The University of Adelaide

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Cooperative Research Centre for Clean Power from Lignite

Physical Modelling of Mixing Between Rectangular Jets Present in Tangentially Fired Brown Coal Boilers

Ph.D. Thesis

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Appendix D

D Evaluation of the Effects of Changes in Boundary Conditions on the Scalar Mixing Field of Three Adjacent Rectangular Jets.

D.1 Introduction

In chapters 4 and 6 mixing within a group of three rectangular jets has been investigated without the complication of the recesses, wall or angle with wall. This approach has been taken because little scalar mixing data has been reported for this three jet system even in a free environment. The present chapter assesses the influence of two slight changes to the geometry of the system on the scalar field.

The nozzles themselves were modified so that two different base plates were used, to simulate the wall of the furnace. The geometry of the nozzles used are explained in Sections 3.4.1.2 and 3.4.1.3 in Figures 3.8 and 3.9. The effects of velocity ratio were assessed with a wall angled at 90 and 60 degrees to the flow. In the studies of Perry and Hausler, (1982) Perry and Pleasance, (1983a; 1983b) and Perry and McIntosh (1985) these modifications were more commonly known as geometries A and B.

Abdel-Rahman *et al.*, (1997) experimentally compared velocity data of a round free jet and a round jet issuing upwards from a wall, and found that the wall jet limits the interaction of jet flow with the surroundings resulting in a reduction in velocity and spread rate, momentum and mass flux of entrained fluid.

Kotsovinos (1978b) compared the effect of a round axissymmetric jet issuing freely with a jet issuing from a flush wall. Kostovinos also found that the entrained fluid into a round jet issuing from a wall enters the jet radially, whereas in the free jet case the entrained fluid enters all directions. The entrained fluid in the wall jet case causes the axial momentum change. Starting with the equation of continuity for a two dimensional axis symmetric jet (Equation D.1), Kotsovinos derived a relationship for the axial momentum (Equation D.2).

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} = 0$$
 Equation D. 1

$$M(x) = M_o + C(x) + H(x)$$
 Equation D. 2

$$C(x) = 2 \int_{0}^{x} \left[-\overline{u}(x, B(x))\overline{v}(x, B(x)) - \overline{u'v'}(x, B(x)) + \left(\frac{dB(x)}{dx}\right) \left\{ \overline{u'}(x, B(x)) + \overline{u'}^{2}(x, B(x)) \right\} \right] dx \qquad \text{Equation D. 3}$$

$$H(x) = \frac{2\int_{0}^{x} \left(\frac{dB(x)}{dx}\right) \overline{p}(x, B(x)) dx}{\rho}$$
 Equation D. 4

where B(x) is the jet boundary, u and v are the axial and radial velocity components. Kotsovinos found that the terms C(x) and H(x) are very much different for the free and flush wall jet cases, proving that a wall jet has a different axial momentum to the free jet this implies that it will also have an effect on the velocity field. Kotsovinos (1978a) also found that the change in the velocity field effects entrainment and derived relationships for the volumetric flux of surrounding fluid into the bulk jet flow.

Zhang (2000; 2003) studied the effect of skew angle of a non circular jet in a cross flow at a fixed pitch angle or inclination from a base wall. By reducing the pitch angle extensively reattachment was observed, causing the jet to 'attach' to a surface (Lai and Lu 1996, Launder and Rodi, 1983).

Selby *et al.*, (1992) examined the effect of angle for a grid of multiple round jets issuing into a cross flow. Bray and Garry (2000) also examined a round jet in a cross flow, varying the pitch angle from 30 to 60 degrees. These authors found that upstream duct geometry and pitch angle both significantly affected velocity distribution and the jet's capacity to re-attach to the wall.

Zhang and Collins (1997) also studied the effect of pitch angle on a jet in a cross flow and found that at lower pitch angles the jet is characterised by high stream wise-momentum near the wall. Zhang and Collins (1997) also observed that a velocity stress distribution makes it less likely to penetrate the boundary layer of the free stream fluid, thereby aiding the reattachment phenomena.

D.2 Experimental Conditions

The general experimental procedures and conditions are analogous to those described in Chapter 3. For the experiments reported here, the laser sheet was orientated parallel to the flow through either plane A, B or C (as explained in Section 4.2) and the primary and secondary jets were individually seeded with sodium flourescein. The fluorescent signal was captured by the video camera at 90 degrees to the laser sheet. The exposure time ranged from 1/50th to 1/125th of a second.

In contrast to the experiments in Chapters 4 and 6 a reduced set of experimental variables were qualitatively investigated. For visualisation of the primary jet the velocity ratios of $\lambda = 0$, 0.55 and 1.4 were investigated, whereas visualisation of the secondary jet was performed for velocity ratios of $\lambda = 0.55$ and 1.4. Table D.1 summarises the experimental conditions.

Dye and Laser	Plane	Velocity ratio
Orientation		(λ)
	A	0, 0.55, 1.4
	A	0.55, 1.4
	В	0.55, 1.4
	С	0, 0.55, 1.4

Table D. 1:Laser orientation and marked stream (green
indicates the marked flow) for the
rectangular nozzles fitted with a base plate
at 90 and 60 degrees to the direction of flow.

D.2.1. Errors and Image Processing Technique

The image processing technique used to obtain the time-averaged normalised concentrations of the jet during this analysis was similar to that used in Chapter 4. Only semi quantitative cross-

stream radial information can be obtained due to the absence of corrections for laser spatial intensity (Chapter 5).

D.3 Scalar Mixing Field of Rectangular Jets with a Base Plate Perpendicular to the Flow

Figures D.1 and D.2 show the normalised intensities of primary fluid planes A and C respectively for velocity ratios of λ =0, 0.55 and 1.4. Primary jet spread appears to be unaffected by the change in velocity ratio from 0 to 0.55. However, at λ =1.4 larger rates of jet spread are encountered at x/D > 6 through Planes A and C. Figures D.3 and D.4 show the normalised intensities of the secondary fluid through planes A and B respectively at velocity ratios of λ =0.55 and 1.4. The secondary jet appears to spread in Plane A and in Plane C as λ is increased from 0.55 to 1.4.



Figure D. 1: Normalised intensity for the primary jet with the laser orientated through Plane A with $\lambda = 0$ a), $\lambda = 0.55$ b), $\lambda = 1.4$ c).



Figure D. 2: Normalised intensity for the primary jet with the laser orientated through Plane C with $\lambda = 0$ a), $\lambda = 0.55$ b), $\lambda = 1.4$ c).



Figure D. 3: Normalised intensity for the secondary jet with the laser orientated through Plane A with $\lambda = 0.55$ a), $\lambda = 1.4$ b).



Figure D. 4: Normalised intensity for the secondary jet with the laser orientated through Plane B with $\lambda = 0.55$ a), $\lambda = 1.4$ b).

D.3.1. Statistical Analysis of the Effects of a Base Plate at 90 Degrees to the Burners

Figures D.5 and D.6 shows the dimensionless half-widths of the primary jet in planes A and C at λ =0, 0.55 and 1.4. The data is compared with the half-widths obtained from the free-jets in Chapter 4. Figures D.7 and D.8 show the dimensionless half-widths of the secondary jet in planes A and B at velocity ratios of 0.55 and 1.4 respectively, compared to the free jets under conditions detailed in Chapter 4.

