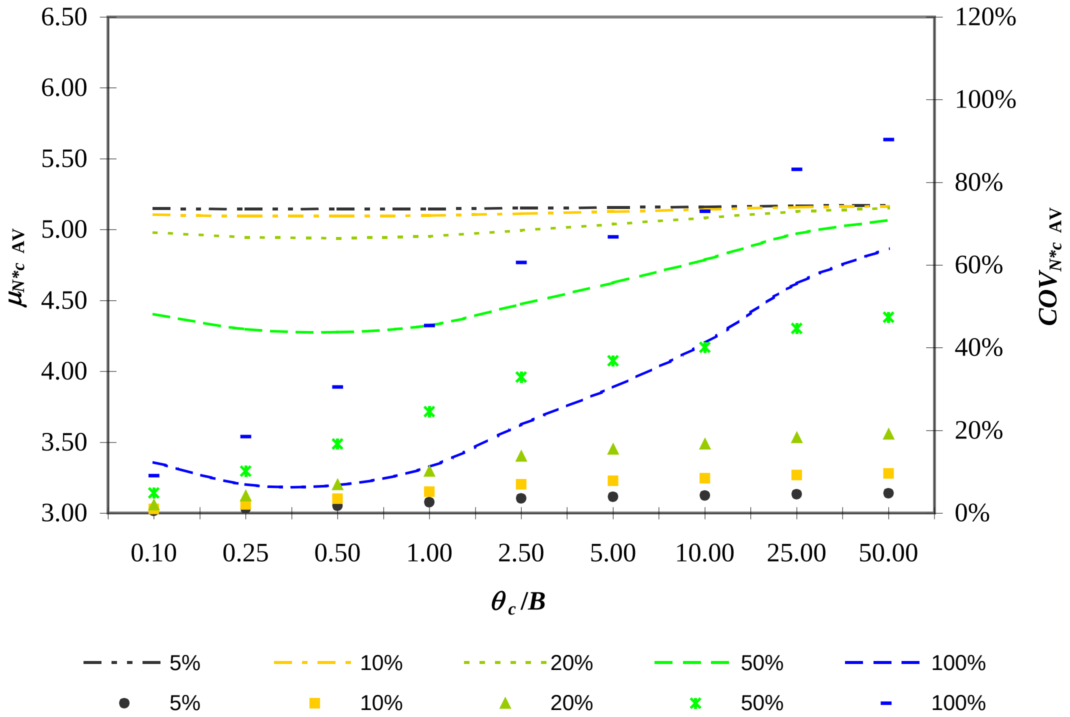

Figure B.49 The variation of $\mu_{N^*c\ \mathrm{AV}}$ and $COV_{N^*c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.05_1.0 case (where $\mu_{c1} / \mu_{c2} = 0.05$ and $H/B = 1.0$).

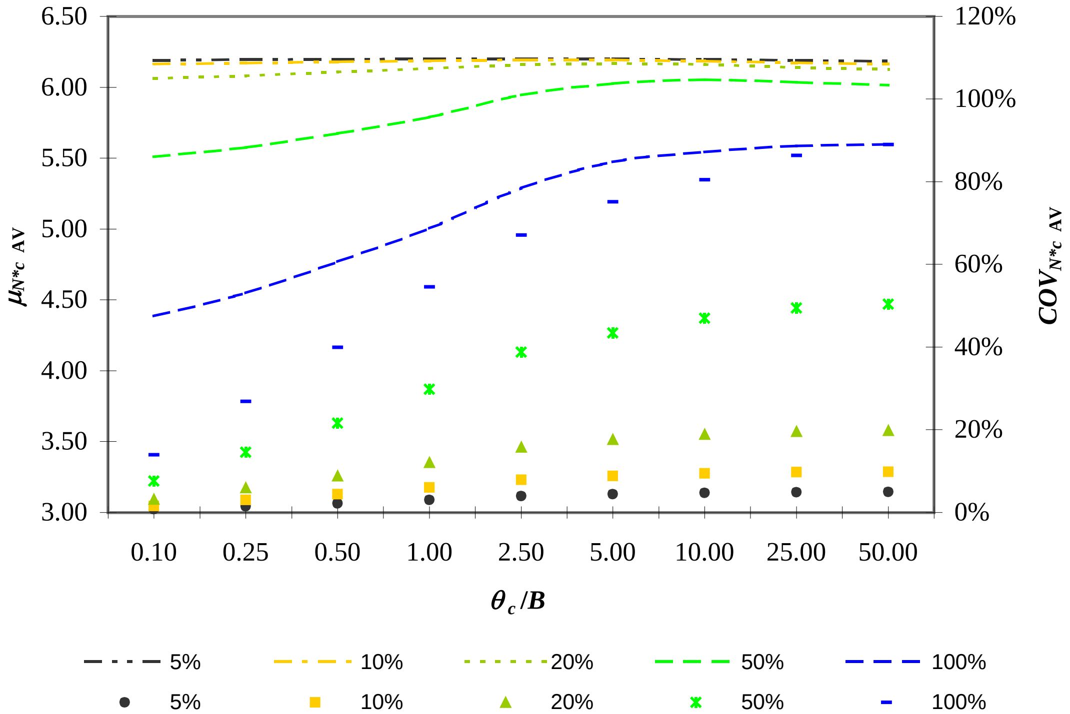

Figure B.50 The variation of $\mu_{N^*c\ \mathrm{AV}}$ and $COV_{N^*c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.1_0.25 case (where $\mu_{c1} / \mu_{c2} = 0.1$ and $H/B = 0.25$).

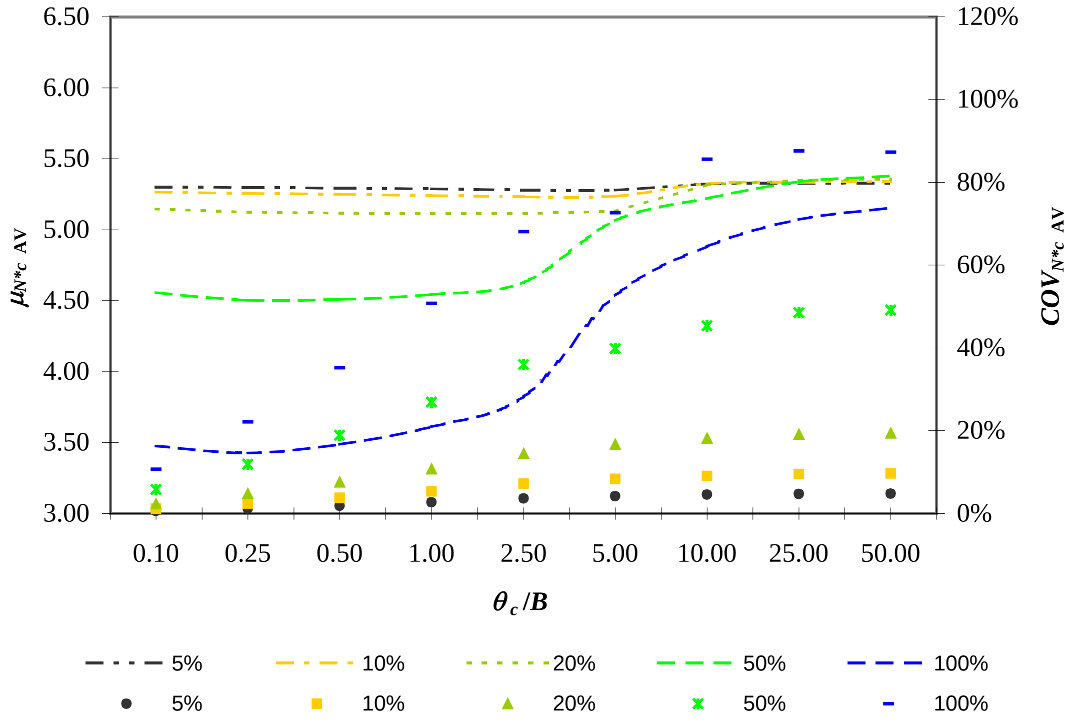

Figure B.51 The variation of $\mu_{N^{*}\mathrm{c\ AV}}$ and $COV_{N^{*}\mathrm{c\ AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.1_0.5 case (where μ_{c1} / μ_{c2} = 0.1 and H/B = 0.5).

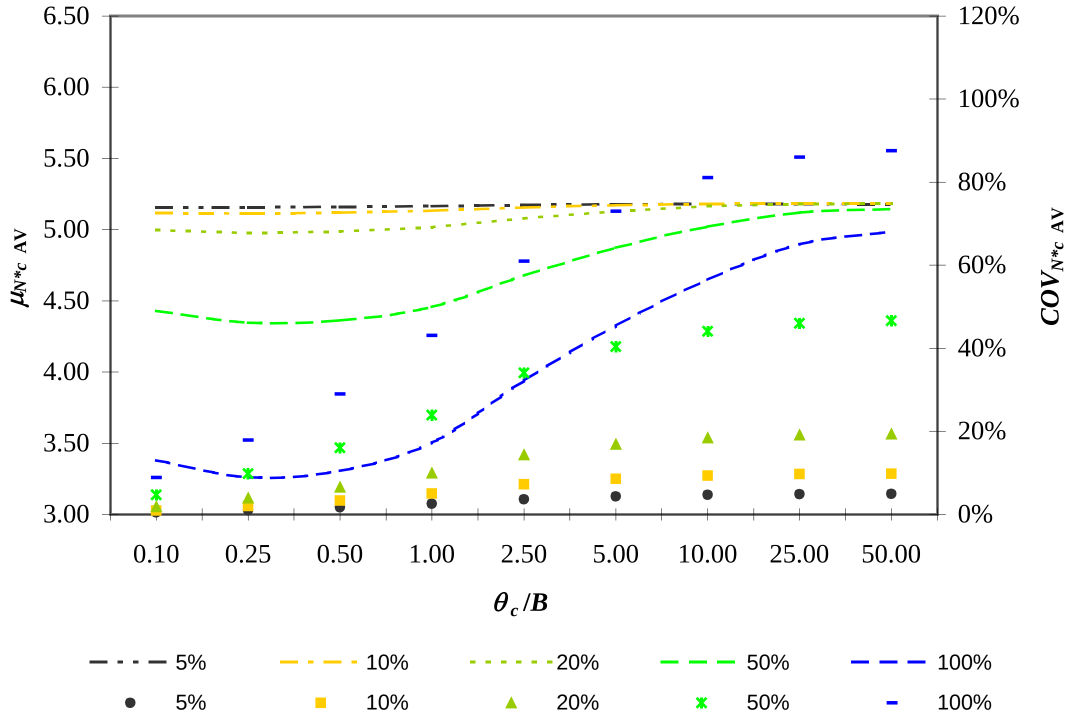

Figure B.52 The variation of $\mu_{N^{*}\mathrm{c\ AV}}$ and $COV_{N^{*}\mathrm{c\ AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.1_1.0 case (where μ_{c1} / μ_{c2} = 0.1 and H/B = 1.0).

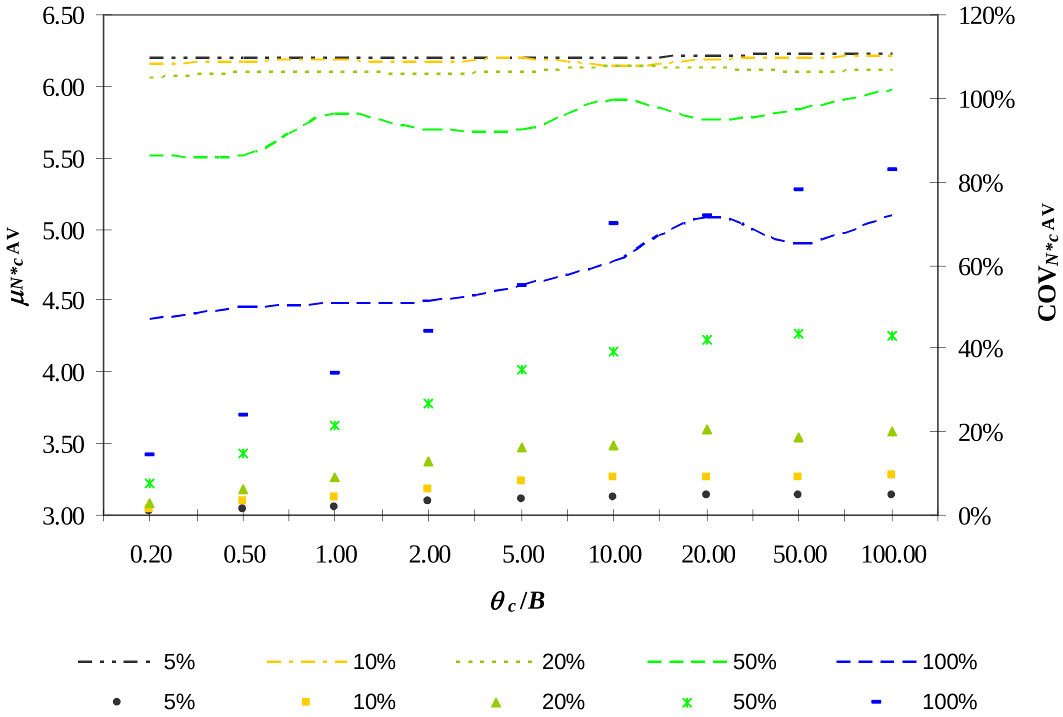

Figure B.53 The variation of $\mu_{N^{*}c\ \mathrm{AV}}$ and $COV_{N^{*}c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.333_0.25 case (where c_{u1}/c_{u2} = 0.333 and H/B = 0.25).

