

# Vector Meson Mass Corrections at $\mathcal{O}(a^2)$ in PQChPT with Wilson and Ginsparg-Wilson quarks

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## Abstract

We derive the mixed as well as unmixed lattice heavy meson chiral Lagrangian up to order  $\mathcal{O}(a^2)$ , with Wilson and Ginsparg-Wilson fermions. We consider two flavor partially quenched QCD and calculate vector meson mass corrections up to order  $\mathcal{O}(a^2)$ , including the corrections associated with the violation of rotational  $O(4)$  symmetry down to the hypercubic group. Our calculations also include the one-loop, phenomenological contribution from the  $\rho \rightarrow \pi\pi$  decay channel. The final result is a chiral-continuum extrapolation formula with model dependent coefficients from which one can recover the physical  $\rho$  meson mass from the large amount of current lattice data. As a verification of our result, the chiral-continuum extrapolation formula is compared with that used in numerical simulations.

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## INTRODUCTION

In the present day lattice simulations the quark masses used are much heavier than those in nature because of the increased computational cost at low mass. Instead of trying to reach to lighter quark masses one runs lattice simulations with heavier quarks and then performs a chiral extrapolation to the physical regime using the predictions of chiral perturbation theory (ChPT) [1, 2]. However, it is not totally consistent to apply ChPT for extrapolation of lattice results because it is a continuum theory which doesn't contain information about the lattice spacing. That is why it is first required to extrapolate lattice data to the continuum limit and then perform the chiral extrapolation to physical regime. In order to consistently include lattice discretization effects, Symanzik's effective theory needs to be applied [3, 4]. This approach is used, for example, in Refs. [5, 6, 7] and generalized in Refs. [8, 9, 10] to study the mixed lattice theories with Wilson fermions as sea and Ginsparg-Wilson fermions as valence quarks. Being relatively expensive to simulate, Ginsparg-Wilson fermions [11], on the other hand, have an exact chiral symmetry even on the lattice. The latter allows one to exploit the good chiral properties of Ginsparg-Wilson fermions by considering them as valence quarks<sup>1</sup> and considering Wilson fermions as sea quarks [12]. The Lagrangian approach used here is necessary because of the large lattice spacing effects in unquenched simulations (which means that the  $\mathcal{O}(a^2)$  effects need to be accounted). It represents an efficient method by which to improve the numerical accuracy of the physical results.

In this paper we calculate the vector meson mass corrections up to order  $\mathcal{O}(a^2)$  in partially quenched chiral perturbation theory (PQChPT) with graded symmetry  $SU(4|2)$ , using Wilson and Ginsparg-Wilson fermions and the following power counting scheme:

$$p^2 \sim a \sim m_q \tag{1}$$

As the small parameters of our double expansion we will choose:  $\epsilon^2 = (a\Lambda, m_q/\Lambda)$ , so that  $m_q \ll \Lambda \ll 1/a$ , (where  $\Lambda$  is a typical QCD scale).

To calculate the corrections coming from lattice discretization one must formulate the corresponding lattice theory and match the new operators with those in chiral perturbation theory (ChPT). The standard procedure consists of the following parts:

1. writing the Lattice QCD action for the fermions one is going to use;
2. finding the corresponding Symanzik action up to a given order in  $a$  based on symmetry constraints from underlying lattice theory;
3. performing a spurion analysis with further projection to ChPT.

If we want to find the mass corrections to the vector mesons, we need to formulate ChPT for both pseudoscalars and vector mesons. ChPT for pseudoscalars with Wilson quarks was formulated, for example, in Ref.[7]. It was combined with Ginsparg-Wilson fermions as a mixed theory in Refs.[8, 10]. Continuum heavy meson ChPT was formulated in Ref.[13]

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<sup>1</sup> such valence fermions will have masses smaller than the valence quark masses accessible using Wilson fermions

and further developed for QChPT and PQChPT in Refs.[14] and [15]. An example of two-flavor PQChPT can be found in Ref.[18]. The main disadvantage of similar heavy vector meson ChPT formulations was that they didn't include terms which explicitly break  $O(4)$  rotational symmetry down to the hypercubic group. Recent studies, particularly by Tiburzi in Ref. [10], with Wilson and Ginsparg-Wilson quarks, showed explicitly the terms in the heavy baryon Lagrangian that break  $O(4)$  symmetry, up to order  $\mathcal{O}(a^2)$ . Following similar lines, we formulate heavy meson partially quenched ChPT (HM PQChPT) for Wilson and Ginsparg-Wilson fermions up to order  $\mathcal{O}(a^2)$ , including chiral as well as continuum symmetry breaking.

