EFFECT OF SOIL VARIABILITY ON THE BEARING CAPACITY OF FOOTINGS ON MULTI-LAYERED SOIL

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To my wife Caryn

and my parents NguongTeck and MeeDing

PREFACE

This work was undertaken between November 2002 and October 2008 within the School of Civil, Environmental and Mining Engineering at the University of Adelaide. Throughout the thesis, any materials, techniques, methods and concepts obtained from other sources have been acknowledged and credited. The following sections list the works which the author claims originality.

In **Chapter 4**:

- The implementation and incorporation of the random field simulator (i.e. local average subdivision (LAS) into finite element limit analysis formulation;

In **Chapter 5**:

- The analyses and quantification of the effect of soil variability on the bearing capacity of footings founded on two-layered, purely cohesive soil;

In **Chapter 6**:

- The analyses of strip footings on four- and ten-layered, purely cohesive soil;
- Development of ANN-based models for predicting the bearing capacity of strip footings on multi-layered, cohesive soil profiles;

In **Chapter 7**:

- The analyses of strip footings on ten-layered, purely cohesive-frictional soil; and

- Development of ANN-based models for predicting the bearing capacity of strip footing on multi-layered cohesive-frictional soil profiles;

Listed below are the publications, which have been published as a direct result of this study:

Kuo, Y. L., Jaksa, M. B., Lyamin, A. V. and Kaggwa, W. S. (2008). ANN-based Model for Predicting the Bearing Capacity of Strip Footing on Multi-layered Cohesive Soil. *Computers and Geotechnics.* doi:10.1016/j.compgeo.2008.07.002.

Jaksa, M. B., Goldsworthy, J. S., Fenton, G. A., Kaggwa, G. W. S., Griffiths, D. V., Kuo, Y. L. and Poulos, H. G. (2005). Towards Reliable and Effective Site Investigations. *Geotechnique,* Vol. 55, No. 2, pp. 109-121.

Kuo, Y. L., Jaksa, M. B., Kaggwa, W. S., Fenton, G. A., Griffiths, D. V. and Goldsworthy, J. S. (2004). Probabilistic Analysis of Multi-layered Soil Effects on Shallow Foundation Settlement. *Proc. 9th Australia New Zealand Conference on Geomechanics,* Auckland, New Zealand, Vol. 2, pp. 541-547.

Goldsworthy, J. S., Jaksa, M. B., Fenton, G. A., Kaggwa, W. S., Griffiths, D. V., Poulos, H. G. and Kuo, Y. L. (2004). Influence of Site Investigations on the Design of Pad Footings. *Proc. 9th Australia New Zealand Conference on Geomechanics,* Auckland, New Zealand, Vol. 1, pp. 282-288.

ABSTRACT

Footings are often founded on multi-layered soil profiles. Real soil profiles are often multi-layered with material constantly varying with depth, which affects the footing response significantly. Furthermore, the properties of the soil are known to vary with location. The spatial variability of soil can be described by random field theory and geostatistics. The research presented in this thesis focuses on quantifying the effect of soil variability on the bearing capacity of rough strip footings on single and two-layered, purely-cohesive, spatially variable soil profiles. This has been achieved by using Monte Carlo analysis, where the rough strip footings are founded on simulated soil profiles are analysed using finite element limit analysis. The simulations of virtual soil profiles are carried out using Local Average Subdivision (LAS), a numerical model based on the random field theory. An extensive parametric study has been carried out and the results of the analyses are presented as normalized means and coefficients of variation of bearing capacity factor, and comparisons between different cases are presented. The results indicate that, in general, the mean of the bearing capacity reduces as soil variability increases and the worstcase scenario occurs when the correlation length is in the range of 0.5 to 1.0 times the footing width.

The problem of estimating the bearing capacity of shallow strip footings founded on multi-layered soil profiles is very complex, due to the incomplete knowledge of interactions and relationships between parameters. Much research has been carried out on single- and two-layered homogeneous soil profiles. At present, the inaccurate weighted average method is the only technique available for estimating the bearing capacity of footing on soils with three or more layers. In this research, artificial neural networks (ANNs) are used to develop meta-models for bearing capacity estimation. ANNs are numerical modelling techniques that imitate the human brain capability to learn from experience. This research is limited to shallow strip footing founded on soil mass consisting of ten layers, which are weightless, purely cohesive and cohesive-frictional.

A large number of data has been obtained by using finite element limit analysis. These data are used to train and verify the ANN models. The shear strength (cohesion and friction angle), soil thickness, and footing width are used as model inputs, as they are influencing factors of bearing capacity of footings. The model outputs are the bearing capacities of the footings. The developed ANN-based models are then compared with the weighted average method. Hand-calculation design formulae for estimation of bearing capacity of footings on ten-layered soil profiles, based on the ANN models, are presented. It is shown that the ANN-based models have the ability to predict the bearing capacity of footings on ten-layered soil profiles with a high degree of accuracy, and outperform traditional methods.

STATEMENT OF ORIGINALITY

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968.

