Computational aspects of generalized continua based on moving least square approximations

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Abstract

In recent years, current engineering technology lead to a renewed interest in generalized continuum theories. In particular, generalized continua are able to address fundamental physical phenomena which are related to the underlying microstructure of the material. Specifically scale-effects are of special interest.

In this work a generalized deformation formulation is developed which allows to incorporate material information from the microscopic and the macroscopic space into an unified constitutive model. The approach is based on a theory developed by Sansour (1998) which was originated in theoretical considerations of Ericksen and Truesdell (1957) and later on Eringen and his co-workers (Eringen 1999). The basic idea is to construct a generalized continuum consisting of macro- and micro-continuum and subsequently to compose the generalized deformation by a macro- and micro-component. This procedure results in a generalized problem formulation. Furthermore, new strain measures as well as corresponding field equations can be identified. Here, it is assumed that the deformation field can only be varied within the macro-continuum so that the balance equations are established for the macro-space. The constitutive law is defined at the microscopic level and the geometrical specification of the micro-continuum is the only material input which goes beyond those needed in a classical description.

A special detail of this approach is that it involves first order strain gradients which are expressed by second order derivatives of the displacement field. It allows to address relative motion between micro- and macro-space without adding extra degrees of freedom. In order to model this formulation this work makes use of a meshfree method based on moving least squares (MLS) which is able to provide the required higher order continuity (Lancaster and Salkauskas 1981).

Examples of meshfree methods are the diffuse element method (Nayroles, Touzot, and Villon 1992), the element-free Galerkin method (Belytschko, Lu and Gu 1994), the reproducing kernel particle method (Liu and Chen, 1995), the partition of unity method (Melenk and Babuska 1997) and the hp-cloud method (Duarte and Oden 1996), just to name a few. It was demonstrated that these kind of methods can deal especially well with problems which are characterized by large deformation or changing domain geometry. The potential in modelling formulations involving higher order derivatives has not been widely recognized yet, with the exception of a few one- respectively two-dimensional case studies (Tang *et al.*, 2003).

This work now aims to illustrate the excellent applicability of the proposed generalized deformation formulation in combination with MLS by modelling elastic and plastic problems which are proven to exhibit size-scale effects (Yang and Lakes 1981; Fleck et al. 1994; Aifantis 1999; Lam et al. 2003). Furthermore, a large-scale case study on underground excavation design reveals the potential and adaptivity of this theory with respect to heterogeneous material such as rock.

Statement of originality

This work contains no material which has been accepted to the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being made available in all forms of media, now or hereafter known.

Student's Signature   Date

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Notation and list of symbols

In the following the general scheme of notation and list of frequently used symbols are assembled:

$\mathbf{a}$ .... roman lower-case bold-face letters denote vectors

$\mathbf{A}$ .... roman upper-case bold-face letters denote tensors

$\mathbf{a}_{,i}$, $\mathbf{A}_{,i}$ ...... partial derivatives of a vector or tensor quantity are denoted by subscripted primed indices

$\mathcal{A}$ .... calligraphic upper-case letter denote sets

$\mathbb{E}(3)$ .... three-dimensional *Euclidian* vector space

$\mathbb{R}$ .... set of real numbers

$:=$ .... definition of equivalence

Grad .... gradient operator with respect to the reference configuration

Div .... divergence operator with respect to the reference configuration

det $(\cdot)$ .... determinant of $(\cdot)$

Further notations are explained as they appear in the thesis. The used operations and relations of tensor calculus are specified in App. D.