The paper is organized in the following way: in Sec.II we review the well developed PQChPT  $SU(4|2)$  Lagrangian for the pseudoscalar meson sector with mixed and unmixed Wilson and Ginsparg-Wilson fermions up to order  $\mathcal{O}(a^2)$ ; in Sec.III the extended heavy meson PQChPT Lagrangian is derived up to order  $\mathcal{O}(a^2)$ , including the terms explicitly breaking  $O(4)$  rotational symmetry down to the hypercubic group (this is actually the adjustment of Ref.[10] applied to vector mesons); in Sec.IV we calculate the  $\rho$  meson mass corrections including phenomenological one-loop correction from  $\rho \rightarrow \pi\pi$  and in Sec.V the chiral-continuum extrapolation formula is derived and compared with the phenomenological expression in Ref. [21], in terms of which the  $\rho$  meson mass was successfully extracted.

## PSEUDOSCALAR MESON SECTOR

Let us denote the quarks of  $SU(4|2)$  as

$$Q = (x, y, u, d, \tilde{x}, \tilde{y})^T, \quad (2)$$

where  $x$  and  $y$  label the valence fermions whose ghost partners  $(\tilde{x}, \tilde{y})$  cancel their effects in the sea loops. Thus, the only sea loop contributions come from unquenched  $u$  and  $d$  quarks. Now, assume that  $m_x = m_y$  and  $m_u = m_d$  which is natural if we want to calculate the correction to the mass of the  $\rho$  meson. The mass and Wilson matrices [19] are then:

$$m_Q = \text{diag}(m_x, m_x, m_u, m_u, m_x, m_x), \quad (3)$$

$$w_Q = \text{diag}(w_v, w_v, w_s, w_s, w_v, w_v) \quad (4)$$

where  $w_v$  refers to valence and  $w_s$  to sea quarks. Note that if quark is a Wilson fermion then:  $(w_Q)_i = 1$ , while if it is GW then:  $(w_Q)_i = 0$ . This could be useful for the construction of mixed types of ChPT, as in Refs.[8, 10].

The leading order (LO) Lagrangian,  $L_2 \sim \mathcal{O}(p^2, a, m_q)$ , which includes chiral symmetry breaking from quark mass and lattice discretization, is given by the expression ([8, 10]):

$$L_2 = \frac{f^2}{8} \langle \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \rangle - \lambda_m \langle m_Q \Sigma^\dagger + m_Q^\dagger \Sigma \rangle - a\lambda_a \langle w_Q \Sigma^\dagger + w_Q^\dagger \Sigma \rangle \quad (5)$$

where the angular brackets stand for the super-trace over the flavor indices,  $f$  is the pion decay constant,  $\lambda_{m(a)}$  are low energy constants and  $\Sigma = \exp(2i\Phi/f)$  contains the matrix of meson fields,  $\Phi \in SU(4|2)$ . The next to LO (NLO) Lagrangian would be of order  $\mathcal{O}(\epsilon^4) \sim \mathcal{O}(p^4, p^2 a, p^2 m_q, a^2, m_q^2, a m_q)$ , which is rather lengthy (see Ref.[7]). However, in calculating the vector meson mass correction the NLO pseudoscalar meson Lagrangian will give corrections of  $\mathcal{O}(\epsilon^5)$  in which we are not interested. We also explicitly omitted from

the Lagrangian the contribution from the  $\eta'$  (the  $SU(2)$  analog of it) which, because of the  $U(1)_A$  anomaly, is integrated out from the theory as a heavy degree of freedom. However, double hairpin vertices involving the  $\eta'$  were used to find disconnected propagators.

The following results from Ref.[10] are needed for our calculations:

$$m_{QQ'}^2 = \frac{4}{f^2} [\lambda_m(m_Q + m_{Q'}) + a\lambda_a(w_Q + w_{Q'})], \quad (6)$$

$$G_{xy} = -\frac{1}{2} \frac{(q^2 + m_{sea}^2)}{(q^2 + m_{val}^2)^2}, \quad G_{vs} = \frac{1}{q^2 + m_{mix}^2}, \quad (7)$$

where  $m_{QQ'}$  is the mass of the pseudoscalar meson with quarks  $Q$  and  $Q'$  and  $m_{sea}$ ,  $m_{val}$  and  $m_{mix}$  are mesons which consist of sea-sea, valence-valence and valence-sea quarks correspondingly. In our case:

$$m_{sea}^2 = \frac{8}{f^2} [\lambda_m m_u + a\lambda_a], \quad (8)$$

$$m_{val}^2 = \frac{8}{f^2} [\lambda_m m_x + a\lambda_a],$$

$$m_{mix}^2 = \frac{4}{f^2} [\lambda_m(m_x + m_u) + 2a\lambda_a].$$

Finally,  $G_{xy}$  is the disconnected meson propagator, which includes all diagrams with double hairpin insertions and  $G_{vs}$  is the connected propagator for the meson lines which consist of one sea and one valence quark.