Signed: Date: 10th October 2008

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CHAPTER 7

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SUMMARY AND CONCLUSIONS

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NOTATION

All variables used in this thesis are defined as they are introduced into the text. For convenience, frequently used variables and their units are described as below. The general convention adopted is that vector and matrix variables are shown in bold print, while scalar variables are shown in italic.

A	surface area/cross sectional area;
$\mathbf{A}$	total matrix of equality constraint gradients (finite element limit analyses);
$\mathbf{a_i}$	vector of constraint variable;
B	width of the footing (m);
B'	effective width of the footing (m);
$\mathbf{b}$	right hand side for linear equalities;
C	rescaled hidden layer threshold;
$C_{y_j d_j}$	the covariance between the model output and measured actual output;
c	cohesion of soil (kPa);
c_i	cohesion of individual soil layer (kPa);
$\mathbf{c}^{\mathbf{T}}$	objective function;
COV	coefficient of variation;
D_f	embedment depth (m);
$\overline{d}$	the mean of measured actual output; and
d_j	the historical or measured actual output;
E	elastic modulus (MPa) (finite element analysis);

E	global error function (artificial neural networks);
$E[\ldots]$	expected value operator (random field theory);
f	yield function (finite element limit analyses);
F	bearing capacity factor (foundations);
F_i	body force (finite element limit analyses);
F_k	yield function (finite element limit analyses);
$G_c(\ldots)$	normally distributed random field, having zero mean, unit variance, and a scale of fluctuation (random field theory);
$G_{\ln c}(\ldots)$	lognormally distributed random field (random field theory);
$\boldsymbol{g}$, g_i	vector/component of prescribed body force;
H	depth of the soil layer (m);
h_i	thickness of individual soil layer (m);
I	number of model inputs;
J_1, J_2, J_3	stress invariants;
K_p	Rankine' passive earth pressure coefficient;
K_s	punching shear coefficient;
L	length of the strip footing (m);
n	number of data.
N^{*}_{c}	modified non-dimensional bearing capacity factor for multi-layered soil;
$\tilde{N}_c$	non-dimensional bearing capacity factor for footings on multi-layered purely-cohesive soil profiles;
$\tilde{N}_{c-\phi}$	non-dimensional bearing capacity factor for footings on multi-layered cohesive-frictional soil profiles
N_c	non-dimensional bearing capacity factor;
N_g	non-dimensional bearing capacity factor;

N_q	non-dimensional bearing capacity factor;
$\dot{\varepsilon}^{\mathrm{p}}$	strain rate vector;
p'	surcharge (kN/m^2);
P_p	passive force (kN);
q	load per unit area (kN/m^2);
$\mathbf{q}$, q_i	vector/components of optimisable surface traction;
q_b	bearing capacity of bottom soil layer (kN/m^2);
Q_u	ultimate bearing capacity (kN);
q_u	ultimate load per unit area (kN/m^2);
$q_{u(1)}$	first failure load per unit area (kN/m^2);
$q_{u(c)}$	ultimate load per unit area of footing on purely-cohesive soil (kN/m^2);
$q_{u(c\text{-}\phi)}$	ultimate load per unit area of footing on cohesive-frictional soil (kN/m^2);
r	correlation coefficient;
s	vector/components of optimisable surface traction;
Δu	tangential velocity jump;
$\dot{u}$	displacement rate;
T_i	connection weight of hidden nodes (artificial neural networks);
T_i	external surface tractions (finite element limit analyses);
V	volume (m^3);
v	Poisson's ratio of soil;
Δv	normal velocity jump;
w_i	connection weight of node i;
$\mathbf{X}$	global vector of unknown stresses;
$\mathbf{x}$	problem variables, vector of stress variables;
x_n	scaled value;

x_{min}	minimum values;
x_{max}	maximum values;
y_j	the predicted output by the network;
$\overline{y}$	the mean of model output;
z	depth below the soil surface (m);
α	load-spread angle (°);
β	load-spread angle (°);
δ	scale of fluctuation (random field theory);
ϕ	friction angle of the soil (°);
ϕ_ι	friction angle of individual soil layer (°);
γ	bulk unit weight of the soil (kN/m^3);
η	learning rate (artificial neural networks);
λ_c	normalised overburden pressure;
λ_q	normalised bearing capacity;
$\dot{\lambda}$	plastic multiplier rate;
λ_F^s	scalar loading multiplier for body force;
λ_T^s	scalar loading multiplier for external surface tractions;
μ	momentum term (artificial neural networks);
μ	mean (random field theory);
$\mu_{ln\ \mathrm{c}}$	mean of lognormal variables (random field theory);
θ_c	correlation length of soil cohesion (Local average subdivision);
ρ	strength gradient;
σ	normal stress vector (finite element limit analyses);

σ	standard deviation (random field theory);
σ_{d_j}	the standard deviation of measured actual output (artificial neural networks);
$\sigma_{ln\ c}$	standard deviation of lognormal variables (random field theory);
σ_{y_j}	the standard deviation of model output (artificial neural networks);
σ_z	vertical stress at the base of the foundation (kN/m^2) (foundations);
τ	distance vector (random field theory);
τ	shear stress vector (finite element limit analyses);