

Decision–Feedback Equalisation with Fixed–Lag Smoothing in Nonlinear Channels

by

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Contents

Contents	iii
Abstract	vii
Statement of Originality	ix
Acknowledgments	xi
Publications	xiii
List of Figures	xv
List of Tables	xix
Glossary	xxi
Chapter 1. Introduction	1
1.1 Literature Review	4
1.2 Original Contributions	5
Chapter 2. The FLSDFE Algorithm	9
2.1 Introduction	9
2.2 Narrowband signal models	10
2.3 MPSK and MQAM signalling, $M \geq 4$	12
2.3.1 Problem definition	13
2.3.2 Derivation of FLSDFE formulae	14
2.3.3 Probability distribution function of FLSDFE estimator	18
2.4 BPSK signalling	23
2.4.1 Problem definition	24
2.4.2 Derivation of FLSDFE formulae	24
2.4.3 Probability distribution function of FLSDFE estimator	25
2.5 Conclusion	27

Chapter 3. State Space Models	29
3.1 Atomic State Space Models—Introduction	30
3.2 Atomic State Space Models (Filtering only)	32
3.2.1 Model definition	32
3.2.2 Lexicographical ordering of states (BPSK, $N \geq 0$)	33
3.2.3 State transition probability matrix (BPSK, $N \geq 0$)	36
3.2.4 State transition probability matrix (BPSK, $N = 0$)	42
3.2.5 State transition probability matrix (BPSK, $N = 1$)	44
3.2.6 Stationary distribution vector	48
3.2.7 Error recovery time (BPSK, $N = 0$)	52
3.2.8 Error recovery time (BPSK, $N = 1$)	56
3.2.9 Error recovery time (BPSK, $N \geq 0$)	65
3.2.10 Error recovery time and the theory of integer partitions	69
3.3 Atomic State Space Models (Smoothing only)	75
3.3.1 Model definition	75
3.3.2 Lexicographical ordering of states (BPSK, $N \geq 1$)	77
3.3.3 State transition probability matrix (BPSK, $N \geq 1$)	78
3.4 Aggregated state space models	85
3.4.1 Introduction	85
3.4.2 Set partitions and Bell numbers	86
3.4.3 Restricted growth strings	87
3.4.4 Optimal state aggregation	87
3.4.5 A suboptimal aggregation	91
3.4.6 Choy and Beaulieu’s state space models	96
Chapter 4. Results	101
4.1 Introduction	101
4.2 Underwater acoustic channel models	102
4.2.1 Computing pressure fields rigorously	102
4.2.2 Computing pressure fields with rays	105
4.2.3 Ray-based channel models	108

4.3	Generic BPSK channel models	114
4.3.1	Model description	114
4.3.2	BER versus SNR	115
4.3.3	State transition probabilities	116
4.3.4	Error recovery time	124
4.3.5	Error bursts	126
4.4	Conclusion	130
Chapter 5. Conclusions and Future Work		131
Appendix A. Terms in decomposition of (2.16)		141
A.1	First-Order terms	141
A.2	Third-Order Terms	142
Appendix B. Terms in (2.22) to third-order		145
Appendix C. Terms in (2.27) to third-order		147
Appendix D. Terms in (2.30) to third-order		151
Appendix E. Terms in decomposition of (2.51)		155
E.1	First-Order terms	155
E.2	Second-Order terms	156
E.3	Third-Order terms	156
Appendix F. Terms A_{t-k} and B_{t-k} in (2.52)		159
Appendix G. Terms $\hat{A}_{t-k t-k}$ and $\hat{B}_{t-k t-k}$ in (2.53)		161
Appendix H. State Transition Probability : $n = 0$		163
Appendix I. State Transition Probability : $n \in \{1, \dots, N\}$		167
Appendix J. Program HANKEL		173
Bibliography		179

Abstract

This thesis is concerned with an aspect of the field of digital signal processing, namely, the application of fixed-lag smoothing (FLS) to decision-feedback equalisation (DFE). The resultant algorithm was introduced in a recent paper and is termed herein the Fixed-Lag-Smoothing Decision-Feedback Equalisation (FLSDFE) algorithm. The motivation for studying the FLSDFE algorithm is that it may potentially improve the performance of a standard DFE algorithm, providing more reliable digital communication and data storage. This thesis extends previous results by applying the FLSDFE algorithm to linear and nonlinear channels, of minimum- or nonminimum-phase.