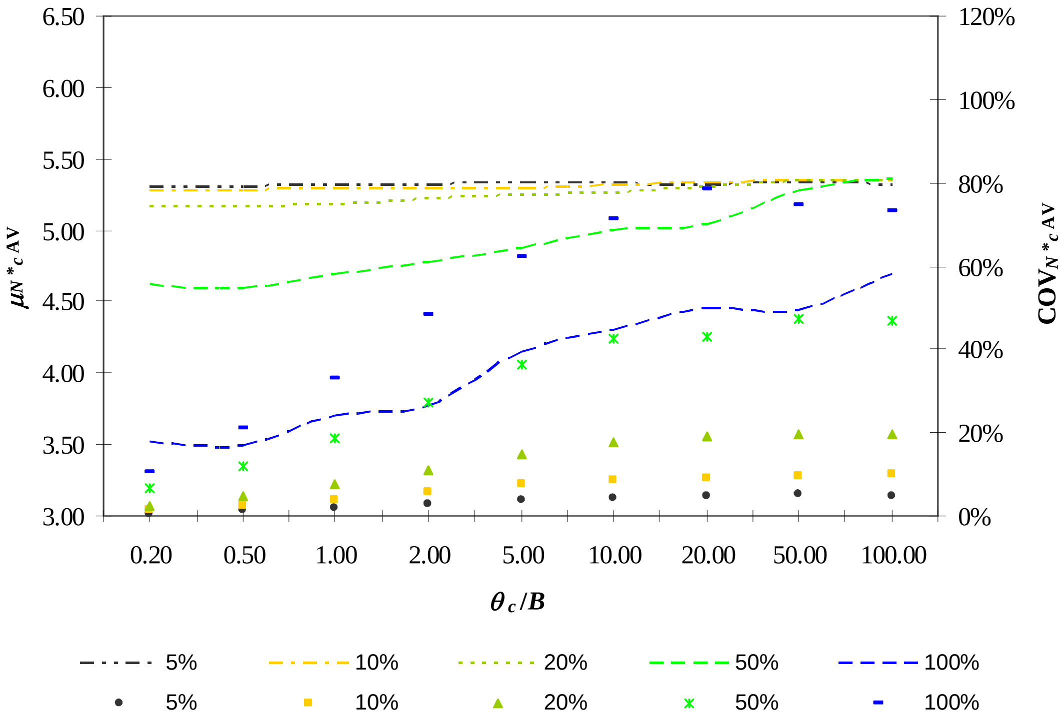

Figure B.54 The variation of $\mu_{N^{*}c\ \mathrm{AV}}$ and $COV_{N^{*}c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.333_0.5 case (where c_{u1}/c_{u2} = 0.333 and H/B = 0.5).

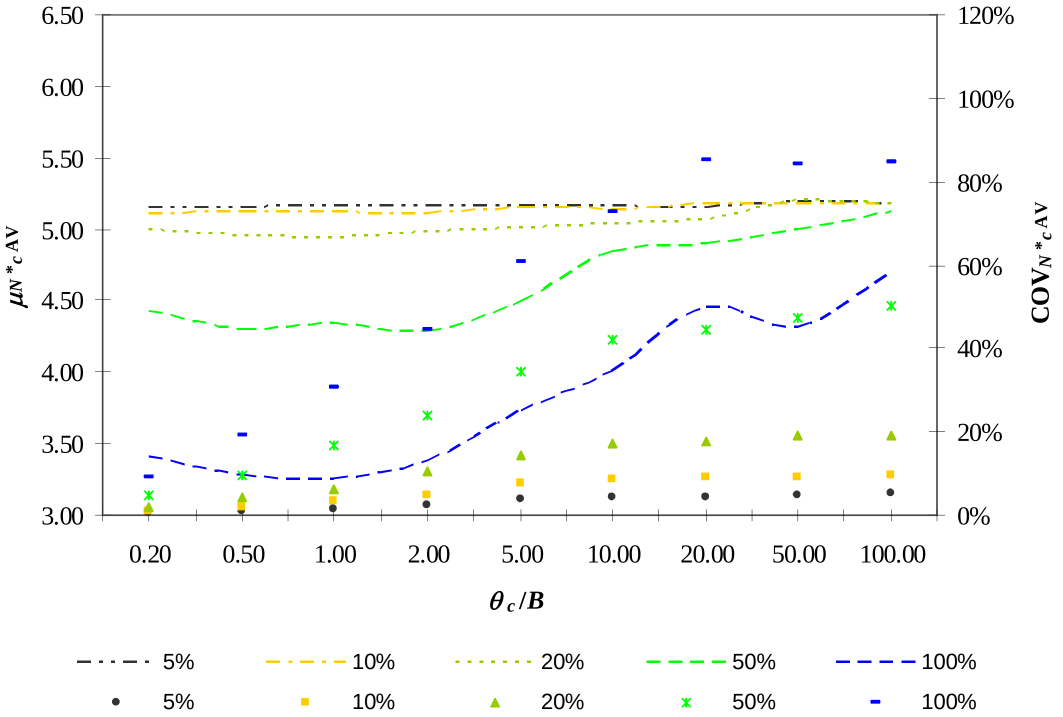

Figure B.55 The variation of $\mu_{N^{*}c\ \mathrm{AV}}$ and $COV_{N^{*}c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.333_1.0 case (where c_{u1}/c_{u2} = 0.333 and H/B = 1.0).

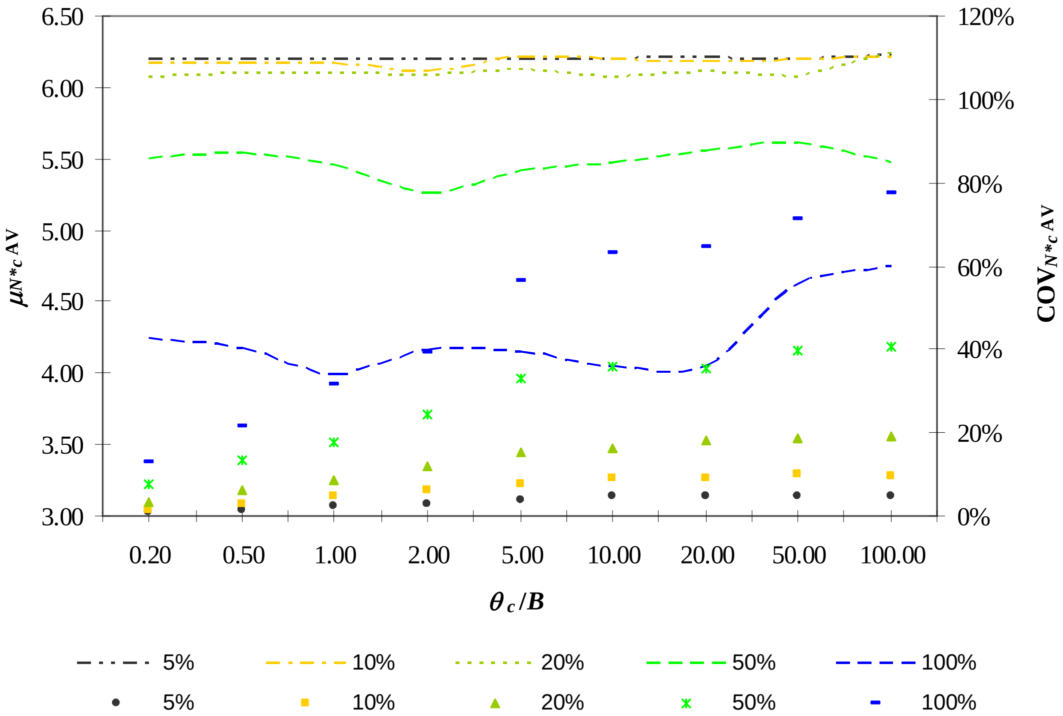

Figure B.56 The variation of $\mu_{N^{*}c\ \mathrm{AV}}$ and $COV_{N^{*}c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.50_0.25 case (where c_{u1}/c_{u2} = 0.50 and H/B = 0.25).

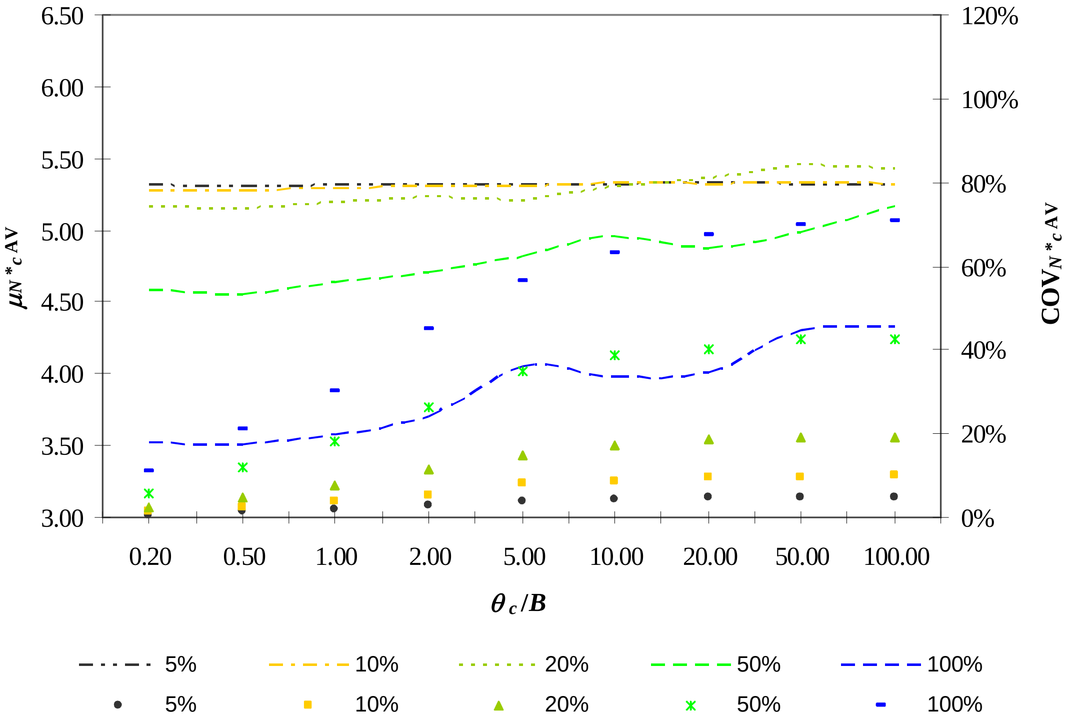

Figure B.57 The variation of $\mu_{N^*c\ \mathrm{AV}}$ and $COV_{N^*c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.50_0.5 case (where c_{u1}/c_{u2} = 0.50 and H/B = 0.5).

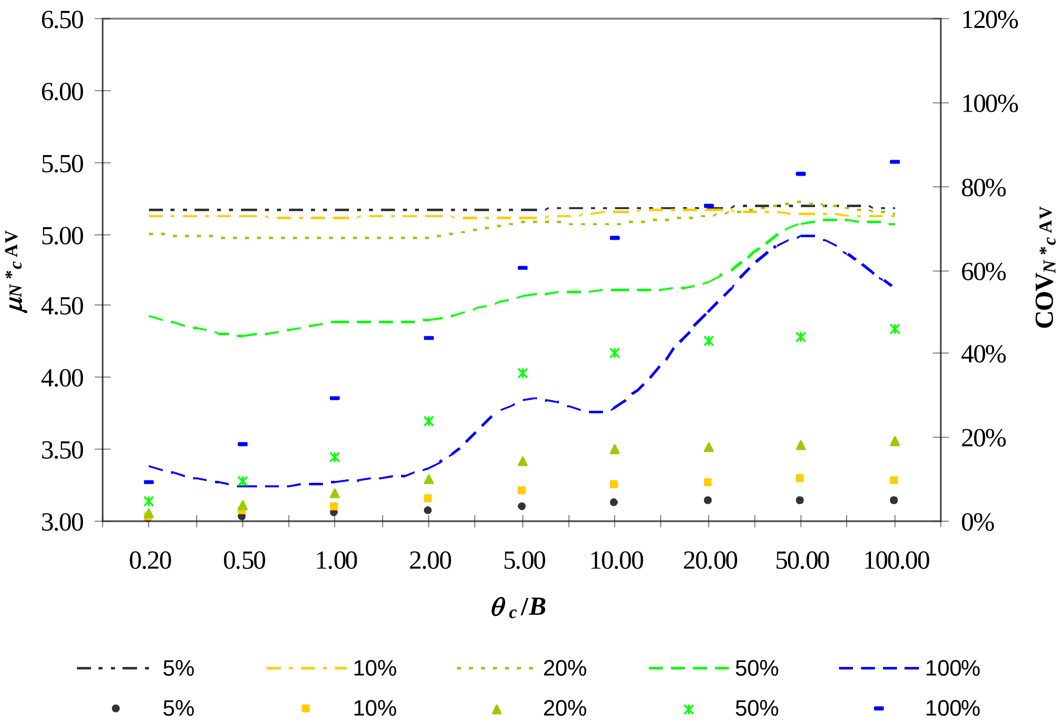

Figure B.58 The variation of $\mu_{N^*c\ \mathrm{AV}}$ and $COV_{N^*c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_0.50_1.0 case (where c_{u1}/c_{u2} = 0.50 and H/B = 1.0).