Figure D.5 clearly shows that the primary jet is unaffected by the presence of the base plate at 90 degrees to the exit plane. For λ =1.4 however the 90 degree base plate increases jet spread. The half-widths of the primary jet in plane C and the secondary in plane B (Figures D.6 and D.8 respectively) are not affected by the presence of the 90 degree base plate. An explanation of the increased spread for the primary jet in plane A at λ =1.4 is uncertain without the benefit of quantitative velocity field data. However a speculative hypothesis can be formulated:

The secondary jets share only one interface with the free stream fluid in plane A. The base plate may delay entrainment of ambient fluid into the free stream sides of the secondary jets, allowing more of the secondary jet momentum to be available for entrainment of the primary jet, thereby increasing the spread of the primary jet.

Figure D.9 is a schematic diagram of the entrainment characteristics of both the primary and secondary jets in plane A illustrating the above hypothesis. The secondary jet shear layers that share an interface with the primary jet move towards the secondary jet geometric axis, creating a small low-pressure region between the two jets, which causes the primary jet to expand.

Figures D.10 and D.11 show the primary jet second order decay constants K_2 and secondary virtual origin $x_{o,2}/D$ as a function of λ . Without the base plate, where K_2 of the primary jet in planes A and C, falls marginally from $\lambda=0$ to $\lambda=0.55$ and rises slightly at $\lambda=1.4$. With the 90 degree base plate, a similar pattern prevails, except that of $\lambda=1.4$, K_2 increases significantly in Plane A. With the base plate, the virtual origin of the primary jet is much lower than is the case without the base plate. In plane C, the base plate has little effect on $x_{o,2}/D$.

Figures D.12 and D.13 show the second order decay constants and secondary virtual origin of the secondary jet. The reduction in second order decay constant as λ is increased from 0.55 to 1.4 is due to a reduction in spread. In plane A the base plate reduces the second order decay constant (remembering that K_2 and $x_{o,2}/D$ are both averages of both interfaces of the secondary jets). In plane B the base plate second order constant increases at λ =1.4. The presence of the base plate has little effect at λ =0.55 however, at λ =1.4 K_2 increases significantly in Plane A whilst reducing in Plane B.

D.3.2. Conclusion

The presence of the base plate causes an increase in the time-averaged spread of the primary jet in plane A at a velocity ratio of $\lambda=1.4$ for x/D > 6. The most plausible physical mechanism behind this phenomena is related to the increased secondary entrainment appetite at $\lambda=1.4$. The entrained ambient fluid enters the secondary jets later than in the case without the base plate allowing greater entrainment of primary fluid into the near field of the secondary jet which in turn causes the primary jet to expand.



Figure D. 5: Normalised half widths of the primary jet in Plane A, with (*WP*) and without the base plate at 90 degrees to the flow.



Figure D. 6: Normalised half widths of the primary jet in Plane C, with (*WP*) and without the base plate at 90 degrees to the flow.



Figure D. 7: Normalised half widths of the secondary jet in Plane A, with (*WP*) and without the base plate at 90 degrees to the flow.



Figure D. 8: Summary and comparison of the normalised half width versus axial distance of the secondary jet on plane B under free conditions and with a base plate at 90 degrees to the flow.



Figure D. 9: Time-averaged entrainment pattern of the surrounding fluid into the primary jet through plane C.



Figure D. 10: Comparison of the secondary decay constant K_2 versus velocity ratio for the primary jet under free conditions and with a base plate at 90 degrees to the flow (*WP*) through planes A and C.



Figure D. 11: Comparison of the secondary virtual origin $\frac{x_{o,2}}{D}$ versus velocity ratio for the primary jet under free conditions and with a base plate at 90 degrees to the flow (*WP*) through planes A and C.



Figure D. 12: Comparison of the secondary decay constant K_2 versus the inverse velocity ratio for the secondary jet under free conditions and with a base plate at 90 degrees to the flow (*WP*) through planes A and B.



D.4 Scalar Mixing Field of The Rectangular Jets with a Base Plate Inclined at 60 Degrees

Figure D.14 is the time-averaged normalised intensity of the primary jet through plane A at velocity ratios of λ =0, 0.55 and 1.4. Figure D.15 is the time-averaged normalised intensity of the primary jet through plane C at velocity ratios of λ =0 and 0.55. Figure D.16 and D.17 are the time-averaged normalised intensity of the secondary jet through planes A and B at velocity ratios of λ =0, and 0.55.

The time averaged normalised intensities of the primary jet through plane A in Figures D.14a) to D.14c) clearly shows that as λ progressively increases greater rates of spread are achieved for x/D>6. Similarly on plane C greater rates of spread are encountered for x/D>6 as λ is increased from 0 to 0.55. The time averaged intensities of the secondary jet through planes A and B in Figures D.16 and D.17 show no significant change is incurred with an increase in λ .



Figure D. 14:

Normalised intensity for the primary jet with the laser orientated through Plane A with $\lambda = 0$ a), $\lambda = 0.55$ b), $\lambda = 1.4$ c).







Figure D. 16:

Normalised intensity for the secondary jet with the laser orientated through Plane A with $\lambda = 0.55$ a), $\lambda = 1.4$ b).



Figure D. 17:

Normalised intensity for the secondary jet with the laser orientated through Plane B with $\lambda = 0.55$ a), $\lambda = 1.4$ b).

D.4.1. Statistical Analysis

The dimensionless half-widths of the primary jet through planes A and C versus axial distance (x/D) with an inclined base plate degree are compared to the free stream flows in Figures D.18 and D.19. The dimensionless half widths of the secondary jets versus axial distance through

plane A and B with an inclined base plate are also compared to the free stream values in Figures D.20 and D.21.

The spread of the primary jet through plane A illustrates very similar results to Figure D.5, showing that the influence of the base plate still has a large effect on the primary flow field at λ =1.4 for x/D>6. The spread of the primary jet through plane C in Figure D.19 is somewhat different. At a velocity ratio of λ =0.55 the half width of the primary jet moves towards the negative region, which coincides with the more 'acute' side of the primary jet interfaces. Figure D.22 is a schematic diagram showing how one side of the primary jet favours the more acute angle of the base plate. Although this flow is similar to a re-attaching one, the low pressure region is developed by the change in entrainment pattern between the primary jet and mean stream fluid. This phenomenon is only present when the secondary jet is functioning, ie at λ =0.55. A plausible explanation behind this is that at λ =0.55 the lower momentum secondary jet reduces the momentum of the primary jet as mixing occurs, in fact this shifting phenomena begins at approximately x/D > 4, where the mixing mechanisms are momentum controlling.

The half-width spread of the secondary jets through plane A in Figure D.20 show no particular effect of an angled base plate on the mean flow field. The half-width spread of the secondary jets through plane B in Figure D.22 show how they drift to the more acute side using a similar mechanism to the primary jet through plane C at λ =0.55.

The second order statistical values of the primary and secondary flow, specially K_2 and $x_{o,2}/D$ versus λ through planes A, B, and C are illustrated in Figures D.23 to D.26. The secondary decay constant for the primary jet in Figure D.23 show that for $\lambda=0$ to 1.4, the value of K_2 is higher than the free stream case and does not significantly increase with λ . Similarly the value of K_2 through plane C is also higher than the free stream case. However, a drop in K_2 is experienced with an increase in λ . The secondary virtual origins of the primary jet in Figure D.24 illustrates that the virtual origin is significantly lower with an inclined base plate for $\lambda = 0 - 1.4$ through plane A. Through Plane C however, the virtual origin of the inclined base plate is only significantly different for $\lambda=0$. At the other velocity ratios no huge differences are observed.

Significant changes for values of K_2 , of the secondary jets in Figure D.25 clearly show that the rate of spread of the secondary jet only changes through plane B with the insertion of the base

plate. The rate of spread is higher through plane B for λ =0.55 and 1.4. The secondary virtual origin of the secondary jets in Figure D.26 shows that the presence of the inclined base plate only significantly changes the virtual origin at λ =0.55 through planes A and B.

