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The strong-coupling limit of lattice Landau gauge

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We report on our recent study of the gluon and ghost propagators of pure SU(2) minimal lattice Landau gauge in the strong-coupling limit. In this limit, we find evidence of the conformal infrared behaviour of these propagators as predicted by functional continuum methods. However, in the strong-coupling limit this happens for lattice momenta with $a^2q^2 > 1$, in units of the lattice spacing a. Deviations from conformal scaling for $a^2q^2 < 1$ are well parameterised by a transverse gluon mass. A comparison of various lattice definitions of gauge potentials, all equivalent in the continuum limit, shows that (a) both the critical exponent and coupling can be extracted unambiguously from the high-momentum data in the strong-coupling limit, in good agreement with the continuum predictions; but that on the other hand (b) the massive branch depends on the definition of lattice gluon fields and is thus not unambiguously defined. We demonstrate that this ambiguity is also present in the low-momentum region for commonly used values of the lattice coupling in SU(2).

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1. Introduction

Continuum functional methods favour QCD's gluon and ghost propagators in Landau-gauge to show a conformal infrared behaviour where their respective dressing functions behave as [1–4]

$$Z(p^2) \sim (p^2/\Lambda^2_{\mathrm{QCD}})^{2\kappa_Z}\,, \qquad G(p^2) \sim (p^2/\Lambda^2_{\mathrm{QCD}})^{-\kappa_G} \qquad \text{for} \quad p^2 \to 0\,, \tag{1.1}$$

which are both determined by an unique critical infrared exponent $\kappa_Z = \kappa_G \equiv \kappa$ with $0.5 < \kappa < 1$. Under a mild regularity assumption on the ghost-gluon vertex [2], the value of this exponent is furthermore obtained as $\kappa \approx 0.595$ [2, 3]. The conformal nature of this infrared behaviour in the pure Yang-Mills sector of Landau gauge QCD is evident in the generalisation to arbitrary gluonic correlations [5]. In particular, in the limit $p \to 0$ the ghost-gluon vertex is infrared finite, and the non-perturbative running coupling in Eq. (1.2) [1] approaches an infrared fixed-point, $\alpha_s \to \alpha_c$ whose maximum value is $\alpha_c \approx 4.46$ for SU(2) [2]. It has been shown that in presence of the single scale, Λ_{QCD}, the solution with such an infrared behaviour is unique [6].

$$\alpha_s(p^2) = \frac{g^2}{4\pi} Z(p^2) G^2(p^2) \tag{1.2}$$

To observe this *scaling solution*, at least in an approximation, in lattice simulations in a finite box of extend L, a wide separation of scales, $\pi/L \ll p \ll \Lambda_{\mathrm{QCD}}$, is necessary such that a reasonably large number of modes with momenta p sufficiently far below Λ_{QCD} are accessible whose corresponding wavelengths are at the same time much shorter than L. Despite tremendous efforts [7–9] the majority of lattice investigations, however, could not confirm this scaling solution. Rather, compelling agreement between lattice Landau gauge and continuum results has been found when the restriction to the first Gribov region is implemented. In such a case gluons and ghosts decouple at low momenta, due to the appearance of a transverse gluon mass (i.e. an infrared-finite gluon propagator) which leads to an essentially free ghost propagator with a free massless-particle singularity at zero momentum. This type of solution is not within the class of scaling solutions, and it is termed the *decoupling solution* in contradistinction [10].

In [11] we recently reported on our study of the gluon and ghost propagators in the strong-coupling limit, $\beta \to 0$, of pure $SU(2)$ lattice Landau gauge. This unphysical limit, which can be interpreted as the formal limit $\Lambda_{\mathrm{QCD}} \to \infty$, allows us to assess whether the predicted conformal behaviour can be seen for the larger lattice momenta p, after the upper bound $p \ll \Lambda_{\mathrm{QCD}}$ has been removed, in a range where the dynamics due to the gauge action would otherwise dominate and cover it up completely. Furthermore, the strong-coupling limit provides a powerful tool to study the non-perturbative measure for gauge-orbit space in Landau gauge. It is this measure that is being assessed when the gauge-field dynamics is switched off. It turns out that there is a discretisation ambiguity which manifests itself in dependencies on the lattice definition of gauge fields underlying the respective lattice Landau gauges and their measures. The strong-coupling limit serves to isolate this ambiguity which noticeably affects the decoupling branch at $a^2q^2 < 1$. Nonetheless, it is possible to extract infrared critical exponent and coupling at large a^2q^2 consistent with the scaling solution, and unaffected by the discretisation ambiguity.