In chapter 2 the FLSDFE formulae are derived for two classes of nonlinear digital communication channel, both described by truncated Volterra series. Section 2.3 treats the case of MPSK and MQAM signalling, and section 2.4 treats the case of BPSK signalling. The discrete probability distribution function of the FLSDFE output estimator is given in each case.

Chapter 3 introduces state space models that capture the exact transient dynamics of the FLSDFE algorithm. The atomic model was introduced by other authors earlier, but a suboptimal aggregation of the atomic model is given that is closely linked to an existing steady-state model. Connections are shown between the state space models and the theory of integer partitions, as well as between linear recurrence relations that generalise that of the Fibonacci series.

Examples of the performance of the FLSDFE algorithm are provided in chapter 4. Two underwater-acoustic communication channels are simulated, and it is shown that the FLSDFE performs well there, giving lower bit error rates than the DFE alone. Generic channel models, both linear and cubic, minimum- and nonminimum-phase, were then simulated. These showed some peculiar characteristics of the FLSDFE algorithm's behaviour. In particular, using the steady-state models of chapter 3, we observe a pseudo-resonance with increasing SNR (that is, the existence of an optimum SNR)—this is related to the presence of previous equalisation errors.

As a caveat to the blind use of the FLSDFE over the DFE, we illustrate that it may be necessary to determine the optimum smoothing lag to use before applying the FLSDFE algorithm, especially on difficult nonlinear channels. On such channels, increasing the

lag beyond this optimum may produce worse equalisation performance than at the optimum lag. Despite this word of caution, however, the FLSDFE seems to provide a robust improvement over the DFE across a broad range of channels.

Statement of Originality

This work contains no material that has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published written by another person, except where due reference has been made in the text.

I give consent to this copy of the thesis, when deposited in the University Library, being available for loan, photocopying and dissemination through the library digital thesis collection.

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Date

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Thanks must also surely go to my family, who provided me with the necessary backdrop of sanity and normalcy to which I could anchor. When thinking about the undertaking of a PhD I have often thought of the parody of the person who gets to be an expert on certain diseases of the back legs of a rare subspecies of Patagonian sand flea. Only a fellow expert would seem to know of or care about such things.

Lastly, but not leastly, I am grateful to DSTO for providing me with the opportunity to undertake this doctoral training. Without this support I would certainly have never undertaken such a venture in mid-career.

Publications

- [1] D. W. Bartel and L. B. White, *A novel nonlinear equalizer–application to underwater acoustic communication*, Proceedings of ACOUSTICS 2002 Conference, Adelaide, South Australia, 13–15 November 2002.
- [2] D. W. Bartel and L. B. White, *Decision–feedback equalization with fixed–lag smoothing in nonlinear channels*, Proceedings of Defense Applications of Signal Processing Conference, Midway, Utah, March 27–April 1, 2005.
- [3] D. W. Bartel, *Generalised differential encoding for underwater acoustic communication*, Proceedings of ACOUSTICS 2005 Conference, Busselton, Western Australia, 9–11 November 2005.

List of Figures

2.1	Basic block diagram of a decision–feedback equaliser (DFE).	18
2.2	The DFE in more detail.	19
2.3	Basic block diagram of the FLSDFE.	19

3.1	Valid atomic state transitions for BPSK channels with memory $N = 0$. . .	53
3.2	Valid atomic state transitions for BPSK channels with memory $N = 0$, simplified.	54
3.3	Atomic state transitions for BPSK channels with memory $N = 1$	59
3.4	Atomic state transitions for BPSK channels with memory $N = 1$, simpli- fied.	60
3.5	Figure 3.4 with the grouping of atomic states given by (3.121)–(3.123). . .	65
3.6	Error recovery for general BPSK channels of memory $N \geq 0$	66
3.7	Error recovery for general BPSK channels of memory $N = 4$	67
3.8	Steady–state transition diagram for Volterra communication channels using BPSK signalling and with memory $N = 4$	72
3.9	State transition diagram for the optimal FSMP for the linear BPSK chan- nel of memory $N = 1$	89
3.10	State transition diagram for a suboptimal FSMP for the linear BPSK channel of memory $N = 1$	92
3.11	Choy and Beaulieu’s single–errors state space model.	99