D.4.2. Conclusion

The inclination of the base plate only affects the primary and secondary jets through planes C and B respectively. The change in entrainment patterns causes the jets to move from their geometric axes towards the acute side of the plate. Although the flow is definitely not an attaching one however, there is distinct evidence that primary and secondary jets favour the side with the more acute angle.



Figure D. 18: Summary and comparison of the normalised half width versus axial distance of the primary jet on plane A, under free conditions and with a base plate orientated at 60 degrees to the flow (π).



Figure D. 19: Summary and comparison of the normalised half width versus axial distance of the primary jet on plane C under free conditions and with a base plate orientated at 60 degrees to the flow (π).



Figure D. 20: Summary and comparison of the normalised half width versus axial distance of the secondary jet on plane A under free conditions and with a base plate orientated at 60 degrees to the flow (π).



Figure D. 21: Summary and comparison of the normalised half width versus axial distance of the secondary jet on plane B under free conditions and with a base plate orientated at 60 degrees to the flow (π).



Figure D. 22: Schematic diagram of the entrainment of free stream fluid into the primary jet through plane C.

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Figure D. 23: Comparison of the secondary decay constant K_2 versus velocity ratio for the primary jet under free conditions and with a base plate orientated at 60 degrees to the flow through planes A and C.



Figure D. 24: Comparison of the secondary virtual origin $\frac{x_{o,2}}{D}$ versus velocity ratio for the primary jet under free conditions and with a base plate orientated at 60 degrees to the flow through planes A and C.



Figure D. 25: Comparison of the secondary decay constant K_2 versus the inverse velocity ratio for the secondary jet under free conditions and with a base plate orientated at 60 degrees to the flow (*WP*) through planes A and B.



Figure D. 26: Comparison of the secondary virtual origin $\frac{x_{o,2}}{D}$ versus the inverse velocity ratio for the secondary jet under free conditions with a base plate orientated at 60 degrees to the flow through planes A and B.

Appendix E

Sections E.1 and E.2 present the formulae and parameters for calculation of the regression coefficients *a*, *b*, *c*, *d*, *e*, y_o and z_o as function of λ and x/D. Once calculated, these parameters can be inserted into Equation 7.4 to calculate the primary or secondary jet mixture fraction.

E. Modelling Parameters for Prediction of Time-Averaged Scalar Mixing

Generic Model

$$\frac{\overline{C}}{\overline{C_c}} = a \exp\left\{-0.5\left[\left(\frac{|y/D-y_o|}{b}\right)^d - \left(\frac{|z/D-z_o|}{c}\right)^e\right]\right\}$$
Equation E. 1

E.1 Primary Jet Regression Parameters

for 0 < x/D < 8 and $0 < \lambda < 3.6$

$a = 0.0057 * \frac{x}{D} + 1.0082$	Equation E. 2
D	

$$y_o = -0.0143 \frac{x}{D} - 0.0219$$
 Equation E. 3

$$z_o = 0.0042 \left(\frac{x}{D}\right)^2 - 0.0204 \frac{x}{D} + 0.0018$$
 Equation E.

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Parameters for the Determination of the b-Regression Coefficient E.1.1

$$\lambda = 0$$

$$b = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

Lower Limit Upper Limit β δ α γ (x/D)(x/D)0.2 -0.727 0.479 0.1 --0.2 0.5 0.0707 0.32 _ _ 0.5 1 -0.043 0.3771 -_ 1 8 0.0015 -0.013 0.139 0.229

Table E. 1: Parameters and intervals for the determination of b regression coefficient at $\lambda = 0$.

0.55<λ<3.6,

L

$$b = a'^* \lambda + b'$$
$$a' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

Equation E.7

Equation E.8

Equation E. 6

$b' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$
--

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-0.012	0.001
0.2	1	-	-0.097	0.102	-0.017
1	2	-	-	-0.006	-0.006
2	8	0.0019	-0.0268	0.124	-0.175

Table E. 2: Parameters and intervals for the determination of a' regression coefficient for 0.55<*λ*<1.4.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-0.667	0.475
0.2	1	-	0.0287	-0.052	0.351
1	8	-0.002	0.0316	-0.07	0.391

Table E. 3: Parameters and intervals for the determination of b' regression coefficient for 0.55<*λ*<1.4.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	0.006	-0.004
0.2	4	0.0001	0.0048	-0.039	0.0034
4	8	-	-0.006	0.096	-0.374

Table E. 4:Parameters and intervals for the determination of a' regression
coefficient for $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	02	-	-	-0.692	0.483
0.2	0.5	-	-	0.08	0.329
0.5	8	0.0002	0.0043	0.048	0.328

Table E. 5:Parameters and intervals for the determination of b' regression
coefficient for $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	-0.005	-0.036	-
0.5	1	-	-	0.035	-0.037
1	2	-	-	0.0051	-0.007
2	4	-	-	-0.023	0.049
4	8	_	-0.0179	0.215	-0.62

Table E. 6:Parameters and intervals for the determination of a' regression
coefficient for $2.8 < \lambda < 3.6$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-0.567	0.472
0.2	1	-	-0.332	0.294	0.315
1	8	0.0027	-0.0168	0.0923	0.1855

Table E. 7:Parameters and intervals for the determination of b' regression
coefficient for $2.8 < \lambda < 3.6$.

E.1.2 Parameters for the Determination of the *c*-Regression Coefficient

 $\lambda = 0$,

$$c = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

Equation E.9

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Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-0.405	0.347
0.2	0.5	-	_	0.088	0.249
0.5	2	-	0.104	-0.17	0.352
2	8	-0.002	0.0488	-0.177	0.607

Table E. 8: Parameters and intervals for the determination of *c* regression coefficient at $\lambda=0$.

0.55<λ<3.6,

$$c = a'^* \lambda + b'$$
$$a' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

Equation E. 11

Equation E. 10

 $b' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$

Equation E. 12

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	0.27	-0.079	0.0112
0.5	4	-0.001	0.0021	-0.055	0.0663
4	8	-	0.0013	-0.020	-0.069

Table E. 9:Parameters and intervals for the determination of a' regression
coefficient for $0.55 < \lambda < 1.4$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-0.401	0.348
0.2	0.5	-	-	-0.053	0.278
0.5	1	-	-	0.0794	0.213
1	8	0.0018	-0.0177	0.142	0.173

Parameters and intervals for the determination of b' regression Table E. 10: coefficient for $0.55 < \lambda < 1.4$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-0.112	0.0236
0.2	0.5	-	-	0.069	-0.012
0.5	2	-	-0.012	0.0182	0.016
2	4	-	-	-0.015	0.0274
4	8	-	0.0209	-0.172	0.336

Table E. 11: Parameters and intervals for the determination of a' regression coefficient for $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-0.242	0.323
0.2	2	0.0301	-0.0526	0.0307	0.27
2	6	-	0.0093	-0.036	0.398
6	8	_	_	-0.014	0.597

Table E. 12: Parameters and intervals for the determination of b' regression coefficient for $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	0.164	-0.024
0.2	0.5	-	-	0.029	0.0022
0.5	1	-	-	0.0014	0.0162
1	2	-	-	-0.014	0.031
2	6	-	0.0124	-0.066	0.087
6	8	-	-	-0.154	1.061

Table E. 13: Parameters and intervals for the determination of a' regression coefficient for $2.8 < \lambda < 3.6$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-1.013	0.458
0.2	0.5	-	-	0.1197	0.231
0.5	1	-	-	0.0008	0.291
1	2	-	-	0.0737	0.218
2	6	-	0.0083	-0.084	0.501
6	8	-	-	0.755	-4.238

Table E. 14:Parameters and intervals for the determination of b' regression
coefficient for $2.8 < \lambda < 3.6$.