2. Infrared exponents

We simulate pure $SU(2)$ gauge theory in the strong-coupling limit by generating random link configurations $\{U\}$. These are sets of $SU(2)$ gauge links, $U_{x\mu} = u^0_{x\mu}\mathbf{1} + i\sigma^a u^a_{x\mu}$, equally distributed over $(u^0, \vec{u})_{x\mu} \in S^3$. Those configurations are fixed to the minimal lattice Landau gauge and gluon and ghost propagators are then calculated in momentum space employing standard techniques (see [11] for further details).

The gluon propagator in the strong-coupling limit is observed to increase with momentum, while it plateaus at low momenta. This massive behaviour sets in, irrespective of the lattice size ($N = L/a$), at around $x \equiv a^2q^2 \approx 1$, and the observed mass behaves as $M^2 \equiv \lim_{x\to 0} D^{-1}(x) \propto 1/a^2$ with hardly any significant dependence on N. In particular, if there is a systematic N dependence at all, the zero momentum limit of the gluon propagator tends to slowly increase with the volume. It certainly extrapolates to a finite value $\propto 1/a^2$ in the infinite-volume limit, $N \to \infty$.

In order to assess the asymptotic form of the gluon dressing function at large lattice momenta we have fitted the gluon propagator data to different fitting formulas. The results of these fits for the gluon exponent κ_Z from different fit models and lattice sizes are shown in Fig. 1. The observed dependencies on either one are rather small. There is a general trend for κ_Z to slightly increase with a/L (the dark grey band in Fig. 1) though this is within the systematic uncertainty due to the fit model.

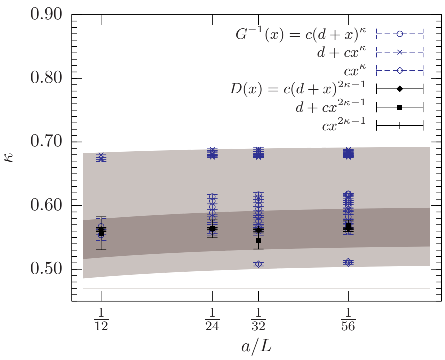

Figure 1: κ versus a/L for the ghost and gluon propagators. Grey-coloured bands mark the variation of κ with the fit model.

Similar fits were performed to extract the exponent κ_G from the ghost dressing function G. These fits are less robust with a more pronounced model dependence (light grey band in Fig. 1). This is mainly due to the wider transition region, from $G =$ const. at small x to $G \sim x^{-\kappa_G}$ at large x, which is under less control here. The exponent can nevertheless be estimated as $\kappa_G = 0.60(7)$. The results are consistent with the scaling relation $\kappa_Z = \kappa_G$.

3. Different gauge-field definitions on the lattice

The strong-coupling limit is an ideal testbed for different lattice definitions of gauge-fields which correspond to different choices of coordinates that agree only near the identity, or in the continuum limit. The definition of the standard lattice Landau gauge (SLG), e.g., corresponds to choosing separate coordinates for the Northern (NH) and Southern Hemispheres (SH) of S^3 in the case of $SU(2)$. Strictly speaking, the SLG gluon propagator therefore corresponds to an average for each link of the contributions from NH and SH to the expectation value. The maximal chart is provided by stereographic projection which covers the whole sphere except for the South Pole. A definition of $SU(2)$ gauge fields on the lattice based on stereographic projection is possible (see [11]). It agrees with the standard definition near the North Pole, and in the continuum limit, but the South Pole is now at infinity and the gauge fields are non-compact. The associated Landau gauge is the modified lattice Landau gauge (MLG) of Ref. [12].