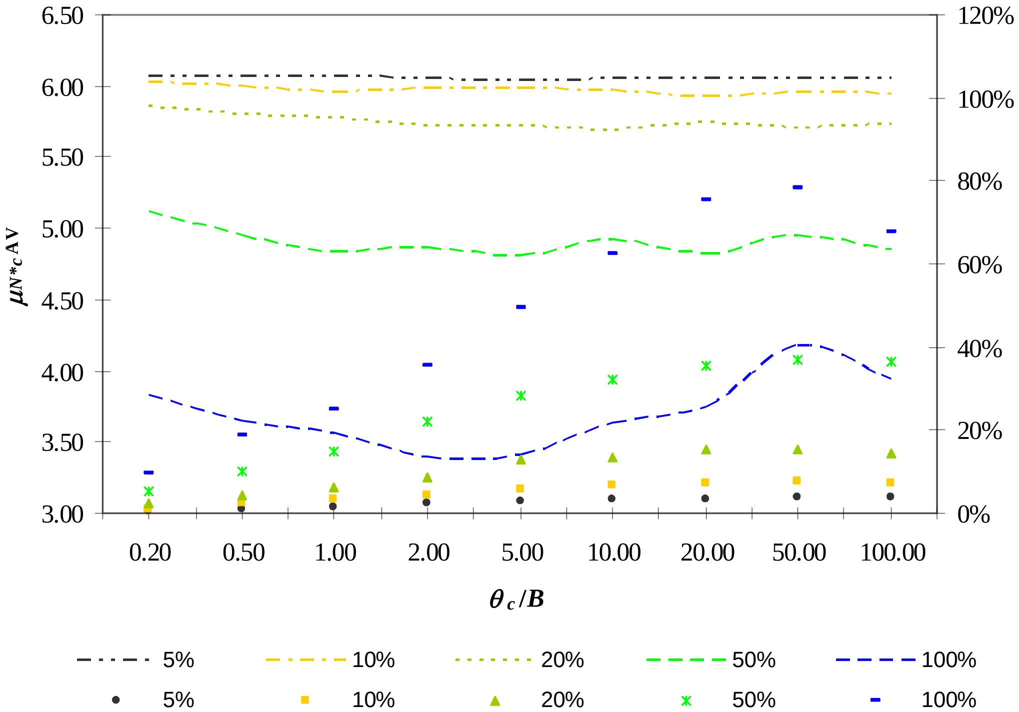

Figure B.59 The variation of $\mu_{N^*c\ AV}$ and $COV_{N^*c\ AV}$ with respect to COV_c and θ_c/B for COHESIVE_0.75_0.25 case (where c_{u1}/c_{u2} = 0.75 and H/B = 0.25).

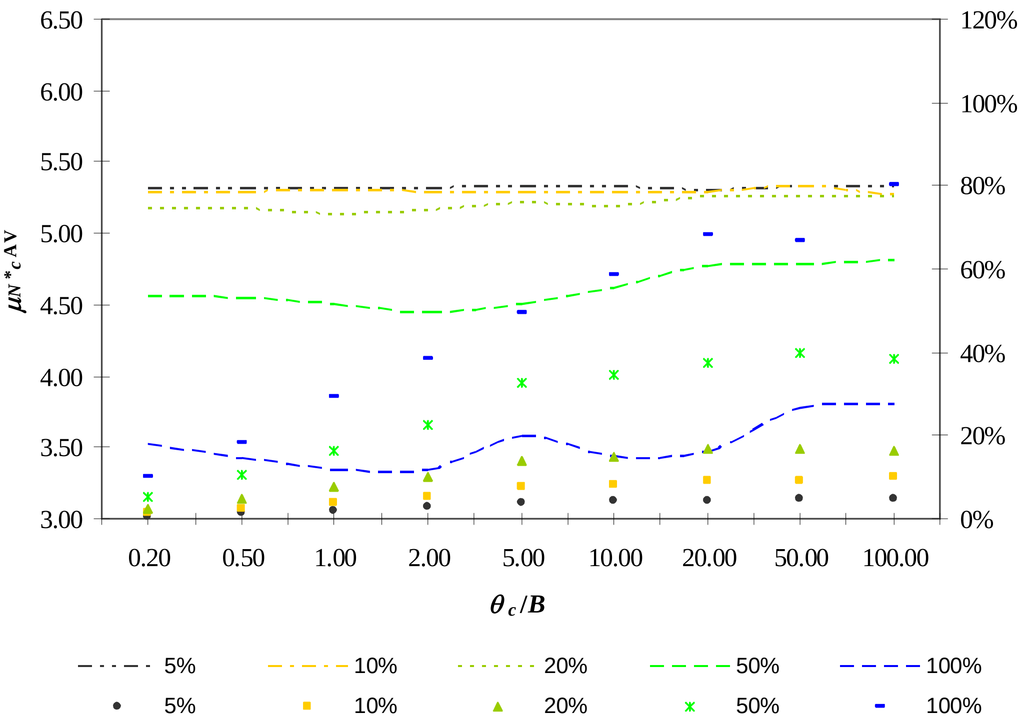

Figure B.60 The variation of $\mu_{N^*c\ AV}$ and $COV_{N^*c\ AV}$ with respect to COV_c and θ_c/B for COHESIVE_0.75_0.5 case (where c_{u1}/c_{u2} = 0.75 and H/B = 0.5).

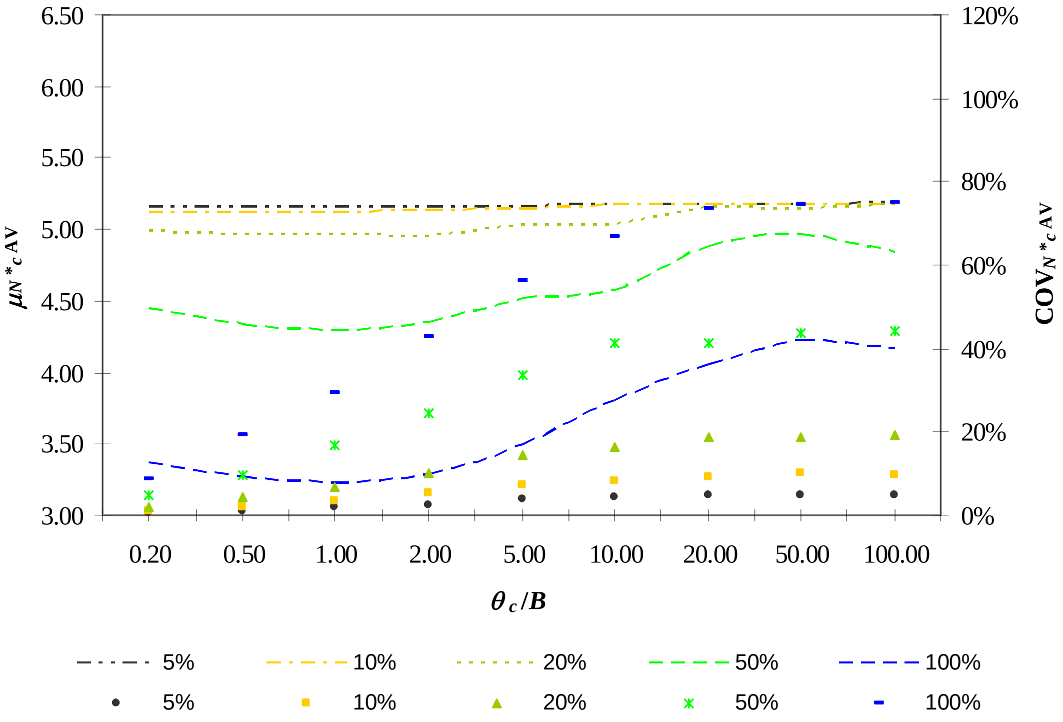

Figure B.61 The variation of $\mu_{N^*c\,AV}$ and $COV_{N^*c\,AV}$ with respect to COV_c and θ_c/B for COHESIVE_0.75_1.0 case (where c_{u1}/c_{u2} = 0.75 and H/B = 1.0).

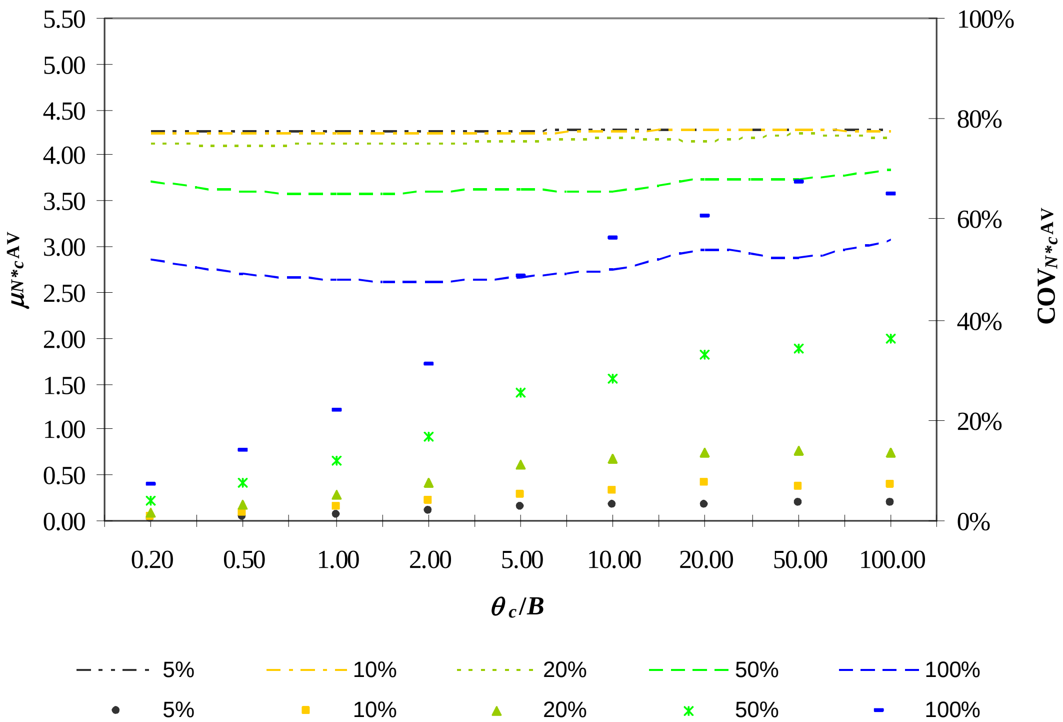

Figure B.62 The variation of $\mu_{N^*c\,AV}$ and $COV_{N^*c\,AV}$ with respect to COV_c and θ_c/B for COHESIVE_1.333_0.25 case (where c_{u1}/c_{u2} = 1.333 and H/B = 0.25).

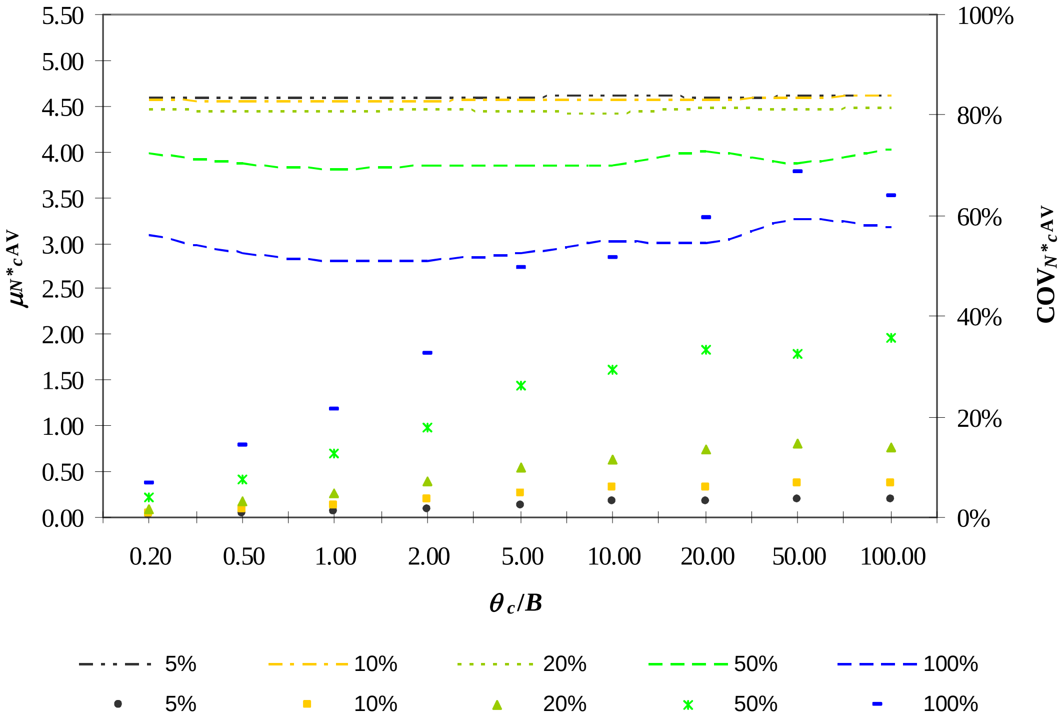

Figure B.63 The variation of $\mu_{N^{*}c\ AV}$ and $COV_{N^{*}c\ AV}$ with respect to COV_c and θ_c/B for COHESIVE_1.333_0.5 case (where c_{u1}/c_{u2} = 1.333 and H/B = 0.5).