## VECTOR MESON SECTOR

Standard chiral perturbation theory for vector mesons has been formulated by Jenkins, Manohar and Wise in Ref. [13], the quenched counterpart by Booth, Chiladze and Falk in Ref. [14] and the partially quenched counterpart by C. K. Chow and S. J. Rey in Ref. [15]. The study of vector meson masses within the context of ChPT<sup>2</sup> can be found in Ref. [17]. Here we will construct the partially quenched theory using mixed type of fermions and including the lattice discretization effects up to order  $\mathcal{O}(a^2)$ .

The heavy vector meson Lagrangian, up to order  $\mathcal{O}(\epsilon^4)$ , consists of the following parts:

$$L^{(1)} \sim \mathcal{O}(p), \quad (9)$$

$$L^{(2)} \sim \mathcal{O}(a, m_q),$$

$$L^{(4)} \sim \mathcal{O}(a^2, m_q^2, am_q).$$

The Lagrangian  $L^{(3)}$  is omitted here because it contains derivatives which can be eliminated using the equations of motion [20]. In the partially quenched (PQ) case the vector meson

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<sup>2</sup> performing an expansion in terms of the momenta, quark masses and  $1/N_c$

multiplet (in the large  $N_c$  limit) is described by a  $6 \times 6$  matrix field (similar to the Goldstone meson sector):

$$N_\mu = \begin{pmatrix} \mathcal{V} & \Psi \\ \Psi^\dagger & \tilde{\mathcal{V}} \end{pmatrix}_\mu \quad (10)$$

where  $\mathcal{V}$  is a  $(4 \times 4)$  matrix of  $q\bar{q}$  states,  $\tilde{\mathcal{V}}$  is  $(2 \times 2)$  matrix which consists of  $\tilde{q}\tilde{\bar{q}}$  states,  $\Psi$  and  $\Psi^\dagger$  are rectangular matrices involving fermionic bound states with  $q\bar{q}$  or  $\tilde{q}\tilde{\bar{q}}$  content. Under the  $G = SU(4|2)_L \times SU(4|2)_R \times U(1)_V$  graded chiral symmetry group, the heavy meson field transforms as:

$$N_\mu \rightarrow UN_\mu U^\dagger, \quad U \in G \quad (11)$$

and under charge conjugation:

$$CN_\mu C^{-1} = -N_\mu^T. \quad (12)$$

As in conventional ChPT [13] (as well as QChPT [14] and PQChPT [15]), the vector meson multiplet will be treated as heavy, static source. We will also assume that the static vector meson propagates with a fixed four-velocity  $v^\mu$ ,  $v^2 = 1$  and interacts with soft Goldstone multiplets. The three polarization states of the vector mesons are perpendicular to the propagation direction (i.e.  $v \cdot N = 0$ ).

Taking all of these into account, let us now proceed to construct the relevant Lagrangians. The LO heavy-meson chiral Lagrangian will consist of terms of order  $\mathcal{O}(p) \sim \mathcal{O}(\epsilon)$  which are:

$$L_{kin} = -i \langle N_\mu^\dagger (v \cdot D) N_\mu \rangle - iA_1 \langle N_\mu^\dagger \rangle (v \cdot D) \langle N_\mu \rangle \sim \mathcal{O}(\epsilon), \quad (13)$$

$$\begin{aligned} L_{int} = & i g_1 \langle N_\mu^\dagger \rangle \langle N_\nu A_\lambda \rangle v_\sigma \epsilon^{\mu\nu\lambda\sigma} + h.c. + i g_2 \langle \{N_\mu^\dagger, N_\nu\} A_\lambda \rangle v_\sigma \epsilon^{\mu\nu\lambda\sigma} + \\ & i g_3 \langle N_\mu^\dagger \rangle \langle N_\nu \rangle \langle A_\lambda \rangle v_\sigma \epsilon^{\mu\nu\lambda\sigma} + i g_4 \langle N_\mu^\dagger N_\nu \rangle \langle A_\lambda \rangle v_\sigma \epsilon^{\mu\nu\lambda\sigma} \sim \mathcal{O}(\epsilon), \end{aligned} \quad (14)$$

where the following notations are implied:

$$D_\mu = \partial_\mu + [V^\mu, \ ], \quad V^\mu = \frac{1}{2} (\xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi), \quad A^\mu = \frac{1}{2} (\xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi) \quad (15)$$

$$\xi = \sqrt{\Sigma}, \quad M = \frac{1}{2} (\xi^\dagger m_Q \xi^\dagger + \xi m_Q \xi). \quad (16)$$

Here we treat  $A_1$  as a small quantity, so the effects of this term can be reabsorbed in the other parameters by the field redefinition:

$$N'_\mu = N_\mu + \frac{A_1}{2} \langle N_\mu \rangle. \quad (17)$$

This redefined field still transforms under the chiral group in the same way as  $N_\mu$ . As we may notice, there are four-types of chiral invariant interactions in Eq. (14) (consistent with graded chiral symmetry) between vector meson multiplets and pseudoscalar mesons. In the large  $N_c$  limit the coupling constants scale with  $N_c$  as follows:  $g_1 \sim \frac{1}{N_c}$ ,  $g_2 \sim 1$ ,  $g_3 \sim \frac{1}{N_c^2}$ ,

$g_4 \sim \frac{1}{N_c}$ . However, according to Ref. [14], the corresponding quark (and color) flow diagrams are not planar, so they need to be multiplied by an additional factor of  $1/N_c$ .