When comparing MLG to the ever popular SLG, there is no advantage that the SLG has over the MLG. Both lattice definitions of Landau gauge have the same continuum limit, and any differences between MLG and SLG data at finite lattice spacings are lattice artifacts. Furthermore, their

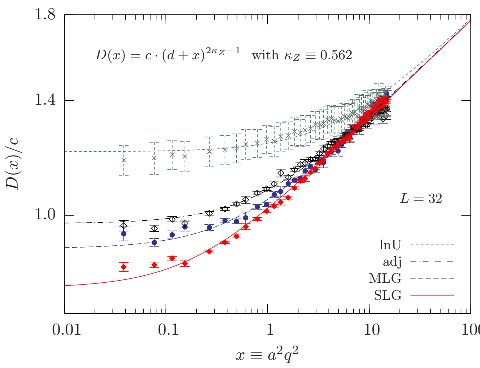

Figure 2: The strong-coupling gluon propagator over a^2q^2 for the various definitions of gauge fields. All on 32^4 lattices and normalised to the scaling branch after fitting to $D(x) = c(d+x)^{2\kappa_Z-1}$; all with $\kappa_Z = 0.562$ from the fit to the SLG data.

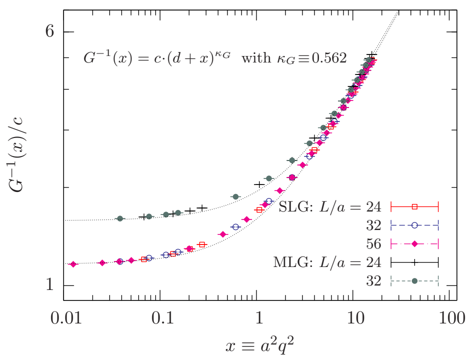

Figure 3: Inverse ghost dressing functions in the strong-coupling limit of minimal lattice Landau gauge using SLG and MLG gauge fields/conditions. Data is normalised to the scaling branch after fitting to $G^{-1}(x) = c(d+x)^{\kappa_G}$; all with $\kappa_G = 0.562$

lattice Landau gauge conditions define gluon fields that are transverse in the physical momentum $aq_\mu = 2\sin(\pi k_\mu/N_\mu)$ (with $k_\mu \in (-N_\mu/2, N_\mu])$ at any finite lattice spacing a.

The data for the gluon propagator of SLG (red filled diamonds) is compared to that of MLG (blue filled circles) in Fig. 2. There we also show data for the gluon propagator where either $aA^{\mathsf{adj}}_{x\mu} = u^0_{x\mu} u^a_{x\mu} \sigma^a$ (no sum μ), or $aA^{\mathsf{ln}}_{x\mu} = \phi^a_{x\mu}\sigma^a/2$ from $U_{x\mu} = e^{i\phi^a_{x\mu}\sigma^a/2}$ were used to define lattice gluon fields based on the adjoint representation [13], A^{adj} (black open diamonds), or on the tangent space at the identity A^{ln} (green crosses). In these two cases, A^{adj} and A^{ln}, for the purpose of a qualitative comparison, we simply use the gauge configurations of the SLG to calculate the gluon propagator. Especially for A^{ln} this implies, however, that the condition $q_\mu(k)A_\mu(k) = 0$ is satisfied at best approximately and nowhere near the precision of SLG or MLG. This uncertainty then causes the somewhat larger errors for this definition as seen in Fig. 2.

First, we fit the data from all four definitions to $D(x) = c(d+x)^{2\kappa_Z-1}$ which provides the best overall description in the full momentum range. In order to demonstrate how the other definitions compare to the SLG, we keep its value for the exponent fixed when fitting the other data, i.e., $\kappa_Z = 0.562$ as obtained for $N = 32$ in SLG is used in all fits. Relative to the scaling branch for large x we then observe a strong definition dependence in the (transverse) gluon mass term at small x (see Fig. 2). The relative weight of the two asymptotic branches, scaling at large x and massive at small, is clearly discretisation dependent and can not be compensated by finite renormalisations. A first indication that the massive branch might indeed be the ambiguous one is the observed $M \propto 1/a$. This is consistent with the fact that the definitions of gauge fields on the lattice, which agree at leading order, all differ at order a^2, and so do their corresponding Jacobian factors which leads to lattice mass counter-terms of different strengths. We find a similar behaviour of the ghost dressing function whose inverse is shown in Fig. 3.