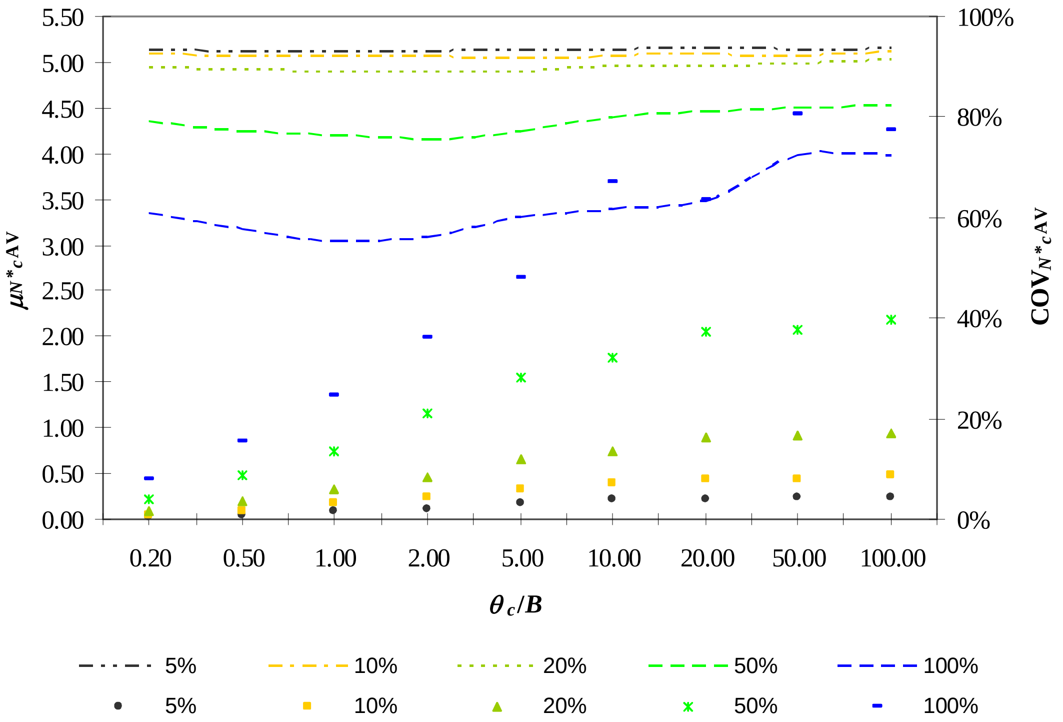

Figure B.64 The variation of $\mu_{N^{*}c\ AV}$ and $COV_{N^{*}c\ AV}$ with respect to COV_c and θ_c/B for COHESIVE_1.333_1.0 case (where c_{u1}/c_{u2} = 1.333 and H/B = 1.0).

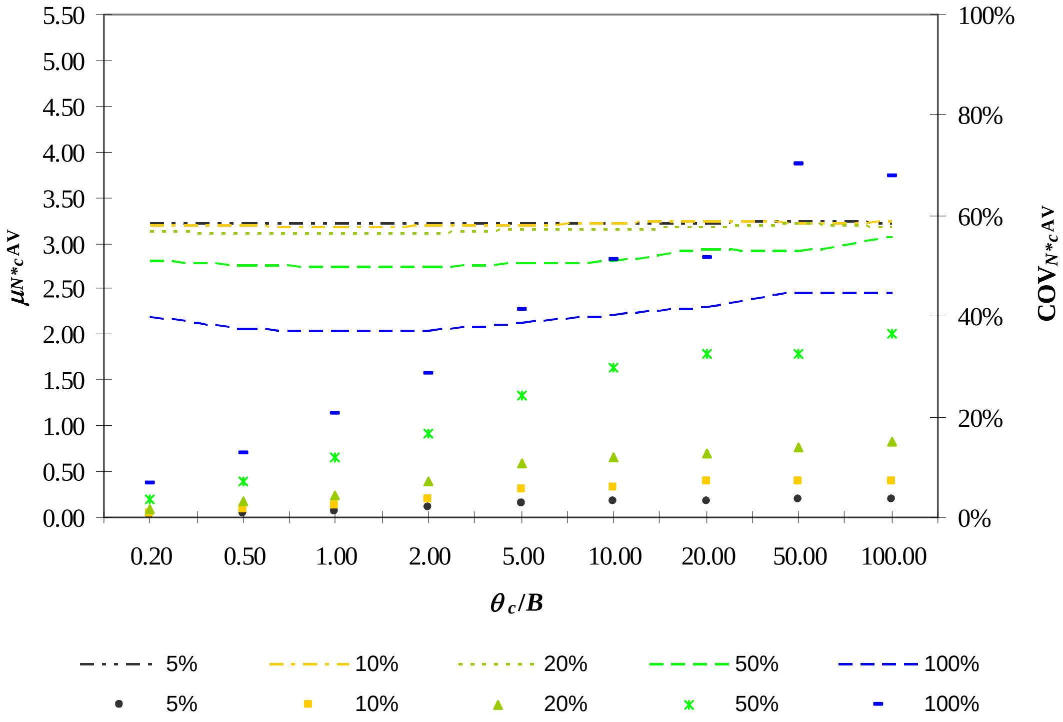

Figure B.65 The variation of $\mu_{N^*c\ \mathrm{AV}}$ and $COV_{N^*c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_2.0_0.25 case (where c_{u1}/c_{u2} = 2.00 and H/B = 0.25).

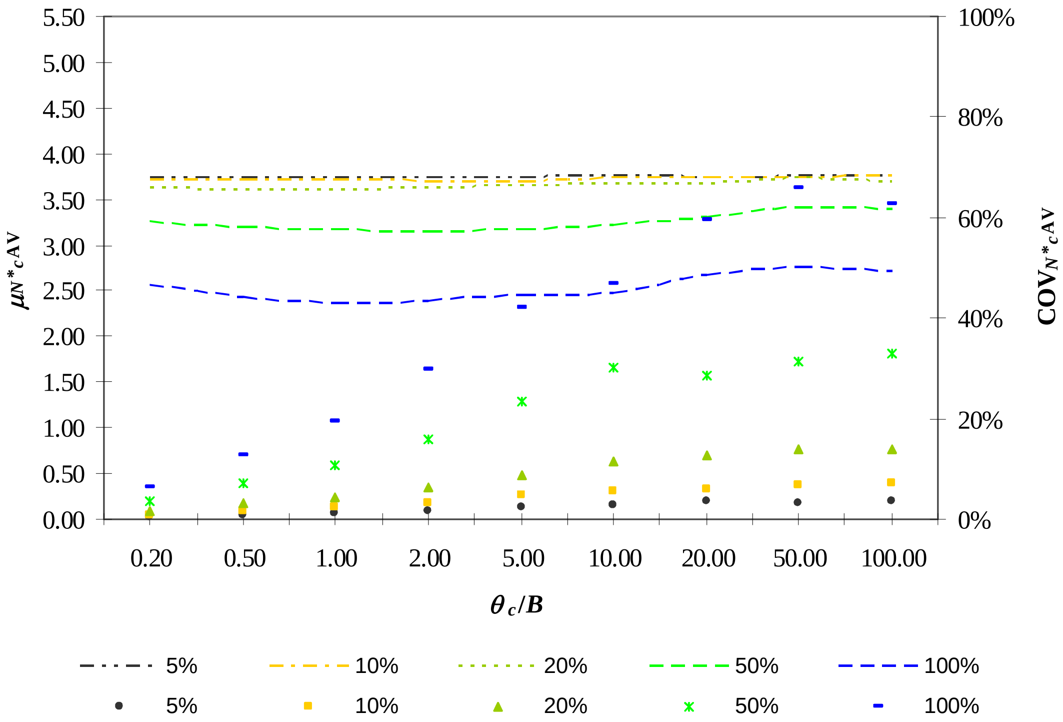

Figure B.66 The variation of $\mu_{N^*c\ \mathrm{AV}}$ and $COV_{N^*c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_2.0_0.5 case (where c_{u1}/c_{u2} = 2.00 and H/B = 0.5).

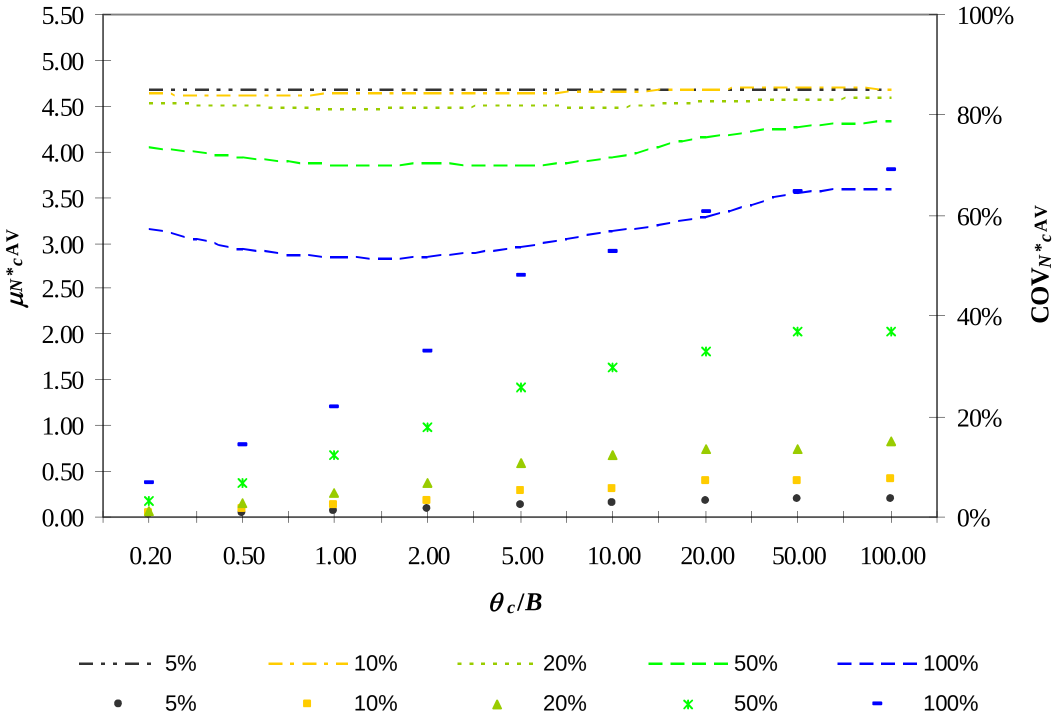

Figure B.67 The variation of $\mu_{N^{*}\mathrm{c\,AV}}$ and $COV_{N^{*}\mathrm{c\,AV}}$ with respect to COV_c and θ_c/B for COHESIVE_2.0_1.0 case (where c_{u1}/c_{u2} = 2.00 and H/B = 1.0).