E.1.3 Parameters for the Determination of the *d*-Regression Coefficient

$$\lambda = 0$$
,

$$d = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

Equation E. 13

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-111.7	28.12
0.2	0.5	-	-	-7.73	7.32
0.5	1	-	-	1.06	2.922
1	2	-	-	-2	5.98
2	8	-0.017	0.293	-1.374	3.69

Table E. 15: Parameters and intervals for the determination of *a*' regression coefficient for $\lambda=0$.

 $d = a'^* \lambda + b'$ Equation E. 14 $a' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$ Equation E. 15 $b' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$ Equation E. 16

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-15.85	2.62
0.2	0.5	-	-	-4.5	0.351
0.5	2	-	-0.851	4.19	-3.78
2	4	-	-	-0.626	2.45
4	6	-	-	-0.069	0.223
6	8	-	-	0.035	-0.401

Table E. 16:Parameters and intervals for the determination of a' regression
coefficient for $0.55 < \lambda < 1.4$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-124.6	30.212
0.2	0.5	-	-	4.46	4.196
0.5	2	-	0.386	-4.64	8.65
2	6	-	0.0338	0.0275	0.73
6	8	-	-	-0.11	2.77

Table E. 17:Parameters and intervals for the determination of b' regression
coefficient for $0.55 < \lambda < 1.4$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	17.24	-3.46
0.2	0.5	-	-	-1.81	0.34
0.5	2	-	0.0467	0.078	-0.611
2	4	-	-	0.0755	-0.42
4	6	-	-	0.009	-0.154
6	8	-	-	0.162	-1.075

Table E. 18:Parameters and intervals for the determination of a' regression
coefficient for $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-171.9	38.72
0.2	0.5	-	-	0.7	4.2
0.5	2	-	-0.866	1.12	4.2
2	4	-	-	-0.755	4.49
4	8	-	-0.135	1.605	-2.79

Table E. 19:Parameters and intervals for the determination of b' regression
coefficient $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-16.94	2.969
0.2	0.5	-	-	0.593	-0.537
0.5	2	-	0.0266	0.1081	-0.301
2	4	-	-	0.0255	-0.03
4	8	-	-0.0861	1.047	2.74

Table E. 20:Parameters and intervals for the determination of a' regression
coefficient for $2.8 < \lambda < 3.6$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-76.16	20.71
0.2	0.5	-	-	-6.1	6.7
0.5	2	-	-1.183	1.97	2.95
2	6	-	0.0945	-1.18	4.167
6	8	-	-	0.605	-3.174

Table E. 21:Parameters and intervals for the determination of b' regression
coefficient for $2.8 < \lambda < 3.6$.

E.1.4 Parameters for the Determination of the *e*-Regression Coefficient

for $\lambda = 0$,

$$e = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

Equation E. 17

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	32.193	1.46
0.2	0.5	-	-	-17.67	11.58
0.5	1	-	-	0.969	2.26
1	2	-	-	-1.55	4.78
2	4	-	-	-0.012	1.696
4	8	-	-0.010	0.218	0.94

Table E. 22: Parameters and intervals for the determination of *a*' regression coefficient for $\lambda=0$.

0.55<λ<3.6,

 $e = a'*\lambda + b'$

$$a' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$
$$b' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

Lower Limit	Upper Limit	a	в	ν	δ
(r/D)	(r/D)	0.	P	/	Ū
(λ/D)	(<i>X/D</i>)		14 5 60	2.12	0.1.(0
0.1	0.5	-	-14.569	2.43	-0.163
0.5	2	-	-1.67	6.51	-5.42
2	4	-	-	-0.637	2.17
4	6	-	-	-0.022	-0.28
6	8	-	-	0.216	-1.71

Table E. 23:Parameters and intervals for the determination of a' regression
coefficient for $0.55 < \lambda < 1.4$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	24.73	2.274
0.2	1	-	-2.29	-1.76	7.664
1	2	-	-	-2.401	6.006
2	8	-0.009	0.0715	0.0813	0.826

Table E. 24:Parameters and intervals for the determination of b' regression
coefficient for $0.55 < \lambda < 1.4$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-25.98	2.68
0.2	0.5	-	-	7.48	-4.003
0.5	1	-	-	1.098	-0.811
1	4	-	-0.061	0.189	0.158
4	8	-	0.0175	-0.099	0.0614

Table E. 25:Parameters and intervals for the determination of a' regression
coefficient for $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	58.51	-1.331
0.2	0.5	-	-	-24.77	15.325
0.5	1	-	-	-1.14	3.51
1	6	-	0.0903	-0.915	3.52
6	8	-	-	-0.127	2.038

Table E. 26:Parameters and intervals for the determination of b' regression
coefficient for $1.4 < \lambda < 2.8$.

Equation E. 20

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-3.712	0.382
0.2	0.5	-	-	1.35	-0.63
0.5	1	-	-	0.123	-0.016
1	6	-0.01	0.124	-0.386	0.379
6	8	-	-	-0.002	0.263

Table E. 27:Parameters and intervals for the determination of a' regression
coefficient for $2.8 < \lambda < 3.6$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	-9.58	-0.925	4.93
0.5	1	-	-	1.6	1.28
1	4	-	-0.328	0.924	2.28
4	8	-	0.057	-0.524	1.904

Table E. 28:Parameters and intervals for the determination of b' regression
coefficient for $2.8 < \lambda < 3.6$.

E.1.5 Centreline Mixture Fraction Regression Parameters

$$\frac{1}{\overline{\xi_c}} = \alpha \left(\frac{x}{D}\right)^2 + \beta \left(\frac{x}{D}\right) + \gamma$$
 Equation E. 21

 $\lambda = 0$

$$\frac{1}{\overline{\xi_c}} = 0.0027 \left(\frac{x}{D}\right)^2 + 0.0045 \left(\frac{x}{D}\right) + 1.033$$
 Equation E. 22

0.55<λ <3.6

$$\alpha = a' * \left(\frac{x}{D}\right) + b'$$
 Equation E. 23

Lower Limit	Upper Limit	<i>a</i> '	<i>b</i> '
(λ)	(λ)		
0.55	1.4	-0.01	0.0325-
1.4	2.8	0.0141	-0.0014
2.8	3.6	0.0344	-0.0583

Table E. 29: Parameters and intervals for the determination of α , for centreline mixture fraction determination

$$\beta = a' \ast \left(\frac{x}{D}\right) + b'$$

Equation E. 24

Equation E. 25

Lower Limit	Upper Limit	<i>a</i> '	<i>b</i> '
(λ)	(λ)		
0.55	1.4	0.0615	-0.1688
1.4	2.8	-0.001	-0.0817
2.8	3.6	-0.134	0.291

Table E. 30: Parameters and intervals for the determination of β ,for centreline mixture fraction determination.



Lower Limit	Upper Limit	<i>a</i> '	<i>b</i> '
(λ)	(λ)		
0.55	1.4	0.078	1.13
1.4	2.8	0.035	0.972
2.8	3.6	0.025	1

Table E. 31:Parameters and intervals for the determination of γ ,
for centreline mixture fraction determination

E.2 Secondary Jet Regression Parameters

for 0 < x/D < 8 and $0.55 < \lambda < 3.6$

$$a = 0.0186 * \frac{x}{D} + 1.0443$$

E.2.1 Parameters for the Determination of the *b*-Regression Coefficient

0.55<λ<3.6,

$$b = a'^* \lambda + b'$$
Equation E. 27
$$a' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$
Equation E. 28
$$b' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$
Equation E. 29

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	8	-0.001	0.0083	-0.043	-0.029

Table E. 32:Parameters and intervals for the determination of a' regression
coefficient for $0.55 < \lambda < 1.4$.