4.1	Transmission loss curves at 30 Hz.	103
4.2	Transmission loss surface at 30 Hz computed by simple ray tracing. . . .	104
4.3	Transmission loss surface at 30 Hz computed by wavenumber integration.104	
4.4	Transmission loss curves for Silt bottom.	107

4.5	Transmission loss curves for Coarse Sand bottom.	107
4.6	Channel model for Silt bottom.	109
4.7	Channel model for Coarse Sand bottom.	109
4.8	Pole-zero plot for silt bottom.	110
4.9	Pole-zero plot for coarse sand bottom.	111
4.10	FLSDFE performance for silt bottom.	111
4.11	FLSDFE performance for coarse sand bottom.	112
4.12	State transition probabilities for silt bottom.	113
4.13	State transition probabilities for coarse sand bottom.	113
4.14	Bit error rate curves for $p = 1$ and $m = 0$	117
4.15	Bit error rate curves for $p = 1$ and $m = 3$	117
4.16	Bit error rate curves for $p = 3$ and $m = 0$	118
4.17	Bit error rate curves for $p = 3$ and $m = 3$	118
4.18	State transition probabilities α_k for $p = 1$ and $m = 0$	119
4.19	State transition probabilities β_k for $p = 1$ and $m = 0$	119
4.20	State transition probabilities α_k for $p = 1$ and $m = 3$	120
4.21	State transition probabilities β_k for $p = 1$ and $m = 3$	120
4.22	State transition probabilities α_k for $p = 3$ and $m = 0$	121
4.23	State transition probabilities β_k for $p = 3$ and $m = 0$	121
4.24	State transition probabilities α_k for $p = 3$ and $m = 3$	122
4.25	State transition probabilities β_k for $p = 3$ and $m = 3$	122
4.26	Mean time to recover from an error, R_0 , from Choy and Beaulieu's single- distinct-errors state space model.	126
4.27	Error burst statistics for $p = 1$ and $m = 0$	127
4.28	Error burst statistics for $p = 1$ and $m = 3$	127
4.29	Error burst statistics for $p = 3$ and $m = 0$	128
4.30	Error burst statistics for $p = 3$ and $m = 3$	128

List of Tables

2.1	Channel outputs $\{Y_t\}$ and dependent variables $\{X_t\}$ and $\{V_t\}$ for a SISO channel of memory $N = 6$ and an FLSDFE smoothing lag of $n = 6$	15
3.1	Illustration of storage of state vector $\mathbf{s}_{t-1 t-1}$ of (3.5) in the word i	34
3.2	Illustration of storage of state vector $\mathbf{s}_{t t}$ of (3.6) in the word j	35
3.3	Storage of consecutive state vectors $\mathbf{s}_{t-1 t-1}$ and $\mathbf{s}_{t t}$ in words i and j , for the case of BPSK signalling, arbitrary channel memory $N \geq 0$, and filtering only ($n = 0$).	36
3.4	Storage of consecutive state vectors $\mathbf{s}_{t-1 t-1}$ and $\mathbf{s}_{t t}$ in words i and j , for the case of BPSK signalling, channel memory $N = 0$ (memoryless), and FLSDFE smoothing lag $n = 0$ (filtering only).	43
3.5	Storage of consecutive state vectors $\mathbf{s}_{t-1 t-1}$ and $\mathbf{s}_{t t}$ in words i and j , for the case of BPSK signalling, channel memory $N = 1$, and FLSDFE smoothing lag $n = 0$	45
3.6	Mapping of state vector $\mathbf{s}_{t t}$ to state index j , for $N = 0$ and $n = 0$	49
3.7	Elements of state vector $\mathbf{s}_{t t}$ and associated state indices for $N = 1$	51
3.8	Allocation of atomic states $\mathbf{s}_{t t}$ to subsets Φ_0 , Φ_1 and Γ for $N = 1$ and $n = 0$	57
3.9	The partitions of 5 as solutions to (3.154).	70
3.10	State–path sequences from figure 3.8 that have a recovery time of $r_0 = 10$. 72	
3.11	Cycle(s) of the state–path sequences in table 3.10.	73
3.12	Storage of consecutive state vectors $\mathbf{s}_{t-n-1 t-1}$ and $\mathbf{s}_{t-n t}$ in words i and j , for the case of BPSK signalling, arbitrary channel memory $N \geq 1$, and smoothing only ($n \in \{1, \dots, N\}$).	78
3.13	Choy and Beaulieu’s single–distinct–errors model for a channel of memory $N = 4$ (linear or nonlinear), BPSK signalling, and filtering only ($n = 0$). 97	
4.1	Jensen and Kuperman’s representative spectrum of seabed types.	106
4.2	Choy and Beaulieu’s single–distinct–errors model for a channel of memory $N = 6$ (linear or nonlinear), BPSK signalling, and filtering only ($n = 0$).124	