Because we are interested in finding the one-loop correction to the flavor charged vector meson mass in isospin limit there will be no contribution from the terms proportional to  $g_1$  and  $g_3$ . These terms are responsible for quark flow diagrams with disconnected vector meson line, and because we require to have two different flavors at valence lines we cannot have this type of hairpins (as we don't consider any flavor changing interactions). The diagrams from the term proportional to  $g_4$  are one hairpin like diagrams (due to coupling of the vector meson with the two flavor analog of  $\eta'$ ). Sum of these diagrams with vacuum bubble insertions can be shown to vanish after the decoupling of flavor singlet<sup>3</sup>.

The next to LO (NLO) Lagrangian  $-L_2 \sim \mathcal{O}(\epsilon^2)$ , according to our counting scheme, will consist of terms of order  $\mathcal{O}(m_q)$  and  $\mathcal{O}(a)$ . The first Lagrangian  $L_{mass} \sim \mathcal{O}(m_q)$  is:

$$\begin{aligned} L_{mass} = & \bar{\mu} \langle N_\mu^\dagger N_\mu \rangle + \mu_1 \langle N_\mu^\dagger \rangle \langle N_\mu \rangle + \\ & + a_1 \langle N_\mu^\dagger N_\mu \rangle \langle M \rangle + a_2 \langle N_\mu^\dagger \rangle \langle N_\mu \rangle \langle M \rangle \\ & + \lambda_1 (\langle N_\mu^\dagger \rangle \langle N_\mu M \rangle + h.c.) + \lambda_2 \langle \{N_\mu^\dagger, N_\mu\} M \rangle \sim \mathcal{O}(\epsilon^2), \end{aligned} \quad (18)$$

where the first term in Eq. (18) corresponds to residual mass  $\bar{\mu}$  of vector meson multiplets which is always possible to remove by a suitable reparametrization transformation [16], (through the choice of the reference momentum). Here we will use this freedom and assign  $\bar{\mu} = 0$ . The second term is small because of Zweig's rule, therefore we treat it as the same order as the quark masses,  $\mathcal{O}(\epsilon^2)$  [17]. The chirally invariant terms in Eq. (18) proportional to  $a_{1,2}$  are contributing directly to the vector meson mass. For further simplicity and in order to match the conventional ChPT in Ref. [13] in continuum and large  $N_c$  limits we will assign  $a_{1,2} = 0$ . The last two terms in  $L_{mass}$ , proportional to  $\lambda_{1,2}$ , correspond to  $SU(2)$  isospin breaking because of the quark masses. To match with the mass Lagrangian in Ref. [13] (Eq.24) we will take  $\mu_1 = \lambda_1 = 0$  throughout this paper reminding that this particular choice will be justified only by the final results.

From the analysis above it follows that the flavor non-diagonal heavy vector meson propagator is:

$$G_{\mu\nu} = \frac{(v^\mu v^\nu - g^{\mu\nu})}{v \cdot k + i\epsilon} \quad (19)$$

where  $k^\mu = p_V^\mu - (M_V - \bar{\mu})v^\mu$  is its residual momentum,  $p_V^\mu$  and  $M_V$  are the four-momentum and mass of the vector meson correspondingly.

The second NLO term  $L_a \sim \mathcal{O}(a)$  could be written in the following form:

$$\begin{aligned} L_a = & \alpha_1 (\langle N_\mu^\dagger \rangle \langle N_\mu W_+ \rangle + h.c.) + \alpha_2 \langle \{N_\mu^\dagger, N_\mu\} W_+ \rangle \\ & + \alpha_3 \langle N_\mu^\dagger \rangle \langle N_\mu \rangle \langle W_+ \rangle + \alpha_4 \langle N_\mu^\dagger N_\mu \rangle \langle W_+ \rangle \sim \mathcal{O}(\epsilon^2), \end{aligned} \quad (20)$$

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<sup>3</sup> For example, in Ref. [15] it is assumed that  $g_{1,3,4} = 0$  and  $g_2 = 0.75$ . In principle, our considerations are more general and we could leave these couplings as free parameters which are to be determined by further lattice simulations.

with the definition

$$W_+ \equiv \frac{a\Lambda^2}{2} (\xi^\dagger w_Q \xi^\dagger + \xi w_Q \xi), \quad (21)$$

where  $\alpha_{1,2,3,4}$  are undetermined low energy constants. Notice that the terms in  $L_a$  break chiral symmetry in the same way as masses and that the dominant (with respect to  $N_c$ ) term is one proportional to  $\alpha_2$ . The terms proportional to  $\alpha_{1,3}$  don't give any contribution to the vector meson mass because (similar to  $g_{1,3}$ ) these interactions give hairpin insertions in vector meson line.