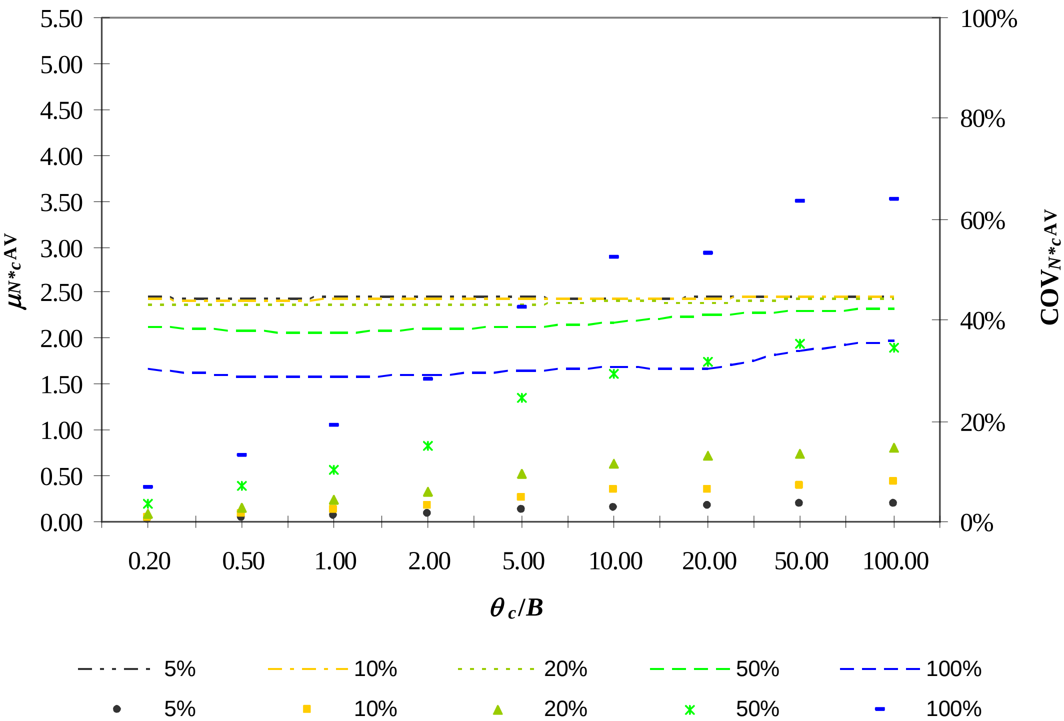

Figure B.68 The variation of $\mu_{N^{*}\mathrm{c\,AV}}$ and $COV_{N^{*}\mathrm{c\,AV}}$ with respect to COV_c and θ_c/B for COHESIVE_3.0_0.25 case (where c_{u1}/c_{u2} = 3.00 and H/B = 0.25).

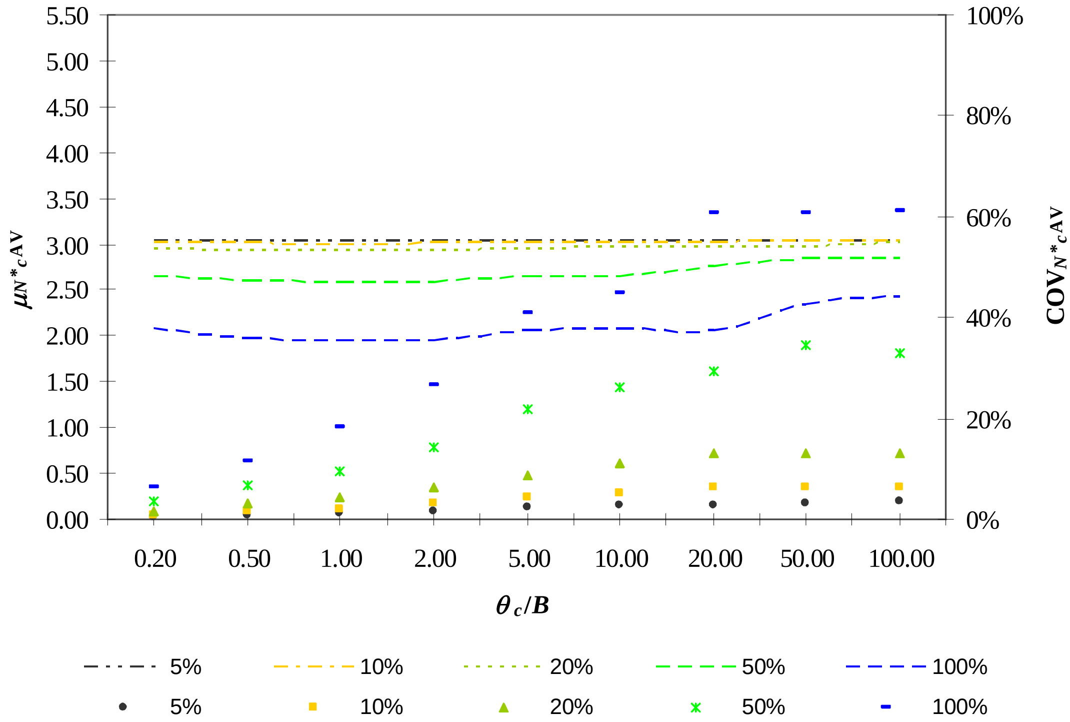

Figure B.69 The variation of $\mu_{N^{*}c\ \mathrm{AV}}$ and $COV_{N^{*}c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_3.0_0.5 case (where c_{u1}/c_{u2} = 3.00 and H/B = 0.5).

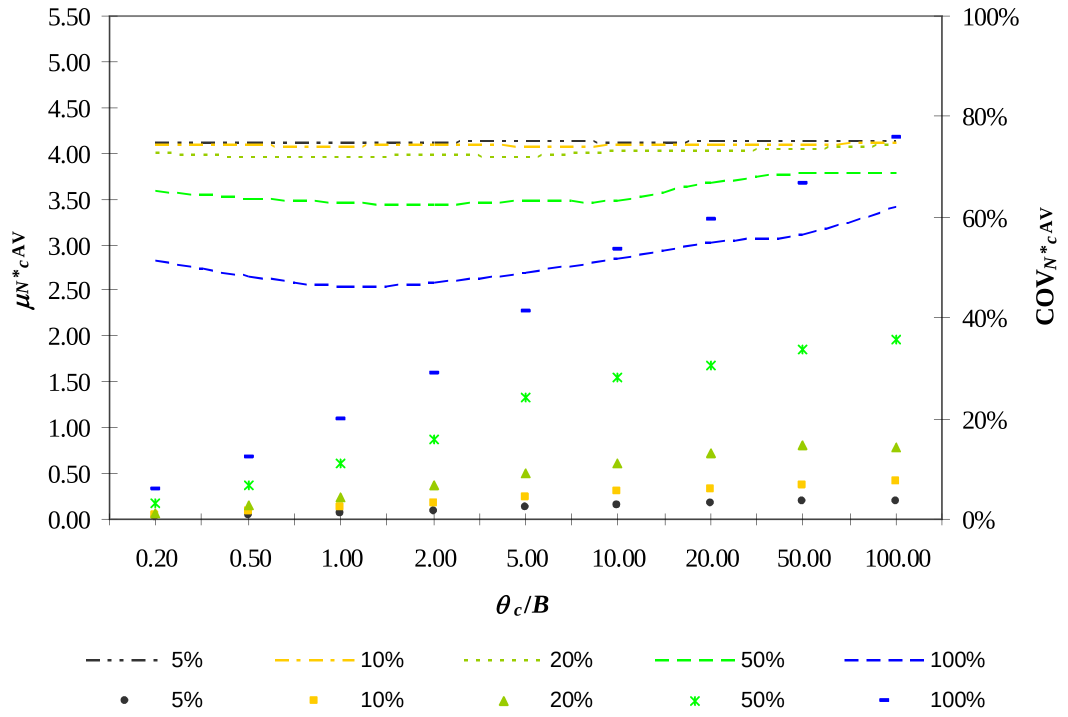

Figure B.70 The variation of $\mu_{N^{*}c\ \mathrm{AV}}$ and $COV_{N^{*}c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_3.0_1.0 case (where c_{u1}/c_{u2} = 3.00 and H/B = 1.0).

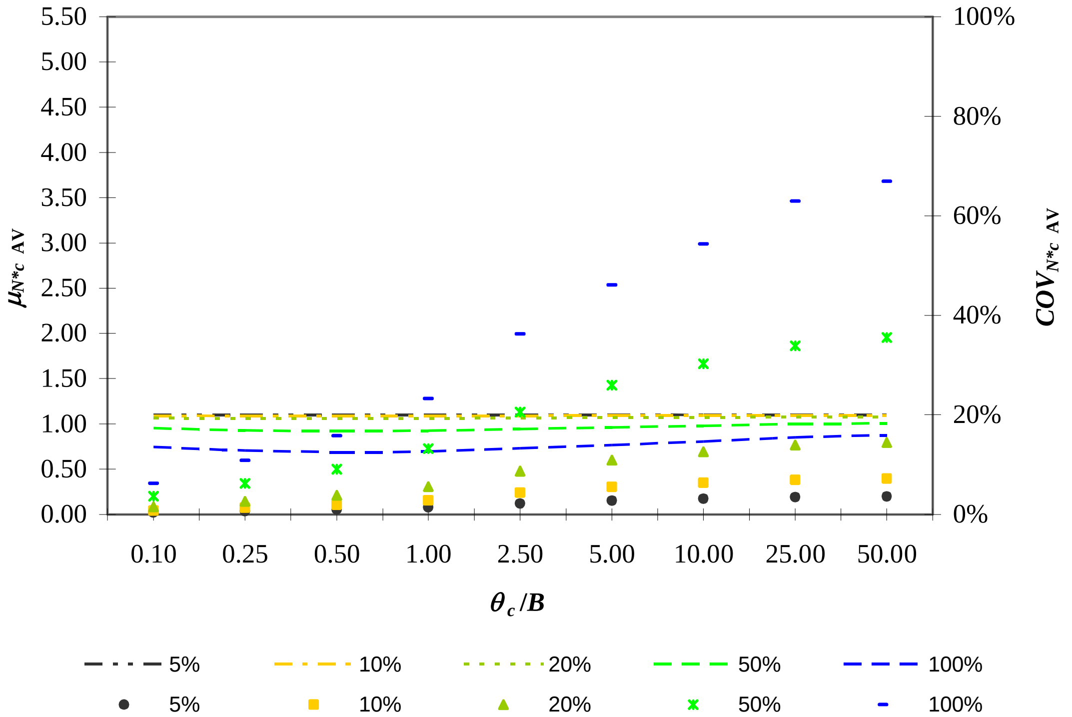

Figure B.71 The variation of $\mu_{N^{*}c\ \mathrm{AV}}$ and $COV_{N^{*}c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_10.0_0.25 case (where c_{u1}/c_{u2} = 10.0 and H/B = 0.25).

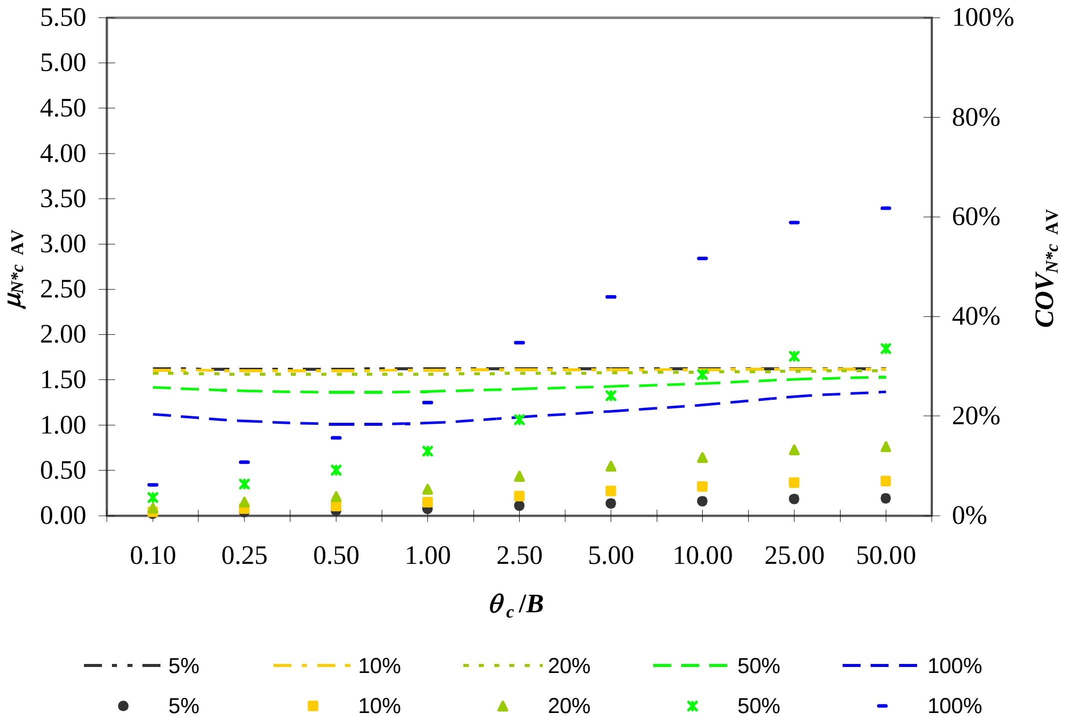

Figure B.72 The variation of $\mu_{N^{*}c\ \mathrm{AV}}$ and $COV_{N^{*}c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_10.0_0.5 case (where c_{u1}/c_{u2} = 10.0 and H/B = 0.5).