Lower Limit (<i>x/D</i>)	Upper Limit (x/D)	α	β	γ	δ
0.1	8	-0.001	0.0446	-0.063	0.4092

Table E. 33:Parameters and intervals for the determination of b' regression
coefficient for $0.55 < \lambda < 1.4$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	0.283	-0.055
0.2	1	-	-0.08	0.2256	-0.003
1	2	-	-	-0.04	-0.008
2	4	-	-	-0.04	-0.008
4	8	-	-0.0007	-0.024	0.0829

Table E. 34:Parameters and intervals for the determination of a' regression
coefficient for $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	1	1.509	2.801	-1.608	0.642
1	8	0.000	0.0274	0.0047	0.309

Table E. 35:Parameters and intervals for the determination of b' regression
coefficient for $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	0.303	-0.02
0.2	2	-0.011	0.074	-0.132	0.0636
2	6	-	0.0115	-0.084	0.1288
6	8	-	-	0.012	-0.038

Table E. 36:Parameters and intervals for the determination of a' regression
coefficient for $2.8 < \lambda < 3.6$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-0.927	0.4968
0.2	8	-	0.0113	0.0498	0.3

Table E. 37:Parameters and intervals for the determination of b' regression
coefficient for $2.8 < \lambda < 3.6$.

for $\lambda = \infty$,

$$b = -0.002 \left(\frac{x}{D}\right)^3 + 0.04 \left(\frac{x}{D}\right)^2 - 0.09 \left(\frac{x}{D}\right) + 0.423$$

Equation E. 30

E.2.2 Parameters for the Determination of the *c*-Regression Coefficient

$$c = a'^* \lambda + b'$$
Equation E. 31
$$a' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$
Equation E. 32
$$b' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$
Equation E. 33

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	1.422	-0.268
0.2	1	-	-0.0139	0.0101	0.0143
1	8	0.0008	-0.0147	0.0274	0.0017

Table E. 38:Parameters and intervals for the determination of a' regression
coefficient for $0.55 < \lambda < 1.4$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	8	-0.003	0.0598	-0.107	0.2246

Table E. 39:Parameters and intervals for the determination of b' regression
coefficient for $0.55 < \lambda < 1.4$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-0.574	0.1142
0.2	0.5	-	-	0.0323	-0.007
0.5	4	0.011	-0.006	-0.005	0.0132
4	6	-	-	0.014	-0.092
6	8	-	-	-0.039	0.2397

Table E.40:Parameters and intervals for the determination of a' regression
coefficient for $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	1.985	-0.211
0.2	1	-	-0.2929	0.334	0.1305
1	8	0.0016	-0.0043	0.1001	0.073

Table E.41:Parameters and intervals for the determination of b' regression
coefficient for $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	0.22	-0.264	0.0773
0.5	1	-	-	-0.001	0.001
1	2	-	-	0.0149	-0.015
2	6	-	-0.0061	0.0305	-0.022
6	8	-	-	0.0738	-0.500

Table E.42:Parameters and intervals for the determination of a' regression
coefficient for $2.8 < \lambda < 3.6$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	-1.017	1.23	-0.163
0.5	6	0.0003	0.0425	-0.148	0.303
6	8	-	-	-0.043	1.271

Table E.43:Parameters and intervals for the determination of b' regression
coefficient for $2.8 < \lambda < 3.6$.

for $\lambda = \infty$,

 $d = a'^*\lambda + b'$

 $a' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$

 $b' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$

$$c = -0.002 \left(\frac{x}{D}\right)^3 + 0.0427 \left(\frac{x}{D}\right)^2 + -0.059 \left(\frac{x}{D}\right) + 0.207$$
 Equation E. 34

E.2.3 Parameters for the Determination of the *d*-Regression Coefficient

$$\begin{array}{|c|c|c|c|c|c|c|c|} Lower Limit & Upper Limit & \alpha & \beta & \gamma & \delta \\ \hline (x/D) & & & & & \\ \hline 0.1 & 1 & - & 3.05 & -2.69 & -0.221 \\ \hline 1 & 4 & - & -0.287 & 1.25 & -0.837 \\ \hline 4 & 8 & - & -0.128 & 1.55 & -4.57 \\ \hline \end{array}$$

Table E. 44: Parameters and intervals for the determination of *a*' regression coefficient for $0.55 < \lambda < 1.4$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	2	0.316	-0.506	2.21	-0.689
2	8	0.035	-0.5	2.21	-0.689

Table E.45: Parameters and intervals for the determination of b' regression coefficient
for $0.55 < \lambda < 1.4$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-369.8	74.11
0.2	2	-0.115	1.37	-2.35	0.565
2	6	-	-0.1405	0.911	-0.843
6	8	-	-	0.279	-2.109

Table E.46: Parameters and intervals for the determination of a' regression coefficient
for $1.4 < \lambda < 2.8$.

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Equation E. 35

Equation E. 36

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-518.8	-99.96
0.2	1	-	1.213	0.964	3.553
1	6	-0.124	1.864	-8.305	12.297
6	8	-	-	-0.413	5.196

Table E.47: Parameters and intervals for the determination of b' regression coefficient
for $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	557.12	111.86
0.2	0.5	-	-	-0.88	-0.262
0.5	1	-	-	0.59	-1
1	6	-0.025	0.342	-0.994	0.274
6	8	-	-	-0.424	3.57

Table E.48: Parameters and intervals for the determination of a' regression coefficient
for $2.8 < \lambda < 3.6$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-2077	420.77
0.2	6	0.0237	-0.4024	0.403	5.35
6	8	-	-	1.55	-10.71

Table E.49: Parameters and intervals for the determination of b' regression coefficient
for $2.8 < \lambda < 3.6$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-368.1	77.54
0.2	1	-	5.27	-4.74	4.65
1	8	-0.02	0.383	-2.416	7.107

Table E.50: Parameters and intervals for the determination of *d* regression coefficient for $\lambda = \infty$.

E.2.4 Parameters for the Determination of the *e*-Regression Coefficient

$$e = a'^*\lambda + b'$$

$$a' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

$$b' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

Equation E. 38

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	-14.569	2.43	-0.163
0.5	2	-	-1.67	6.51	-5.42
2	4	-	-	-0.637	2.17
4	6	-	-	-0.022	-0.28
6	8	-	-	0.216	-1.71

Table E.51:Parameters and intervals for the determination of a' regression
coefficient for $0.55 < \lambda < 1.4$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	24.73	2.274
0.2	1	-	-2.29	-1.76	7.664
1	2	-	-	-2.401	6.006
2	8	-0.009	0.0715	0.0813	0.826

Table E.52:Parameters and intervals for the determination of b' regression
coefficient for $0.55 < \lambda < 1.4$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-25.98	2.68
0.2	0.5	-	-	7.48	-4.003
0.5	1	-	-	1.098	-0.811
1	4	-	-0.061	0.189	0.158
4	8	-	0.0175	-0.099	0.0614

Table E.53:Parameters and intervals for the determination of a' regression
coefficient for $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	58.51	-1.331
0.2	0.5	-	-	-24.77	15.325
0.5	1	-	-	-1.14	3.51
1	6	-	0.0903	-0.915	3.52
6	8	-	-	-0.127	2.038

Table E.54:Parameters and intervals for the determination of b' regression
coefficient for $1.4 < \lambda < 2.8$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-3.712	0.382
0.2	0.5	-	-	1.35	-0.63
0.5	1	-	-	0.123	-0.016
1	6	-0.01	0.124	-0.386	0.379
6	8	-	-	-0.002	0.263

Table E.55:Parameters and intervals for the determination of a' regression
coefficient for $2.8 < \lambda < 3.6$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	-9.58	-0.925	4.93
0.5	1	-	-	1.6	1.28
1	4	-	-0.328	0.924	2.28
4	8	-	0.057	-0.524	1.904

Table E.56:Parameters and intervals for the determination of b' regression
coefficient for $2.8 < \lambda < 3.6$.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-14.92	3.83
0.2	1	-	0.9008	1.41	0.52
1	6	-0.01	0.1992	-1.074	3.729
6	8	-	-	-0.082	2.793

Table E.57: Parameters and intervals for the determination of *e* regression coefficient for $\lambda = \infty$.