4. Conclusion

The Landau-gauge gluon and ghost propagators in the strong-coupling limit do in fact show the scaling behaviour as predicted by the continuum studies mentioned above. A comparison of

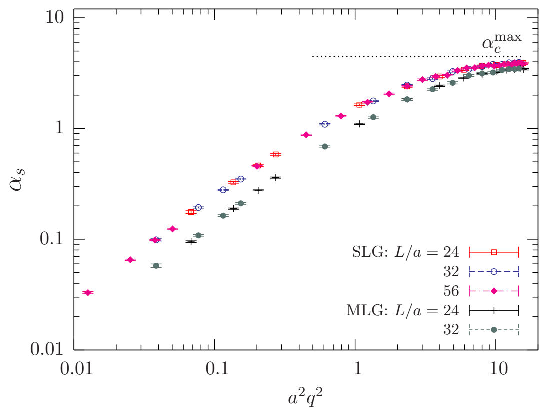

Figure 4: α_s, for the standard (SLG) and modified (MLG) lattice Landau gauge at $\beta = 0$. The dotted line is the critical coupling $\alpha_c^{max} \approx 4.46$ for $N_c = 2$.

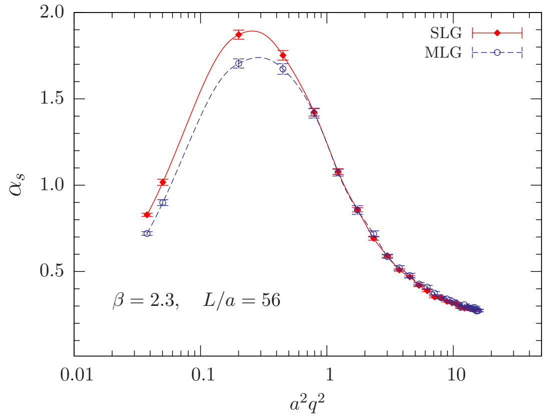

Figure 5: α_s for the standard (SLG) and modified (MLG) lattice Landau gauge at $\beta = 2.3$ on a 56^4 lattice. Lines are spline interpolations to guide the eye.

various lattice definitions of gauge potentials, all equivalent in the continuum limit, shows that critical exponent and coupling can be extracted from the high-momentum data, with $a^2q^2 > 1$, in the strong-coupling limit in good agreement with the continuum predictions, $\kappa_Z = \kappa_G \approx 0.595$. The deviations from this scaling behaviour, on the other hand, depend on the choice of the lattice definition of the gluon fields, i.e., the massive branch observed for $a^2q^2 < 1$.

In complete agreement with this, the coupling (1.2) for large a^2q^2 levels at $\alpha_c \approx 4$, just below the upper bound $\alpha_c^{max} \approx 4.46$ for $SU(2)$, while violations to this conformal scaling, set in as soon as the ambiguity in the definition of minimal lattice Landau gauge does (see Fig. 4). Nonetheless, it is quite compelling that the result, $\alpha_c \approx 4$, is nearly independent of the gauge-field definition.

Indeed, the strong deviations at small momenta are linked to the strong-coupling limit in which discretisation effects are enhanced to the extreme and it is still possible that they disappear in the continuum limit, eventually. But because it is a combination of ultraviolet (mass counter-term) and infrared (breakdown of STIs) effects, this might take very fine lattice spacings in combination with very large volumes and therefore who-knows-how big lattices to verify explicitly. Note that these effects are definitely persist for commonly used β for SU(2) (see Fig. 5 where data from SLG and MLG fixed configurations at $\beta = 2.3$ are shown). These observed differences persist for $\beta = 2.5$.

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