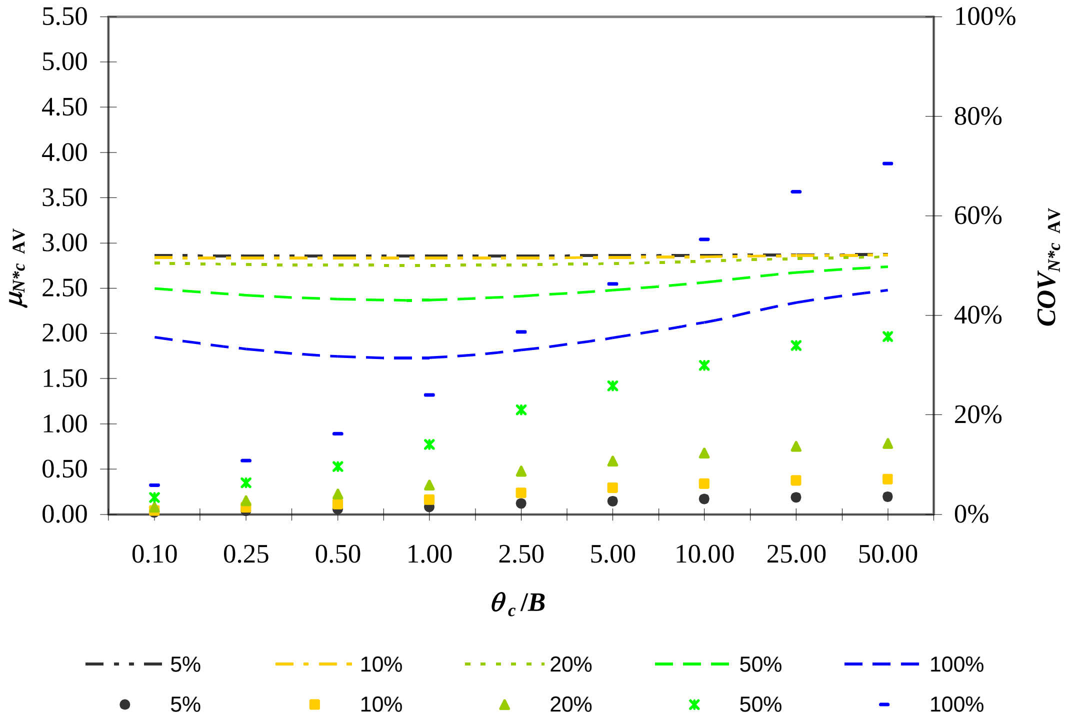

Figure B.73 The variation of $\mu_{N^{*}c\ \mathrm{AV}}$ and $COV_{N^{*}c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_10.0_1.0 case (where c_{u1}/c_{u2} = 10.0 and H/B = 1.0).

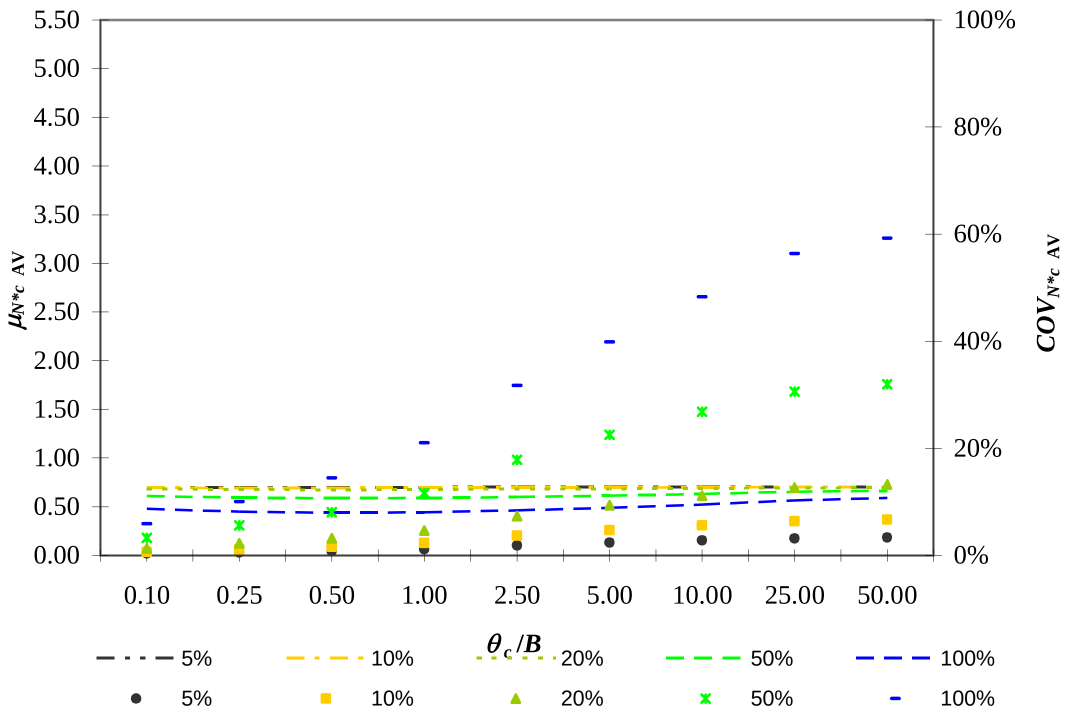

Figure B.74 The variation of $\mu_{N^{*}c\ \mathrm{AV}}$ and $COV_{N^{*}c\ \mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_20.0_0.25 case (where c_{u1}/c_{u2} = 20.0 and H/B = 0.25).

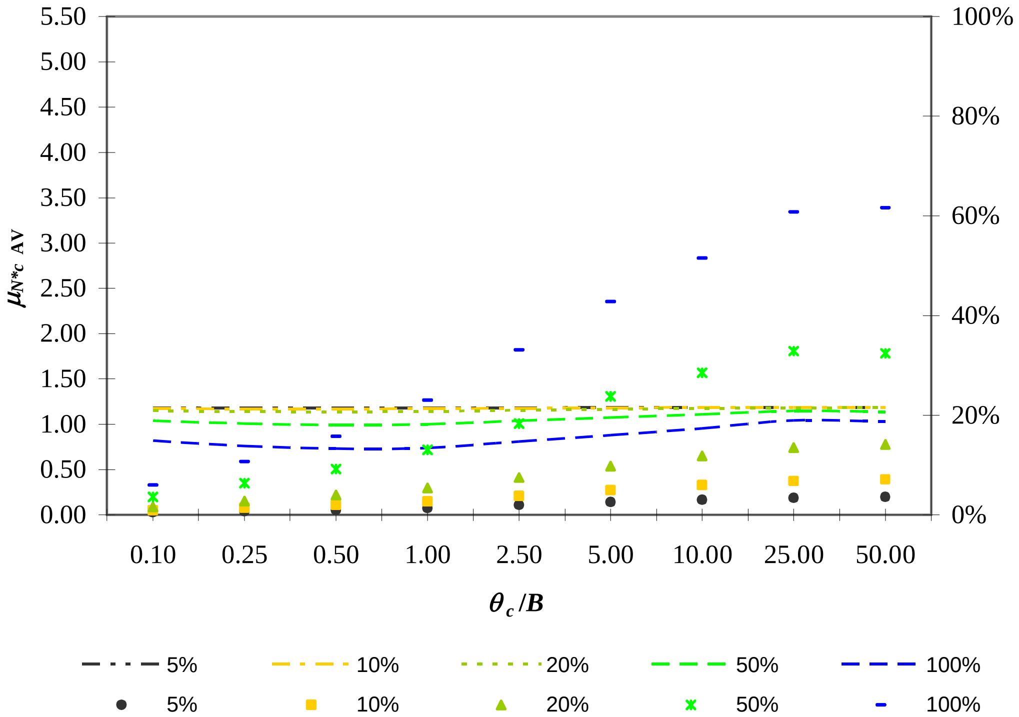

Figure B.75 The variation of $\mu_{N^{*}c\,\mathrm{AV}}$ and $COV_{N^{*}c\,\mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_20.0_0.5 case (where c_{u1}/c_{u2} = 20.0 and H/B = 0.5).

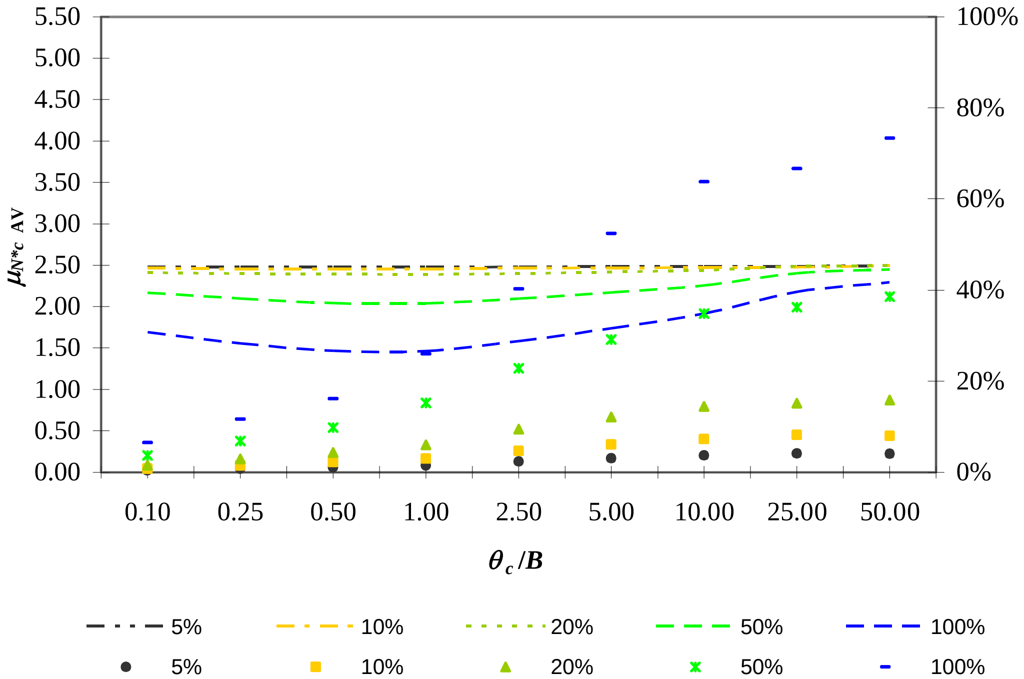

Figure B.76 The variation of $\mu_{N^{*}c\,\mathrm{AV}}$ and $COV_{N^{*}c\,\mathrm{AV}}$ with respect to COV_c and θ_c/B for COHESIVE_20.0_1.0 case (where c_{u1}/c_{u2} = 20.0 and H/B = 1.0).

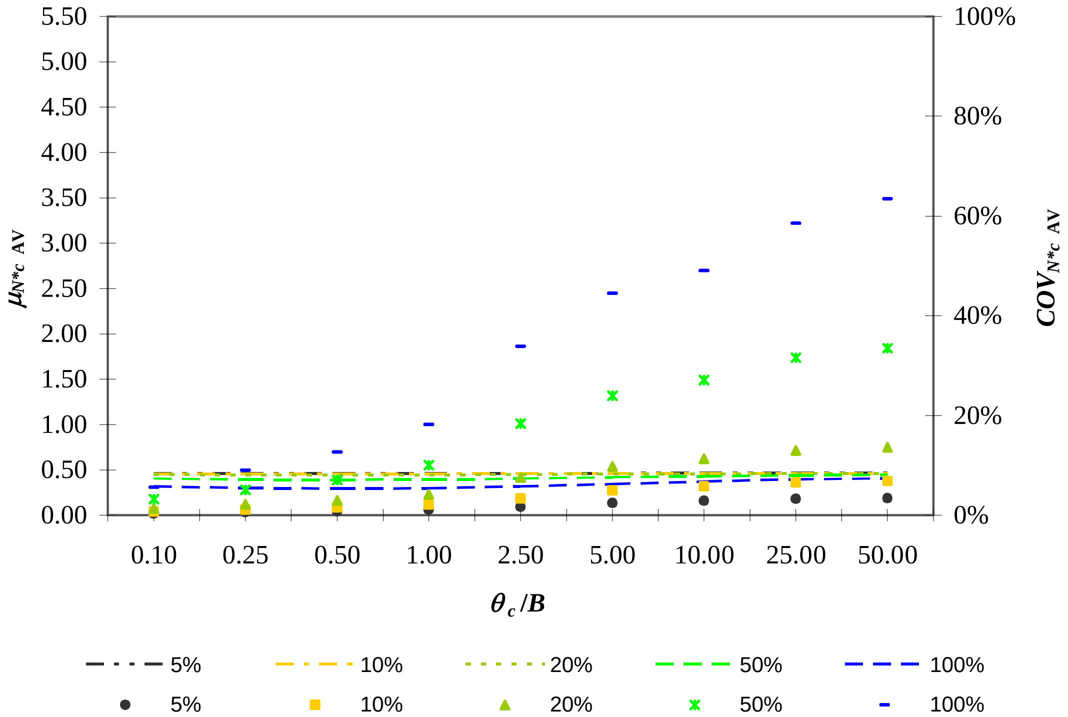

Figure B.77 The variation of $\mu_{N^*c\ AV}$ and $COV_{N^*c\ AV}$ with respect to COV_c and θ_c/B for COHESIVE_40.0_0.25 case (where c_{u1}/c_{u2} = 40.0 and H/B = 0.25).