Now, let us write the next to NLO (NNLO) Lagrangian of order  $\mathcal{O}(\epsilon^4)$ . As we mention in the beginning the NNLO Lagrangian will consists of the following parts:  $L_{am_q} \sim \mathcal{O}(am_q)$ ,  $L_{m_q^2} \sim \mathcal{O}(m_q^2)$  and  $L_{a^2} \sim \mathcal{O}(a^2)$ .

Using the properties of  $M$  and  $W_+$  under chiral transformations the first term is:

$$\begin{aligned} L_{am_q} = & \beta_1 \langle N_\mu^\dagger \rangle \langle N_\mu \rangle \langle W_+ \rangle \langle M \rangle + \beta_2 \langle N_\mu^\dagger N_\mu \rangle \langle W_+ \rangle \langle M \rangle \quad (22) \\ & + \beta_3 \langle N_\mu^\dagger \rangle \langle N_\mu \rangle \langle W_+ M \rangle + \beta_4 (\langle N_\mu^\dagger W_+ \rangle \langle N_\mu \rangle \langle M \rangle + h.c.) \\ & + \beta_5 (\langle N_\mu^\dagger M \rangle \langle N_\mu \rangle \langle W_+ \rangle + h.c.) + \beta_6 \langle N_\mu^\dagger N_\mu \rangle \langle W_+ M \rangle \\ & + \beta_7 (\langle N_\mu^\dagger W_+ \rangle \langle N_\mu M \rangle + h.c.) + \beta_8 \langle \{N_\mu^\dagger, N_\mu\} W_+ \rangle \langle M \rangle \\ & + \beta_9 \langle \{N_\mu^\dagger, N_\mu\} M \rangle \langle W_+ \rangle + \beta_{10} (\langle N_\mu^\dagger \rangle \langle N_\mu \{W_+, M\} \rangle + h.c.) \\ & + \beta_{11} \langle \{N_\mu^\dagger, \{N_\mu, W_+\}\} M \rangle \end{aligned}$$

Similarly, the second term has the structure:

$$\begin{aligned} L_{m_q^2} = & r_1 \langle N_\mu^\dagger \rangle \langle N_\mu \rangle \langle M \rangle \langle M \rangle + r_2 \langle N_\mu^\dagger \rangle \langle N_\mu \rangle \langle MM \rangle \quad (23) \\ & + r_3 \langle N_\mu^\dagger N_\mu \rangle \langle M \rangle \langle M \rangle + r_4 (\langle N_\mu^\dagger W_+ \rangle \langle N_\mu \rangle \langle M \rangle + h.c.) \\ & + r_5 \langle N_\mu^\dagger N_\mu \rangle \langle MM \rangle + r_6 \langle N_\mu^\dagger M \rangle \langle N_\mu M \rangle \\ & + r_7 \langle \{N_\mu^\dagger, N_\mu\} M \rangle \langle M \rangle + r_8 (\langle N_\mu^\dagger \rangle \langle \{N_\mu, M\} M \rangle + h.c.) \\ & + r_9 \langle \{N_\mu^\dagger, \{N_\mu, M\}\} M \rangle, \end{aligned}$$

where  $\beta_{1,\dots,11}$  and  $r_{1,\dots,9} \sim 1/\Lambda_1$  are undetermined constants ( $\Lambda_1$  is some energy scale). The terms  $r_{1,\dots,8}$  are  $1/N_c$  suppressed, according to Ref. [17]. These Lagrangians give only tree level contributions to the mass at  $\mathcal{O}(\epsilon^4)$  which are proportional to  $am_q$  and  $m_q^2$  correspondingly.