5.1	Step 1 of first variant FLSDFE algorithm.	134
5.2	Step 2 of first variant FLSDFE algorithm.	134
5.3	Step 3 of first variant FLSDFE algorithm.	135
5.4	Step 4 of first variant FLSDFE algorithm.	135
5.5	Step 5 of first variant FLSDFE algorithm.	135
5.6	Step 6 of first variant FLSDFE algorithm.	136
5.7	Step 7 of first variant FLSDFE algorithm.	136
5.8	Step 8 of first variant FLSDFE algorithm.	136
5.9	Step 9 of first variant FLSDFE algorithm.	136
5.10	Step 10 of first variant FLSDFE algorithm.	137
5.11	Step 11 of first variant FLSDFE algorithm.	137
5.12	Step 12 of first variant FLSDFE algorithm.	137
5.13	Step 1 of second variant FLSDFE algorithm.	138
5.14	Step 2 of second variant FLSDFE algorithm.	138
5.15	Step 3 of second variant FLSDFE algorithm.	138
5.16	Step 4 of second variant FLSDFE algorithm.	138
5.17	Step 5 of second variant FLSDFE algorithm.	139
5.18	Step 6 of second variant FLSDFE algorithm.	139

Glossary

BER	Bit–Error Rate, 110, 114–116
BPSK	Binary Phase–Shift Keying, vii, 2, 3, 6, 9, 10, 12, 13, 23, 24, 26, 27, 29–36, 41, 45, 52, 56, 65, 66, 68, 77, 78, 85, 86, 88, 90–93, 96, 97, 108, 110, 115, 124, 131, 132, 134
DFE	Decision–Feedback Equalisation, vii, 1, 2, 4, 9, 17, 26, 27, 29, 31, 32, 36, 41, 42, 44, 56, 66, 69, 76, 85, 101, 116, 126, 130–133
DSTO	Australia’s Defence Science and Technology Organisation, xi, 2
EM	Expectation–Maximization, 14
FIR	Finite–Impulse Response, 4–6, 9, 14, 17, 26, 27, 29–31, 41, 52, 56, 76, 85, 86, 90, 92, 106, 108, 110, 115, 131
FLS	Fixed–Lag Smoothing, vii
FLSDFE	Fixed–Lag–Smoothing Decision–Feedback Equalisation, vii, 2–7, 9, 10, 12, 14, 15, 17, 18, 23, 24, 26–32, 43, 45, 75, 76, 78, 82, 84, 90, 101, 102, 106, 108, 110, 114, 116, 123, 125, 126, 130, 131, 133, 134, 137, 139, 140
FSMP	Finite–State Markov Process, 29, 31, 85–88, 90–93, 132
ISI	Intersymbol Interference, 1
LSB	Least–Significant Bit, 34, 82

MPSK	M-ary Phase-Shift Keying, vii, 2, 5, 9, 12, 13, 20, 22, 23, 27, 131, 134
MQAM	M-ary Quadrature-Amplitude Modulation, vii, 5, 9, 12, 13, 20, 22, 27, 131, 134
MSB	Most-Significant Bit, 34, 37, 79
PSK	Phase-Shift Keying, 2
QAM	Quadrature-Amplitude Modulation, 2
QPSK	Quadrature Phase-Shift Keying, also known as <i>Quaternary</i> Phase-Shift Keying, 12, 13, 21, 22, 30, 41, 90
RGS	Restricted-Growth String, 86, 87
SISO	Single-Input Single-Output, 9, 12, 15, 23, 28, 108, 114, 131, 134
SNR	Signal-to-Noise Ratio, vii, 7, 12, 110, 112, 115, 116, 123, 125, 129