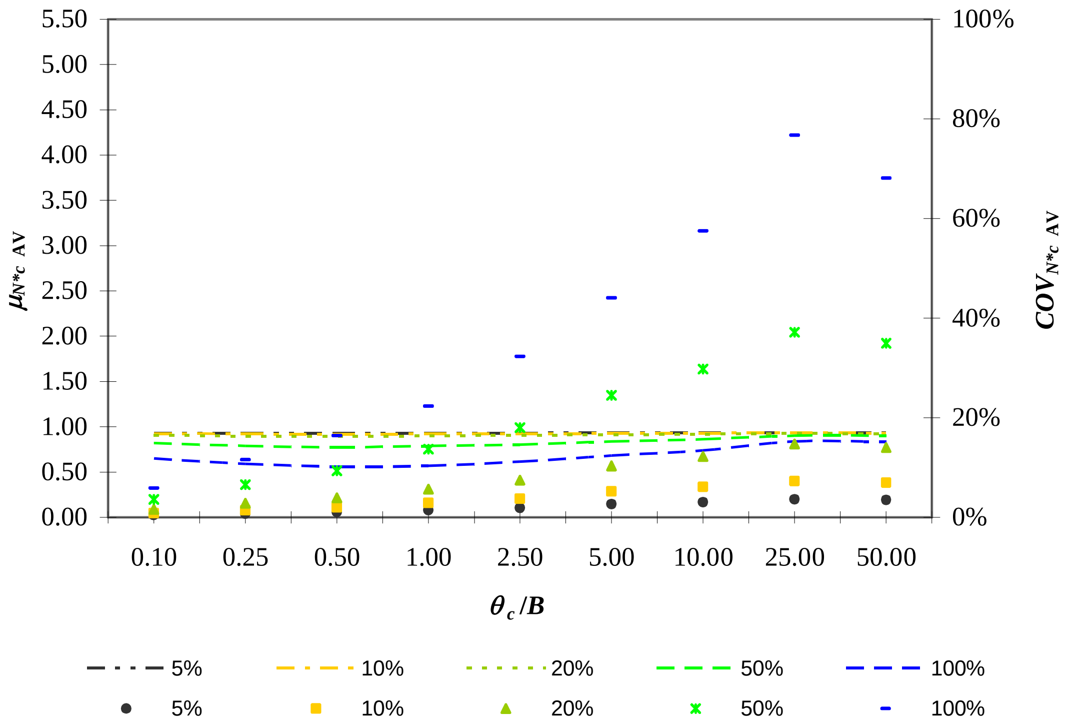

Figure B.78 The variation of $\mu_{N^*c\ AV}$ and $COV_{N^*c\ AV}$ with respect to COV_c and θ_c/B for COHESIVE_40.0_0.5 case (where c_{u1}/c_{u2} = 40.0 and H/B = 0.5).

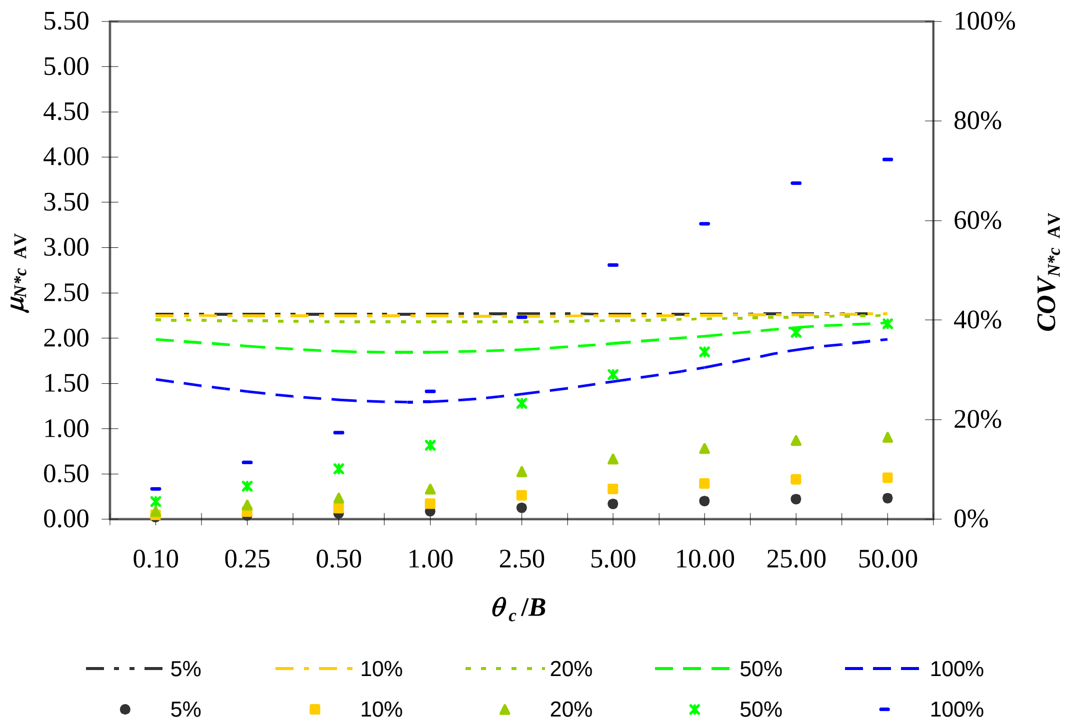

Figure B.79 The variation of $\mu_{N^{*}c\ AV}$ and $COV_{N^{*}c\ AV}$ with respect to COV_c and θ_c/B for COHESIVE_40.0_1.0 case (where c_{u1}/c_{u2} = 40.0 and H/B = 1.0).

APPENDIX C

Table C.1 The results of the ANN input and output statistics.

Model Variable and Data Sets	Mean	Standard Deviation	Maximum	Minimum	Range
Soil cohesion of layer 1 (c_1)					
Training set	5.47	2.60	10.00	1.00	9.00
Testing set	5.29	2.43	9.99	1.12	8.87
Validation set	5.46	2.63	9.99	1.01	8.98
Soil cohesion of layer 2 (c_2)					
Training set	5.42	2.54	10.00	1.00	9.00
Testing set	5.53	2.59	9.99	1.02	8.97
Validation set	5.29	2.63	9.97	1.01	8.96
Soil cohesion of layer 3 (c_3)					
Training set	5.48	2.63	10.00	1.00	9.00
Testing set	5.51	2.64	9.99	1.02	8.97
Validation set	5.74	2.57	9.98	1.01	8.97
Soil cohesion of layer 4 (c_4)					
Training set	5.54	2.63	10.00	1.00	9.00
Testing set	5.27	2.59	9.99	1.01	8.98
Validation set	5.38	2.62	9.99	1.01	8.98
Soil cohesion of layer 5 (c_5)					
Training set	5.51	2.55	10.00	1.00	9.00
Testing set	5.38	2.58	9.97	1.01	8.96
Validation set	5.42	2.59	10.00	1.00	9.00
Soil cohesion of layer 6 (c_6)					
Training set	5.70	2.58	9.99	1.01	8.98
Testing set	5.52	2.52	9.99	1.02	8.97
Validation set	5.42	2.70	9.99	1.02	8.97
Soil cohesion of layer 7 (c_7)					
Training set	5.35	2.53	10.00	1.00	9.00
Testing set	5.64	2.66	9.96	1.01	8.95
Validation set	5.41	2.59	10.00	1.02	9.98
Soil cohesion of layer 8 (c_8)					
Training set	5.67	2.62	10.00	1.00	9.00
Testing set	5.31	2.49	9.99	1.03	8.96
Validation set	5.47	2.59	9.99	1.02	8.97
Soil cohesion of layer 9 (c_9)					
Training set	5.58	2.60	10.00	1.00	9.00
Testing set	5.41	2.54	9.97	1.02	8.95
Validation set	5.52	2.63	9.99	1.00	8.99

Table C.1 The results of the ANN input and output statistics. ***(Continued)***

Model Variable and Data Sets	Mean	Standard Deviation	Maximum	Minimum	Range
Soil cohesion of layer 10 (c_{10})					
Training set	5.41	2.64	9.98	1.01	8.97
Testing set	5.60	2.74	9.97	1.05	8.92
Validation set	5.49	2.62	9.97	1.01	8.96
Friction angle of layer 1 (ϕ_1)					
Training set	12.57	4.32	20.00	5.01	14.99
Testing set	12.68	4.24	19.99	5.02	14.97
Validation set	12.62	4.29	19.99	5.01	14.98
Friction angle of layer 2 (ϕ_2)					
Training set	12.48	4.34	20.00	5.01	14.99
Testing set	12.45	4.33	19.99	5.03	14.96
Validation set	12.23	4.36	19.91	5.04	14.87
Friction angle of layer 3 (ϕ_3)					
Training set	12.40	4.35	19.98	5.00	14.98
Testing set	12.25	4.32	19.93	5.09	14.84
Validation set	12.29	4.46	19.98	5.01	14.97
Friction angle of layer 4 (ϕ_4)					
Training set	12.70	4.41	19.99	5.01	14.98
Testing set	12.56	4.36	19.99	5.15	14.84
Validation set	12.19	4.29	19.98	5.01	14.97
Friction angle of layer 5 (ϕ_5)					
Training set	12.39	4.39	20.00	5.00	15.00
Testing set	12.33	4.25	19.98	5.01	14.97
Validation set	12.50	4.41	19.97	5.02	14.95
Friction angle of layer 6 (ϕ_6)					
Training set	12.83	4.27	19.99	5.00	14.99
Testing set	12.60	4.19	19.99	5.10	14.89
Validation set	12.39	4.33	19.96	5.01	14.95
Friction angle of layer 7 (ϕ_7)					
Training set	12.39	4.38	19.99	5.00	14.99
Testing set	12.45	4.25	19.99	5.09	14.90
Validation set	12.34	4.35	19.97	5.01	14.96
Friction angle of layer 8 (ϕ_8)					
Training set	12.37	4.39	19.99	5.00	14.99
Testing set	12.28	4.33	19.86	5.01	14.85
Validation set	12.48	4.33	19.98	5.04	14.94

Table C.1 The results of the ANN input and output statistics. ***(Continued)***

Model Variable and Data Sets	Mean	Standard Deviation	Maximum	Minimum	Range
Friction angle of layer 9 (ϕ_9)					
Training set	12.62	4.35	20.00	5.00	15.00
Testing set	12.42	4.37	19.98	5.00	14.98
Validation set	12.37	4.32	19.94	5.04	14.90
Friction angle of layer 10 (ϕ_{10})					
Training set	12.50	4.36	19.99	5.00	14.99
Testing set	12.88	4.37	20.00	5.03	14.97
Validation set	12.07	4.38	19.94	5.10	14.84
Soil thickness of layer 1 (h_1)					
Training set	0.61	0.29	1.00	0.20	0.80
Testing set	0.58	0.28	1.00	0.20	0.80
Validation set	0.61	0.28	1.00	0.20	0.80
Soil thickness of layer 2 (h_2)					
Training set	0.62	0.28	1.00	0.20	0.80
Testing set	0.60	0.29	1.00	0.20	0.80
Validation set	0.58	0.27	1.00	0.20	0.80
Soil thickness of layer 3 (h_3)					
Training set	0.60	0.29	1.00	0.20	0.80
Testing set	0.61	0.29	1.00	0.20	0.80
Validation set	0.62	0.29	1.00	0.20	0.80
Soil thickness of layer 4 (h_4)					
Training set	0.60	0.28	1.00	0.20	0.80
Testing set	0.61	0.28	1.00	0.20	0.80
Validation set	0.60	0.29	1.00	0.20	0.80
Soil thickness of layer 5 (h_5)					
Training set	0.59	0.28	1.00	0.20	0.80
Testing set	0.58	0.28	1.00	0.20	0.80
Validation set	0.61	0.29	1.00	0.20	0.80
Soil thickness of layer 6 (h_6)					
Training set	0.60	0.28	1.00	0.20	0.80
Testing set	0.61	0.27	1.00	0.20	0.80
Validation set	0.60	0.29	1.00	0.20	0.80
Soil thickness of layer 7 (h_7)					
Training set	0.61	0.28	1.00	0.20	0.80
Testing set	0.57	0.28	1.00	0.20	0.80
Validation set	0.61	0.28	1.00	0.20	0.80

Table C.1 The results of the ANN input and output statistics. ***(Continued)***

Model Variable and Data Sets	Mean	Standard Deviation	Maximum	Minimum	Range
Soil thickness of layer 8 (h_8)					
Training set	0.59	0.28	1.00	0.20	0.80
Testing set	0.59	0.28	1.00	0.20	0.80
Validation set	0.59	0.29	1.00	0.20	0.80
Soil thickness of layer 9 (h_9)					
Training set	0.60	0.29	1.00	0.20	0.80
Testing set	0.61	0.28	1.00	0.20	0.80
Validation set	0.60	0.29	1.00	0.20	0.80
Footing width (B)					
Training set	2.50	0.93	4.00	1.00	3.00
Testing set	2.51	0.93	4.00	1.00	3.00
Validation set	2.51	0.92	4.00	1.00	3.00
Bearing capacity of strip footing (q_u) for c-ϕ case					
Training set	39.77	14.49	93.31	7.26	86.05
Testing set	39.60	14.19	87.94	11.67	76.27
Validation set	39.28	14.51	80.19	9.04	71.15
$\tilde{N}_c$					
Training set	0.858	0.313	2.260	0.174	2.085
Testing set	0.868	0.304	2.106	0.216	1.890
Validation set	0.862	0.321	2.175	0.255	1.920
$\tilde{N}_{c-\phi}$					
Training set	1.043	0.255	3.035	0.524	2.511
Testing set	1.038	0.243	1.951	0.578	1.373
Validation set	1.018	0.224	1.830	0.561	1.269

Table C.2 The results of null hypothesis tests inputs and outputs.