E.2.5 Centreline Mixture Fraction Regression Parameters

$$\frac{1}{\overline{\xi_c}} = \alpha \left(\frac{x}{D}\right)^2 + \beta \left(\frac{x}{D}\right) + \gamma$$

$$\alpha = a' \ast \left(\frac{x}{D}\right) + b'$$

Equation E. 42

Equation E. 43

Equation E. 41

Upper Limit	<i>a</i> '	b'
(λ)		
1.4	-0.01	0.0325-
2.8	0.0141	-0.0014
3.6	0.0344	-0.0583
	$ \begin{array}{r} \text{Upper Limit} \\ \hline (\lambda) \\ \hline 1.4 \\ \hline 2.8 \\ \hline 3.6 \\ \end{array} $	Upper Limit a' (λ) -0.01 1.4 -0.0141 2.8 0.0141 3.6 0.0344

Table E.58: Values of a' and b' for calculation of α at different intervals of λ .

$$\beta = a' \ast \left(\frac{x}{D}\right) + b'$$

Lower Limit	Upper Limit	<i>a</i> '	<i>b</i> '
(λ)	(λ)		
0.55	1.4	0.0615	-0.1688
1.4	2.8	-0.001	-0.0817
2.8	3.6	-0.134	0.291

Table E.59: Values of a' and b' for calculation of β at different intervals of λ .

$$\gamma = a' \ast \left(\frac{x}{D}\right) + b'$$

Equation E. 44

Lower Limit	Upper Limit	<i>a</i> '	<i>b</i> '
(λ)	(λ)		
0.55	1.4	0.078	1.13
1.4	2.8	0.035	0.972
2.8	3.6	0.025	1

Table E.60:Values of a' and b' for calculation of γ at
different intervals of λ .

E.2.6 Parameters for the Determination of the *y_o* Regression Coefficient

$$y_o = a'\lambda + b'$$

$$a' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

$$b' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	5.19	1.204	0.0214
0.5	1	-	-	-3.01	2.22
1	2	-	-	0.706	-1.49
2	4	-	-	0.0763	-0.234
4	8	-	-0.0088	0.0353	0.0706

Table E.61:Parameters and intervals for the determination of a' regression
coefficient for $0.55 < \lambda < 1.4$, for z/D < 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	-2.26	0.2868	0.0199
0.5	1	-	-	1.64	-1.288
1	2	-	-	-0.405	0.825
2	6	-	0.0375	-0.301	0.468
6	8	-	-	0.113	-0.67

Table E.62:Parameters and intervals for the determination of b' regression
coefficient for $0.55 < \lambda < 1.4$, for z/D < 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	5.87	-1.527	0.0822
0.5	1	-	-	-1.48	1.528
1	4	-	-0.0547	0.1406	-0.038
4	6	-	-	0.387	-1.902
6	8	-	-	-0.159	1.377

Table E.63:Parameters and intervals for the determination of a' regression
coefficient for $0.55 < \lambda < 1.4$, for z/D > 0.

Equation E. 45

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Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	-2.72	0.488	-0.005
0.5	1	-	-	0.814	-0.85
1	2	-	-	-0.047	0.011
2	4	-	-	0.2885	-0.659
4	6	-	-	-0.563	2.74
6	8	-	-	0.212	-1.9

Table E.64:Parameters and intervals for the determination of b' regression
coefficient for $0.55 < \lambda < 1.4$, for z/D > 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-0.572	0.0858
0.2	0.5	-	-	-1.381	0.247
0.5	1	-	-	0.871	-0.878
1	2	-	-	0.0071	-0.014
2	4	-	-	-	0
4	8	-	0.0723	-0.851	2.24

Table E.65:Parameters and intervals for the determination of b' regression
coefficient for $1.4 < \lambda < 2.8$, for z/D < 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	0.9	-0.17
0.2	0.5	-	-	4.03	-0.796
0.5	1	-	-	-2.42	2.43
1	2	-	-	-0.11	0.12
2	4	-	-	0.03	-0.16
4	8	-	-0.1038	1.2175	-3.25

Table E.66:Parameters and intervals for the determination of b' regression
coefficient for $1.4 < \lambda < 2.8$, for z/D < 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	-2.14	0.142	0.0072
0.5	1	-	-	0.914	-0.914
1	2	-	-	-0.021	-0.038
2	4	-	-	0.0393	-0.099
4	8	-	0.05	-0.585	1.599

Table E.67:Parameters and intervals for the determination of a' regression
coefficient for $1.4 < \lambda < 2.8$, for z/D > 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	8.5	-1.85	0.1
0.5	1	-	-	-2.54	2.57
1	4	-	0.0067	-0.07	0.0933
4	8	-	-0.0675	0.775	-2.1

Table E.68:Parameters and intervals for the determination of b' regression
coefficient for $1.4 < \lambda < 2.8$, for z/D > 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	0.375	-0.037
0.2	0.5	-	-	-0.125	0.0625
0.5	1	-	-	-0.125	0.0625
1	2	-	-	-	0
2	4	-	-	0.025	-0.025
4	6	-	-	0.1938	-0.787
6	8	-	-	-0.206	1.61

Table E.69:Parameters and intervals for the determination of a' regression
coefficient for $2.8 < \lambda < 3.6$, for z/D < 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-1.75	0.175
0.2	0.5	-	-	0.516	-0.278
0.5	1	-	-	0.02	-0.03
1	2	-	-	-0.16	0.15
2	4	-	-	-0.16	0.15
4	8	-	0.3787	-4.51	11.975

Table E.70:Parameters and intervals for the determination of b' regression
coefficient for $2.8 < \lambda < 3.6$, for z/D < 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	1	-0.1
0.2	0.5	-	-	-0.416	0.1833
0.5	2	-	-0.025	0.1125	-0.075
2	4	_	_	-0.056	0.1625
4	8	-	-0.0516	0.6094	-1.67

Table E.71:Parameters and intervals for the determination of a' regression
coefficient for $2.8 < \lambda < 3.6$, for z/D > 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-3.5	0.35
0.2	0.5	-	-	1.466	-0.643
0.5	2	-	-0.0167	-0.165	0.1767
2	4	-	-	0.237	-0.695
4	8	-	0.2169	-2.571	7.07

Table E.72:Parameters and intervals for the determination of b' regression
coefficient for $2.8 < \lambda < 3.6$, for z/D > 0.

for
$$\lambda = \infty$$
,

$$y_o = 0$$

Equation E. 48

E.2.7 Parameters for the Determination of the *z*_o Regression Coefficient

for $0.55 < \lambda < 1.4$,

$$z_o = a'\lambda + b'$$
 Equation E. 49

$$a' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

$$b' = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

Equation E. 51

Equation E. 50

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.5	-	-0.195	-0.059	0.0432
0.5	1	-	-	3.47	1.77
1	2	-	-	-1.73	3.43
2	4	-	-	0.0059	-0.047
4	8	-	-0.516	6.05	-15.98

Table E.73:Parameters and intervals for the determination of a' regression
coefficient for $0.55 < \lambda < 1.4$, for z/D < 0.