The NNLO Lagrangian of order  $\mathcal{O}(a^2)$  will consist of four parts, the first part has the form:

$$\begin{aligned} L_{a^2}^{WW} = & q_1 \langle N_\mu^\dagger \rangle \langle N_\mu \rangle \langle W_+ \rangle \langle W_+ \rangle + q_2 \langle N_\mu^\dagger \rangle \langle N_\mu \rangle \langle W_+ W_+ \rangle \quad (24) \\ & + q_3 \langle N_\mu^\dagger N_\mu \rangle \langle W_+ \rangle \langle W_+ \rangle + q_4 (\langle N_\mu^\dagger W_+ \rangle \langle N_\mu \rangle \langle W_+ \rangle + h.c.) \\ & + q_5 \langle N_\mu^\dagger N_\mu \rangle \langle W_+ W_+ \rangle + q_6 \langle N_\mu^\dagger W_+ \rangle \langle N_\mu W_+ \rangle \\ & + q_7 \langle \{N_\mu^\dagger, N_\mu\} W_+ \rangle \langle W_+ \rangle + q_8 (\langle N_\mu^\dagger \rangle \langle \{N_\mu, W_+\} W_+ \rangle + h.c.) \\ & + q_9 \langle \{N_\mu^\dagger, \{N_\mu, W_+\}\} W_+ \rangle, \end{aligned}$$

where  $q_{1,\dots,9} \sim -1/\Lambda$ .

In order to construct the second and third parts of  $\mathcal{O}(a^2)$  Lagrangian, taking into account the relation  $\bar{w}_Q \equiv 1 - w_Q$  (from [10]), let us define:

$$P_+ \equiv \frac{1}{2} [\xi^\dagger(w_Q - \bar{w}_Q)\xi + \xi(w_Q - \bar{w}_Q)\xi^\dagger] \quad (25)$$

which transforms under  $G$  as  $UP_+U^\dagger$ . Using this spurion field the second part of the  $\mathcal{O}(a^2)$  Lagrangian will be:

$$\begin{aligned} L_{a^2}^P &= \gamma_0 \langle N_\mu^\dagger N_\mu \rangle + \gamma_1 \langle N_\mu^\dagger \rangle \langle N_\mu \rangle \langle P_+ \rangle + \gamma_2 \langle N_\mu^\dagger N_\mu \rangle \langle P_+ \rangle \\ &+ \gamma_3 \langle \{N_\mu^\dagger, N_\mu\} P_+ \rangle + \gamma_4 (\langle N_\mu^\dagger \rangle \langle N_\mu P_+ \rangle + h.c.), \end{aligned} \quad (26)$$

where  $\gamma_{0,\dots,4} \sim -a^2 \Lambda^3 / \Lambda_1$ . Similar to Eq. (24), the third part of  $\mathcal{O}(a^2)$  Lagrangian will be written in the form:

$$\begin{aligned} L_{a^2}^{PP} &= \delta_1 \langle N_\mu^\dagger \rangle \langle N_\mu \rangle \langle P_+ \rangle \langle P_+ \rangle + \delta_2 \langle N_\mu^\dagger \rangle \langle N_\mu \rangle \langle P_+ P_+ \rangle \\ &+ \delta_3 \langle N_\mu^\dagger N_\mu \rangle \langle P_+ \rangle \langle P_+ \rangle + \delta_4 (\langle N_\mu^\dagger P_+ \rangle \langle N_\mu \rangle \langle P_+ \rangle + h.c.) \\ &+ \delta_5 \langle N_\mu^\dagger N_\mu \rangle \langle P_+ P_+ \rangle + \delta_6 \langle N_\mu^\dagger P_+ \rangle \langle N_\mu P_+ \rangle \\ &+ \delta_7 \langle \{N_\mu^\dagger, N_\mu\} P_+ \rangle \langle P_+ \rangle + \delta_8 (\langle N_\mu^\dagger \rangle \langle \{N_\mu, P_+\} P_+ \rangle + h.c.) \\ &+ \delta_9 \langle \{N_\mu^\dagger, \{N_\mu, P_+\}\} P_+ \rangle, \end{aligned} \quad (27)$$

where  $\delta_{1,\dots,9} \sim -a^2 \Lambda^3 / \Lambda_1$ .

Finally, following Ref. [10], we write the last part of  $\mathcal{O}(a^2)$  Lagrangian, which explicitly break  $O(4)$  symmetry, in the form:

$$\begin{aligned} L_{a^2}^c &= k_1 v_\mu^4 \langle N_\nu^\dagger N_\nu \rangle + k_2 v_\mu^4 \langle N_\nu^\dagger \{N_\nu, P_+\} \rangle + k_3 v_\mu^4 \langle N_\nu^\dagger N_\nu \rangle \langle P_+ \rangle \\ &+ k_4 v_\mu^2 \langle N_\mu^\dagger N_\mu \rangle + k_5 v_\mu^2 \langle N_\mu^\dagger \{N_\mu, P_+\} \rangle + k_6 v_\mu^2 \langle N_\mu^\dagger N_\mu \rangle \langle P_+ \rangle, \end{aligned} \quad (28)$$

where  $k_{1,\dots,6} \sim -a^2 \Lambda^3 / \Lambda_1$ . Later we will see that for the vector meson at rest the mass corrections from lattice discretization up to  $\mathcal{O}(\epsilon^4)$  are of the form:  $M_\rho(a) = M_\rho(0) - a^2 \Lambda D$ , where  $D$  is some constant.