Model Variable and Data Sets	t-value	Lower Critical value	Upper Critical value	t-test	F-value	Lower Critical value	Upper Critical value	F-test
Soil cohesion of layer 1 (c_1)								
Training	1.042	-1.961	1.961	Accept	0.996	-0.937	0.937	Accept
Testing	-1.607	-1.961	1.961	Accept	1.024	-0.916	0.916	Accept
Validation	-0.218	-1.961	1.961	Accept	0.973	-0.869	0.869	Accept

Table C.2 The results of null hypothesis tests inputs and outputs. ***(Continued)***

Model Variable and Data Sets	*t*-value	Lower Critical value	Upper Critical value	*t*-test	*F*-value	Lower Critical value	Upper Critical value	*F*-test
Soil cohesion of layer 2 (c_2)								
Training	0.868	-1.961	1.961	Accept	0.991	-0.937	0.937	Accept
Testing	-0.338	-1.961	1.961	Accept	1.010	-0.916	0.916	Accept
Validation	-1.294	-1.961	1.961	Accept	0.942	-0.869	0.869	Accept
Soil cohesion of layer 3 (c_3)								
Training	-0.737	-1.961	1.961	Accept	0.993	-0.937	0.937	Accept
Testing	0.970	-1.961	1.961	Accept	1.005	-0.916	0.916	Accept
Validation	0.334	-1.961	1.961	Accept	1.071	-0.869	0.869	Accept
Soil cohesion of layer 4 (c_4)								
Training	0.193	-1.961	1.961	Accept	0.997	-0.937	0.937	Accept
Testing	-0.508	-1.961	1.961	Accept	1.011	-0.916	0.916	Accept
Validation	0.194	-1.961	1.961	Accept	0.990	-0.869	0.869	Accept
Soil cohesion of layer 5 (c_5)								
Training	-0.674	-1.961	1.961	Accept	1.026	-0.937	0.937	Accept
Testing	0.546	-1.961	1.961	Accept	0.984	-0.916	0.916	Accept
Validation	0.650	-1.961	1.961	Accept	0.991	-0.869	0.869	Accept
Soil cohesion of layer 6 (c_6)								
Training	-0.612	-1.961	1.961	Accept	0.996	-0.937	0.937	Accept
Testing	0.846	-1.961	1.961	Accept	1.050	-0.916	0.916	Accept
Validation	0.228	-1.961	1.961	Accept	0.912	-0.869	0.869	Accept
Soil cohesion of layer 7 (c_7)								
Training	0.895	-1.961	1.961	Accept	1.006	-0.937	0.937	Accept
Testing	-1.927	-1.961	1.961	Accept	1.005	-0.916	0.916	Accept
Validation	0.434	-1.961	1.961	Accept	0.969	-0.869	0.869	Accept
Soil cohesion of layer 8 (c_8)								
Training	0.239	-1.961	1.961	Accept	1.007	-0.937	0.937	Accept
Testing	0.207	-1.961	1.961	Accept	0.978	-0.916	0.916	Accept
Validation	-0.689	-1.961	1.961	Accept	0.967	-0.869	0.869	Accept
Soil cohesion of layer 9 (c_9)								
Training	0.360	-1.961	1.961	Accept	1.009	-0.937	0.937	Accept
Testing	0.440	-1.961	1.961	Accept	0.993	-0.916	0.916	Accept
Validation	-1.168	-1.961	1.961	Accept	0.991	-0.869	0.869	Accept

Table C.2 The results of null hypothesis tests inputs and outputs. ***(Continued)***

Model Variable and Data Sets	***t*-value**	**Lower Critical value**	**Upper Critical value**	***t*-test**	***F*-value**	**Lower Critical value**	**Upper Critical value**	***F*-test**
Soil cohesion of layer 10 (c_{10})								
Training	0.159	-1.961	1.961	Accept	0.990	-0.937	0.937	Accept
Testing	0.634	-1.961	1.961	Accept	1.009	-0.916	0.916	Accept
Validation	-1.015	-1.961	1.961	Accept	1.034	-0.869	0.869	Accept
Soil friction angle of layer 1 (ϕ_1)								
Training	-0.530	-1.961	1.961	Accept	1.006	-0.937	0.937	Accept
Testing	0.788	-1.961	1.961	Accept	0.978	-0.916	0.916	Accept
Validation	0.125	-1.961	1.961	Accept	1.025	-0.869	0.869	Accept
Soil friction angle of layer 2 (ϕ_2)								
Training	0.309	-1.961	1.961	Accept	0.997	-0.937	0.937	Accept
Testing	-0.970	-1.961	1.961	Accept	1.025	-0.916	0.916	Accept
Validation	0.475	-1.961	1.961	Accept	0.995	-0.869	0.869	Accept
Soil friction angle of layer 3 (ϕ_3)								
Training	1.227	-1.961	1.961	Accept	0.972	-0.937	0.937	Accept
Testing	-1.626	-1.961	1.961	Accept	1.059	-0.916	0.916	Accept
Validation	-0.596	-1.961	1.961	Accept	0.912	-0.869	0.869	Accept
Soil friction angle of layer 4 (ϕ_4)								
Training	0.174	-1.961	1.961	Accept	0.992	-0.937	0.937	Accept
Testing	-0.994	-1.961	1.961	Accept	1.021	-0.916	0.916	Accept
Validation	0.764	-1.961	1.961	Accept	1.050	-0.869	0.869	Accept
Soil friction angle of layer 5 (ϕ_5)								
Training	1.130	-1.961	1.961	Accept	0.992	-0.937	0.937	Accept
Testing	-1.373	-1.961	1.961	Accept	1.011	-0.916	0.916	Accept
Validation	-0.642	-1.961	1.961	Accept	0.946	-0.869	0.869	Accept
Soil friction angle of layer 6 (ϕ_6)								
Training	-0.473	-1.961	1.961	Accept	1.003	-0.937	0.937	Accept
Testing	0.935	-1.961	1.961	Accept	0.995	-0.916	0.916	Accept
Validation	-0.142	-1.961	1.961	Accept	0.996	-0.869	0.869	Accept
Soil friction angle of layer 7 (ϕ_7)								
Training	0.050	-1.961	1.961	Accept	1.003	-0.937	0.937	Accept
Testing	0.079	-1.961	1.961	Accept	0.985	-0.916	0.916	Accept
Validation	-0.184	-1.961	1.961	Accept	1.005	-0.869	0.869	Accept

Table C.2 The results of null hypothesis tests inputs and outputs. ***(Continued)***

Model Variable and Data Sets	*t*-value	Lower Critical value	Upper Critical value	*t*-test	*F*-value	Lower Critical value	Upper Critical value	*F*-test
Soil friction angle of layer 8 (ϕ_8)								
Training	0.158	-1.961	1.961	Accept	0.985	-0.937	0.937	Accept
Testing	-1.015	-1.961	1.961	Accept	1.038	-0.916	0.916	Accept
Validation	0.809	-1.961	1.961	Accept	1.011	-0.869	0.869	Accept
Soil friction angle of layer 9 (ϕ_9)								
Training	-0.412	-1.961	1.961	Accept	1.016	-0.937	0.937	Accept
Testing	0.195	-1.961	1.961	Accept	0.974	-0.916	0.916	Accept
Validation	0.561	-1.961	1.961	Accept	0.997	-0.869	0.869	Accept
Soil friction angle of layer 10 (ϕ_{10})								
Training	0.861	-1.961	1.961	Accept	0.992	-0.937	0.937	Accept
Testing	-1.787	-1.961	1.961	Accept	1.024	-0.916	0.916	Accept
Validation	0.325	-1.961	1.961	Accept	0.969	-0.869	0.869	Accept
Soil thickness of layer 1 (h_1)								
Training	-0.027	-1.961	1.961	Accept	0.974	-0.937	0.937	Accept
Testing	-0.611	-1.961	1.961	Accept	1.070	-0.916	0.916	Accept
Validation	0.713	-1.961	1.961	Accept	0.986	-0.869	0.869	Accept
Soil thickness of layer 2 (h_2)								
Training	-0.152	-1.961	1.961	Accept	1.004	-0.937	0.937	Accept
Testing	-0.561	-1.961	1.961	Accept	0.989	-0.916	0.916	Accept
Validation	0.917	-1.961	1.961	Accept	1.054	-0.869	0.869	Accept
Soil thickness of layer 3 (h_3)								
Training	-0.755	-1.961	1.961	Accept	0.994	-0.937	0.937	Accept
Testing	1.393	-1.961	1.961	Accept	1.033	-0.916	0.916	Accept
Validation	-0.088	-1.961	1.961	Accept	0.941	-0.869	0.869	Accept
Soil thickness of layer 4 (h_4)								
Training	0.113	-1.961	1.961	Accept	1.008	-0.937	0.937	Accept
Testing	0.786	-1.961	1.961	Accept	0.983	-0.916	0.916	Accept
Validation	-1.097	-1.961	1.961	Accept	0.968	-0.869	0.869	Accept
Soil thickness of layer 5 (h_5)								
Training	0.346	-1.961	1.961	Accept	0.995	-0.937	0.937	Accept
Testing	-0.265	-1.961	1.961	Accept	1.042	-0.916	0.916	Accept
Validation	-0.365	-1.961	1.961	Accept	0.949	-0.869	0.869	Accept

Table C.2 The results of null hypothesis tests inputs and outputs. ***(Continued)***

Model Variable and Data Sets	***t*-value**	**Lower Critical value**	**Upper Critical value**	***t*-test**	***F*-value**	**Lower Critical value**	**Upper Critical value**	***F*-test**
Soil thickness of layer 6 (h_6)								
Training	-0.151	-1.961	1.961	Accept	0.997	-0.937	0.937	Accept
Testing	-0.285	-1.961	1.961	Accept	1.021	-0.916	0.916	Accept
Validation	0.599	-1.961	1.961	Accept	0.921	-0.869	0.869	Accept
Soil thickness of layer 7 (h_7)								
Training	-0.445	-1.961	1.961	Accept	0.988	-0.937	0.937	Accept
Testing	1.558	-1.961	1.961	Accept	1.010	-0.916	0.916	Accept
Validation	-0.885	-1.961	1.961	Accept	0.978	-0.869	0.869	Accept
Soil thickness of layer 8 (h_8)								
Training	0.653	-1.961	1.961	Accept	1.000	-0.937	0.937	Accept
Testing	-0.795	-1.961	1.961	Accept	1.010	-0.916	0.916	Accept
Validation	-0.362	-1.961	1.961	Accept	0.972	-0.869	0.869	Accept
Soil thickness of layer 9 (h_9)								
Training	0.705	-1.961	1.961	Accept	0.990	-0.937	0.937	Accept
Testing	-0.890	-1.961	1.961	Accept	1.020	-0.916	0.916	Accept
Validation	-0.366	-1.961	1.961	Accept	0.980	-0.869	0.869	Accept
Footing width (*B*)								
Training	-0.760	-1.961	1.961	Accept	0.983	-0.937	0.937	Accept
Testing	0.552	-1.961	1.961	Accept	0.998	-0.916	0.916	Accept
Validation	0.862	-1.961	1.961	Accept	0.973	-0.869	0.869	Accept
Average bearing capacity of strip footing (q_{av})								
Training	1.280	-1.961	1.961	Accept	1.012	-0.937	0.937	Accept
Testing	-1.162	-1.961	1.961	Accept	0.984	-0.916	0.916	Accept
Validation	-1.124	-1.961	1.961	Accept	0.949	-0.869	0.869	Accept
$\tilde{N}_c$								
Training	-0.510	-1.961	1.961	Accept	0.972	-0.937	0.937	Accept
Testing	1.388	-1.961	1.961	Accept	1.037	-0.916	0.916	Accept
Validation	-0.550	-1.961	1.961	Accept	0.919	-0.869	0.869	Accept
$\tilde{N}_\phi$								
Training	0.159	-1.961	1.961	Accept	0.964	-0.937	0.937	Accept
Testing	-0.019	-1.961	1.961	Accept	1.078	-0.916	0.916	Accept
Validation	-0.288	-1.961	1.961	Accept	1.121	-0.869	0.869	Accept