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Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	1.7	-1.11
0.2	0.5	-	-	0.2	-0.81
0.5	1	-	-	0.16	-0.79
1	2	-	-	-0.23	-0.4
2	4	-	-	0.0215	-0.903
4	6	-	_	-0.431	0.909
6	8	-	-	0.868	-6.88

Table E.74:Parameters and intervals for the determination of b' regression
coefficient for $0.55 < \lambda < 1.4$, for z/D < 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	0.471	-0.094
0.2	0.5	-	-	-2.54	1.309
0.5	1	-	-	1.3094	-0.619
1	2	-	-	-0.642	1.33
2	4	-	-	-0.011	0.07
4	6	-	-	-0.871	3.5
6	8	-	-	-1.65	1.13

Table E.75:Parameters and intervals for the determination of a' regression
coefficient for $0.55 < \lambda < 1.4$, for z/D > 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	1.207	-8.96
0.2	1	-	0.125	-0.287	0.852
1	4	-	-0.0208	0.156	0.554
4	6	-	-	0.391	-0.719
6	8	-	-	-0.905	7.06

Table E.76:Parameters and intervals for the determination of b' regression
coefficient for $0.55 < \lambda < 1.4$, for z/D > 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-0.361	0.0075
0.2	0.5	-	-	0.239	-0.11
0.5	1	-	-	-0.014	0.0142
1	2	-	-	0.0143	-0.014
2	4	-	-	0.0143	-0.014
4	6	-	-	-0.018	0.1147
6	8	-	-	0.0925	-0.547

Table E.77:Parameters and intervals for the determination of a' regression
coefficient for $1.4 < \lambda < 2.8$, for z/D < 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	2.1	-1.07
0.2	0.5	-	-	-0.4	-0.57
0.5	1	-	-	0.18	-0.86
1	2	-	-	-0.25	-0.43
2	4	-	-	0.01	-0.95
4	8	-	-0.0438	0.492	-2.18

Table E.78:Parameters and intervals for the determination of b' regression
coefficient for $1.4 < \lambda < 2.8$, for z/D < 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	0.857	-0.092
0.2	0.5	-	-	-0.026	0.131
0.5	1	-	-	0.0714	-0.035
1	2	-	-	-0.078	0.1143
2	8	-0.001	0.0054	0.0172	-0.092

Table E.79:Parameters and intervals for the determination of a' regression
coefficient for $1.4 < \lambda < 2.8$, for z/D > 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-2.2	1.13
0.2	0.5	-	-	0.333	0.623
0.5	1	-	-	-0.3	0.94
1	2	-	-	0.27	0.37
2	6	-	-0.01	0.035	0.88
6	8	-	-	0.025	0.58

Table E.80:Parameters and intervals for the determination of b' regression
coefficient for $1.4 < \lambda < 2.8$, for z/D > 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	1	-	-	-	0
1	4	-	-0.0188	0.0688	-0.05
4	6	-	-	0.0188	-0.15
6	8	-	-	-0.068	0.375

Table E.81:Parameters and intervals for the determination of a' regression
coefficient for $2.8 < \lambda < 3.6$, for z/D < 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-1.7	-0.77
0.2	0.5	-	-	1.36	-1.383
0.5	1	-	-	0.14	-0.82
1	2	-	-	-0.245	-0.435
2	4	-	-	0.172	-1.27
4	6	-	-	-0.047	-0.39
6	8	-	-	0.3325	-2.67

Table E.82:Parameters and intervals for the determination of b' regression
coefficient for $2.8 < \lambda < 3.6$, for z/D < 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-2.375	0.325
0.2	0.5	-	-	0.541	-0.258
0.5	1	-	-	-0.05	0.0375
1	2	-	-	0.05	-0.062
2	4	-	-	-0.062	0.1625
4	6	-	-	-	-0.087
6	8	_	-	-0.018	0.025

Table E.83:Parameters and intervals for the determination of a' regression
coefficient for $2.8 < \lambda < 3.6$, for z/D > 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	6.85	-0.04
0.2	0.5	-	-	-1.916	1.713
0.5	2	-	-0.0867	0.17	0.6917
2	4	-	-	0.23	0.225
4	6	-	-	-0.025	1.245
6	8	-	-	0.0275	0.93

Table E.84:Parameters and intervals for the determination of b' regression
coefficient for $2.8 < \lambda < 3.6$, for z/D > 0.

for $\lambda = \infty$,

$$z_o = \alpha \left(\frac{x}{D}\right)^3 + \beta \left(\frac{x}{D}\right)^2 + \gamma \left(\frac{x}{D}\right) + \delta$$

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	1.6	-1.1
0.2	1	-	0.05	0.065	-0.795
1	2	-	-	-0.25	-0.43
2	4	-	-	0.105	-1.14
4	8	-	0.03	-0.33	0.12

Table E.85:Parameters and intervals for the determination of a' regression
coefficient for $\lambda = \infty$, for z/D < 0.

Lower Limit	Upper Limit	α	β	γ	δ
(x/D)	(x/D)				
0.1	0.2	-	-	-1.5	1.09
0.2	1	-	0.05	-0.135	0.815
1	2	-	-	0.07	0.66
2	6	-	-0.0063	0.0225	0.78
6	8	-	-	0.03	0.51

Table E. 86: Parameters and intervals for the determination of b' regression coefficient for $\lambda = \infty$, for z/D > 0.

Appendix F

F. Analysis of Variance Data

This section represents the full data retrieved from the analysis of variance of the modelled results from the primary and secondary jets. The formulae and techniques used to perform the analysis of variance are illustrated in detail in Section 7.3.3.

Z/D	-0.25	0	0.3	0.35	-0.35
SSE	3.97	1.01	2.36	1.87	1.69
SST	11.61	10.02	8.56	0.72	3.66
SSR	7.64	9.00	6.20	-1.15	1.97
MSR	1.91	2.25	1.55	-0.29	0.49
MSE	0.08	0.02	0.05	0.04	0.04
R^2	0.66	0.90	0.72	-1.61	0.54
Fa	23.12	106.82	31.52	-7.39	14.02

Table F. 1: Analysis of variance data of the primary jet, x/D = 0.1, $\lambda = 0$.

Z/D	0	0.4	-0.4	-0.3	0.3	-0.35	0.35	0.37
SSE	3.91	21.30	10.44	1.10	0.41	2.38	7.83	7.23
SST	7.42	0.23	1.17	10.62	8.94	4.86	2.49	2.24
SSR	3.51	-21.07	-9.26	9.53	8.53	2.47	-5.33	-4.99
MSR	0.88	-5.27	-2.32	2.38	2.13	0.62	-1.33	-1.25
MSE	0.08	0.45	0.22	0.02	0.01	0.05	0.17	0.15
R^2	0.47	-89.98	-7.89	0.90	0.95	0.51	-2.14	-2.22
Fo	10.55	-11.62	-10.43	101.98	246.42	12.21	-8.01	-8.11

Table F.2: Analysis of variance data of the primary jet, x/D = 0.1, $\lambda = 3.6$.

Z/D	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2
SSE	2.30	1.82	1.28	1.02	0.76	0.97
SST	0.97	2.48	4.78	7.36	13.53	18.91
SSR	-1.34	0.67	3.50	6.34	12.77	17.94
MSR	-0.33	0.17	0.87	1.59	3.19	4.48
MSE	0.02	0.01	0.01	0.01	0.01	0.01
R^2	-1.38	0.27	0.73	0.86	0.94	0.95
Fo	-17.8	11.26	83.7	191.88	514.11	566.73
Z/D	0	0.2	0.3	0.4	0.5	0.6
SSE	1.40	1.07	0.80	0.85	1.08	1.58
SST	24.30	17.91	12.7	7.19	4.37	2.30
SSR	22.91	16.84	11.9	6.34	3.29	0.72
MSR	5.73	4.21	2.98	1.59	0.82	0.18
MSE	0.01	0.01	0.01	0.01	0.01	0.01
R^2	0.94	0.94	0.94	0.88	0.75	0.31

Table F.3: Analysis of variance data of the primary jet, x/D = 2, $\lambda = 0$.