## VECTOR MESON MASSES UP TO ORDER $\mathcal{O}(a^2)$

This developed heavy meson (HM) PQChPT with  $SU(4|2)$  graded symmetry allows us to calculate the corresponding vector meson mass correction. The vector meson mass in both chiral and continuum expansions can be written in the form:

$$M_V^2 = \left( M_0(\mu) + \sigma_{tree}^{(2)}(\mu) + \sigma_{tree}^{(3)}(\mu) + \sigma_{tree}^{(4)}(\mu) + \dots \right)^2 + \sigma_{loop}^{(3)}(\mu) + \sigma_{loop}^{(4)}(\mu) \dots, \quad (29)$$

where  $M_0(\mu)$  is the renormalized vector meson mass in continuum and chiral limits,  $\sigma_{tree(loop)}^{(n)}(\mu)$  is the tree (loop) contribution to the self energy of order  $\mathcal{O}(\epsilon^n)$  and  $\mu$  is the renormalization scale. At order  $\mathcal{O}(\epsilon^2)$  we have the following tree level contribution to the mass:

$$\sigma_{tree}^{(2)} = -2\lambda_2 m_x - 2a\Lambda^2 [\alpha_2 w_v + \alpha_4 w_s]. \quad (30)$$



We will not have any contribution of order  $\mathcal{O}(\epsilon^3)$  simply because we don't have Lagrangian of this order, so  $\sigma_{tree}^{(3)}(\mu) = 0$ .

Now, let us write the  $\mathcal{O}(\epsilon^4)$  correction as:

$$\begin{aligned} \sigma_{tree}^{(4)} = & -\frac{1}{\Lambda_1} [a\Lambda^2 (Am_x + Bm_u) + \bar{A}m_x^2 + \bar{B}m_u^2] \\ & - a^2\Lambda^3 (C + Dv_\mu^4 N_\nu^\dagger N_\nu + \bar{D}v_\mu^2 N_\mu^\dagger N_\mu), \end{aligned} \quad (31)$$

where  $A, B, \bar{A}, \bar{B}, C, D$  and  $\bar{D}$  are undetermined coefficients <sup>4</sup>.

The contribution from the tadpole-like loops, with double hairpin insertion is:

$$\sigma_{loop}^{(4)} = \frac{4\alpha_2 a \Lambda^2}{\Lambda_1^2} \left[ (w_s + w_v)F(m_{mix}, \mu) - 2w_v F(m_{val}, \mu) - 2w_v \Delta m^2 \frac{\partial F(m_{val}, \mu)}{\partial m_{val}^2} \right] \quad (32)$$

$$F(m, \mu) \equiv m^2 \ln \frac{m^2}{\mu^2} \quad (33)$$

where  $\Delta m^2 \equiv m_{val}^2 - m_{sea}^2 = \frac{8}{f^2}(m_x - m_u)$ . The corrections  $\sigma_{loop}^{(3)}$  were calculated in Ref. [21], including the loop contribution from  $\rho \rightarrow \pi\pi$  (which is included phenomenologically). Rather than repeating the expressions used in that analysis of data over a wide range of meson mass, which were based on finite range regularization, we show just the leading or next-to-leading non-analytic behavior.

## CHIRAL EXPANSION FORMULA

The complete chiral expansion for the vector meson mass at rest is then:

$$\begin{aligned} M_V^2 = & (M_0(\mu) - \chi_1 m_x - a\Lambda^2 \chi_2 m_u - \bar{\chi}_1 m_x^2 - \bar{\chi}_2 m_u^2 - a\Lambda^2 [\alpha_2 w_v + \alpha_4 w_s] + c_b a^2)^2 \\ & + \frac{4\alpha_2 a \Lambda^2}{\Lambda_1^2} \left[ (w_s + w_v)F(m_{mix}, \mu) - 2w_v F(m_{val}, \mu) - 2w_v \Delta m^2 \frac{\partial F(m_{val}, \mu)}{\partial m_{val}^2} \right] + \Sigma^{tot}(\mu), \end{aligned} \quad (34)$$

where  $\chi_1 = 2\lambda_2 + Aa\Lambda^2/\Lambda_1$ ,  $\chi_2 = B/\Lambda_1$ ,  $\bar{\chi}_1 = \bar{A}/\Lambda_1$ ,  $\bar{\chi}_2 = \bar{B}/\Lambda_1$  and  $c_b = -\Lambda^3 D'$  are new parameters as functions of the previous ones. The term in square brackets comes from tadpole diagrams and  $\Sigma^{tot}$  is the total one-loop correction from Ref.[21] which includes the  $\sigma_{loop}^{(3)}(\mu)$  contribution. If one fixes the masses of the sea quarks and introduces new dimensionful (reparametrized) extrapolation coefficients, then we will have:

$$\begin{aligned} M_V^2 = & (c_0 - \Lambda^2(\alpha_2 w_v + \alpha_4 w_s + \chi_2 m_u)a + c_b a^2 + (\chi + ca)m_\pi^2 + \bar{\chi}m_\pi^4)^2 \\ & - \frac{4\alpha_2 a \Lambda^2}{\Lambda_1^2} \left\{ (w_s + w_v)F(m_{mix}) - 2w_v F(m_{val}) - 2w_v \Delta m^2 \frac{\partial F(m_{val})}{\partial m_{val}^2} \right\} + \Sigma^{tot}(\mu), \end{aligned} \quad (35)$$

where  $m_\pi^2 = \frac{8\lambda_m}{f^2}m_x$  and  $c_0, c, \chi$ , and  $\bar{\chi}$  are new independent parameters which are functions of the old ones.

Now, consider two different cases following from the theory:

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<sup>4</sup> for example,  $A \equiv (2\beta_5 + 2\beta_9)w_s + (2\beta_7 + 4\beta_{10} + 4\beta_{11})w_v$  and  $B \equiv (\beta_1 + \beta_2 + \beta_3 + \beta_6)w_s + (2\beta_4 + 2\beta_8)w_v$

1. when the sea and valence quarks are Wilson-type fermions then the expansion takes the form:

$$M_V^2 = (c_0 - \Lambda^2(\alpha_2 + \alpha_4 + \chi_2 m_u)a + c_b a^2 + (\chi + ca)m_\pi^2 + \bar{\chi} m_\pi^4)^2 \quad (36)$$

$$- \frac{8\alpha_2 a \Lambda^2}{\Lambda_1^2} \left\{ F(m_{mix}) - F(m_{val}) - \Delta m^2 \frac{\partial F(m_{val})}{\partial m_{val}^2} \right\} + \Sigma^{tot}(\mu),$$

2. when the valence quarks are Ginsparg-Wilson fermions and the sea quarks are Wilson fermions we have:

$$M_V^2 = (c_0 - \Lambda^2(\alpha_4 + \chi_2 m_u)a + c_b a^2 + (\chi + ca)m_\pi^2 + \bar{\chi} m_\pi^4)^2 \quad (37)$$

$$- \frac{4\alpha_2 a \Lambda^2}{\Lambda_1^2} F(m_{mix}) + \Sigma^{tot}(\mu),$$

The analysis in Ref.[21], which corresponds to the first case, was not sensitive to the existence of tadpole-like diagrams which are proportional to  $\alpha_2$ , so there is a good reason to believe that  $\alpha_2$  is consistent with zero. Similarly,  $c$  and  $\alpha_4 + \chi_2 m_u$  or  $\alpha_4$  and  $\chi_2$  are also consistent with zero<sup>5</sup> according to the analysis in the same reference. Comparison with the numerical simulations in Ref. [21] tells us that  $\bar{\chi} \approx -0.061 GeV^{-3}$  which gives a negligible contribution to the vector meson mass close to chiral limit.

## CONCLUSION

Based on the symmetries of the mixed (as well as unmixed) lattice QCD Lagrangian with Wilson and Ginsparg-Wilson fermions, heavy vector meson, partially quenched chiral perturbation theory was derived with explicit breaking of continuum symmetry up to order  $\mathcal{O}(a^2)$ . Using this formalism for the two flavor case, we extracted chiral and continuum expansion formulas with model dependent inputs with further comparison to the recently published numerical simulation which incorporated data over a wide range of meson masses.

We perform our calculations in the isospin symmetry limit when  $m_x = m_y$  and  $m_u = m_d$ . In our calculations of vector meson masses we include the phenomenological contribution from the  $\rho \rightarrow \pi\pi$  decay which can't be consistently extracted from heavy meson ChPT (because this is strongly relativistic process). We also didn't include the finite volume effects from the lattice.

The comparison with the extrapolation expression successfully used in Ref. [21] leads us to the conclusion that in our approximation chiral and continuum symmetries are mostly broken by the operators in  $L_{mass}$  and  $L_{a^2}$  and it is consistent to disregard the contributions from the Lagrangians:  $L_a$ ,  $L_{m_q^2}$  and  $L_{am_q}$ . This also gives us a simplified chiral-continuum extrapolation formula for further simulations using Wilson sea and Ginsparg-Wilson valence quarks.

This study is important because it allows us to check the validity of lattice QCD simulations, QCD and ChPT as well, by performing systematic chiral and continuum expansions of lattice data to the real world with further comparison to the experimental data.

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<sup>5</sup>  $\alpha_4 + \chi_2 m_u = 0$  can be achieved if  $\alpha_4 = \chi_2 = 0$  because  $m_u$  is a variable.

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