Z/D	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	0
SSE	0.21	0.16	0.18	0.26	0.32	0.47	0.30
SST	0.42	0.94	2.36	4.71	9.77	13.49	13.75
SSR	0.22	0.78	2.18	4.45	9.45	13.02	13.45
MSR	0.05	0.20	0.55	1.11	2.36	3.26	3.36
MSE	0.00	0.00	0.00	0.00	0.00	0.01	0.00
R^2	0.52	0.83	0.92	0.94	0.97	0.97	0.98
Fo	22.40	105.02	258.3	359.93	615.69	587.16	947.43
Z/D	0.2	0.3	0.4	0.5	0.6	0.7	
SSE	0.38	0.48	0.47	0.40	0.29	0.26	
SST	12.40	9.55	5.18	2.54	1.03	0.43	
SSR	12.01	9.06	4.72	2.14	0.75	0.18	
MSR	3.00	2.27	1.18	0.54	0.19	0.04	
MSE	0.00	0.01	0.01	0.00	0.00	0.00	
R^2	0.97	0.95	0.91	0.84	0.72	0.41	
Fo	658.0	394.50	212.3	112.67	54.36	14.57	

Table F.4: Analysis of variance data of the primary jet, x/D = 2, $\lambda = 3.6$.

z/D	-2.2	-1.80	-1.5	-1	-0.50	0.00	0.50	1.00	1.50	1.80	2.20
SSE	0.33	0.25	0.51	1.11	2.39	2.45	3.65	2.20	0.36	0.36	0.17
SST	0.12	0.37	0.49	1.54	2.39	4.15	2.57	1.36	0.22	0.22	0.07
SSR	-0.21	0.12	-0.03	0.44	0.00	1.69	-1.08	-0.84	-0.14	-0.14	-0.10
MSR	-0.05	0.03	-0.01	0.11	0.00	0.42	-0.27	-0.21	-0.04	-0.04	-0.02
MSE	0.00	0.00	0.01	0.01	0.03	0.03	0.04	0.03	0.00	0.00	0.00
R^2	-1.79	0.33	-0.05	0.28	0.00	0.41	-0.42	-0.62	-0.64	-0.64	-1.37
F_{o}	-13.9	10.68	-1.12	8.56	-0.02	15.01	-6.44	-8.33	-8.46	-8.46	-12.58

Table F.5: Analysis of variance data of the primary jet, x/D = 8, $\lambda = 0$.

z/D	-2.2	-1.80	-1.5	-0.50	0.00	0.50	1.00	1.50	1.80	2.20
SSE	1.00	1.17	1.07	1.92	1.28	1.35	1.22	1.44	1.21	1.06
SST	1.13	1.08	1.52	5.36	8.17	5.43	4.39	1.62	1.55	0.67
SSR	0.13	-0.10	0.45	3.44	6.89	4.08	3.16	0.18	0.35	-0.39
MSR	0.03	-0.02	0.11	0.86	1.72	1.02	0.79	0.05	0.09	-0.10
MSE	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
R^2	0.11	-0.09	0.29	0.64	0.84	0.75	0.72	0.11	0.22	-0.59
Fo	5.83	-3.74	19.09	81.66	245.57	138.3	118.21	5.78	13.21	-16.9

Table F.6: Analysis of variance data of the primary jet, x/D = 8, $\lambda = 3.6$.

y/D	0	0.3	0.4	0.425	0.45
SSE	9.42	15.33	15.57	5.79	7.28
SST	31.38	39.86	18.54	5.17	5.17
SSR	21.96	24.53	2.97	-0.62	-2.11
MSR	5.49	6.13	0.74	-0.16	-0.53
MSE	0.04	0.07	0.07	0.03	0.03
R^2	0.70	0.62	0.16	-0.12	-0.41
Fo	131.68	90.41	10.77	-6.08	-16.37

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y/D	0	0.3	0.4	0.425	0.45
SSE	11.08	8.76	7.75	8.70	71.95
SST	49.14	48.19	37.11	23.48	7.90
SSR	38.06	39.43	29.36	14.78	-64.04
MSR	9.52	9.86	7.34	3.69	-16.01
MSE	0.05	0.04	0.03	0.04	0.32
R2	0.77	0.82	0.79	0.63	-8.10
Fo	194.08	254.22	213.95	95.92	-50.29

Table F.8:

Analysis of variance data of the secondary jet, x/D = 0.1, $\lambda = 3.6$.

y/D	0	0.3	0.4	0.5	0.6	0.7	0.8
SSE	72.38	48.39	41.96	33.61	26.76	18.29	11.98
SST	56.88	45.94	35.50	24.18	14.17	5.89	2.66
SSR	-15.50	-2.45	-6.46	-9.43	-12.59	-12.40	-9.32
MSR	-3.88	-0.61	-1.61	-2.36	-3.15	-3.10	-2.33
MSE	0.20	0.14	0.12	0.09	0.07	0.05	0.03
R^2	-0.27	-0.05	-0.18	-0.39	-0.89	-2.10	-3.50
Fo	19.17	4.52	13.78	25.11	42.11	60.67	69.60

Table F.9:

9: Analysis of variance data of the secondary jet, x/D = 2, $\lambda = 0$

y/D	0	0.3	0.4	0.5	0.6	0.7	0.8
SSE	22.91	17.54	13.65	8.76	5.16	2.82	2.07
SST	72.64	57.70	41.67	25.91	12.95	4.57	1.42
SSR	49.72	40.16	28.02	17.15	7.80	1.74	-0.64
MSR	12.43	10.04	7.01	4.29	1.95	0.44	-0.16
MSE	0.06	0.05	0.04	0.02	0.01	0.01	0.01
R^2	0.68	0.70	0.67	0.66	0.60	0.38	-0.45
Fo	194.24	204.99	183.73	175.27	135.34	55.27	-27.81

Table F.10: Analysis of variance data of the secondary jet, x/D = 2, $\lambda = 3.6$

y/D	0	1	1.5	2	2.5
SSE	41.70	60.15	76.17	89.50	83.35
SSE	41.70	60.15	76.17	89.50	83.35
SST	49.20	45.55	21.85	18.54	9.80
SSR	7.50	-14.61	-54.32	-70.96	-73.55
MSR	1.87	-3.65	-13.58	-17.74	-18.39
MSE	0.10	0.14	0.18	0.21	0.20
R^2	0.15	-0.32	-2.49	-3.83	-7.51
Fo	19.15	-25.86	-75.95	-84.44	-93.98

Table F.11:

Analysis of variance data of the secondary jet, x/D = 8, $\lambda = 0.55$

y/D	0	1	1.5	2	2.5
SSE	26.20	34.46	13.74	6.93	2.50
SST	72.19	56.74	23.35	8.84	1.76
SSR	45.99	22.28	9.61	1.90	-0.74
MSR	11.50	5.57	2.40	0.48	-0.18
MSE	0.06	0.08	0.03	0.02	0.01
R^2	0.64	0.39	0.41	0.22	-0.42
Fo	186.96	68.84	74.51	29.19	-31.43

Table F.12: Analysis of variance data of the secondary jet, x/D = 8, $\lambda = 3.6$