Uncertainty Analysis Methods For Multi-Criteria Decision Analysis

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Abstract

Planning, design and operational decisions are made under complex circumstances of multiple objectives, conflicting interests and participation of multiple stakeholders. Selection of alternatives can be performed by means of traditional economics-based methods, such as benefit-cost analysis. Alternatively, analyses of decision problems, including water resource allocation problems, which involve trade-offs among multiple criteria, can be undertaken using multi-criteria decision analysis (MCDA). MCDA is used to assist decision makers (DMs) in prioritising or selecting one or more alternatives from a finite set of available alternatives with respect to multiple, usually conflicting, criteria.

In the majority of decision problems, MCDA is complicated by input parameters that are uncertain and evaluation methods that involve different assumptions. Consequently, one of the main difficulties in applying MCDA and analysing the resultant ranking of the alternatives is the uncertainty in the input parameter values (i.e. criteria weights (CWs) and criteria performance values (PVs)). Analysing the sensitivity of decisions to various input parameter values is, therefore, an integral requirement of the decision analysis process. However, existing sensitivity analysis methods have numerous limitations when applied to MCDA, including only incorporating the uncertainty in the CWs, only varying one input parameter at a time and only being applicable to specific MCDA techniques.

As part of this research, two novel uncertainty analysis approaches for MCDA are developed, including a distance-based method and a reliability based approach, which enable the DM to examine the robustness of the ranking of the alternatives. Both of the proposed methods require deterministic MCDA to be undertaken in the first instance to obtain an initial ranking of the alternatives. The purpose of the distance-based uncertainty analysis method is to determine the minimum modification of the input parameters that is required to alter the total values of two selected alternatives such that rank equivalence occurs. The most critical criteria for rank reversal to occur are also able to be identified based on the results of the distance-based approach. The proposed stochastic method involves defining the uncertainty in the input values using probability distributions, performing a reliability analysis by Monte Carlo Simulation and undertaking a significance analysis using the Spearman Rank Correlation Coefficient. The outcomes of the stochastic uncertainty analysis approach include a distribution of the total values of each alternative based upon the expected range of input parameter values. The uncertainty analysis methods are implemented using a software program developed as part of this research, which may assist in negotiating sustainable decisions while fostering a collaborative learning process between DMs, experts and the community. The two uncertainty analysis approaches overcome the limitations of the existing sensitivity analysis methods by being applicable to multiple MCDA techniques, incorporating uncertainty in all of the input parameters simultaneously, identifying the most critical criteria to the ranking of the alternatives and enabling all actors preference values to be incorporated in the analysis.

Five publications in refereed international journals have emerged from this research, which constitute the core of the thesis (i.e. PhD by Publication). The publications highlight how uncertainty in all of the input parameters can be adequately considered in the MCDA process using the proposed uncertainty analysis approaches. The methodologies presented in the publications are demonstrated using a range of case studies from the literature, which illustrate the additional information that is able to be provided to the DM by utilising these techniques. Publications 1 and 2 (Journal of Environmental Management and European Journal of Operational Research) demonstrate the benefits of the distance-based uncertainty analysis approach compared to the existing deterministic sensitivity analysis methods. In addition, the benefits of incorporating all of the input parameters in the uncertainty analysis, as opposed to only the CWs, are illustrated. The differences between global and non-global optimisation methods are also discussed. Publications 3 and 4 (Journal of Water Resources Planning and Management and Journal of Multi-Criteria Decision Analysis) present the stochastic uncertainty analysis approach and illustrate its use with two MCDA techniques (WSM and PROMETHEE). Publication 5 (Environmental Modelling & Software) introduces the software program developed as part of this research, which implements the uncertainty analysis approaches presented in the previous publications.

Despite the benefits of the approaches presented in the publications, some limitations have been identified and are discussed in the thesis. Based on these limitations, it is recommended that the focus for further research be on developing the uncertainty analysis methods proposed (and in particular the program, and extension of the program) so that it includes additional MCDA techniques and optimisation methods. More work is also required to be undertaken on the Genetic Algorithm optimisation method in the distance-based uncertainty analysis approach, in order to simplify the specification of input parameters by decision analysts and DMs.

Declaration

I, **Kylie Marie Hyde**, declare that the work presented in this thesis is, to the best of my knowledge and belief, original and my own work, except as acknowledged in the text, and that the material has not been submitted, either in whole or in part, for a degree at this or any other university.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

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Dated: ………………………………………………………….

Acknowledgements

The balance of personal life with doctoral research is a complex multi-criteria decision analysis problem. A doctoral candidate is forced to trade-off recreation time against time spent with a computer and a ceiling high stack of journal papers.

(Hajkowicz, 2000)

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Publications

The following publications and conference presentations have arisen from this research:

Journal Papers:

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Hyde, K.M., Maier, H.R., Colby, C.B., (2006), New Distance-based Uncertainty Analysis Approach to Multi-Criteria Decision Analysis, European Journal of Operational Research, Under Review.

Hyde, K.M., Maier, H.R., Colby, C.B., (2005), A Distance-Based Uncertainty Analysis Approach to Multi-Criteria Decision Analysis for Water Resource Decision-making, Journal of Environmental Management, Vol 77, Iss 4, pp 278-290.

Hyde, K.M., Maier, H.R., Colby, C.B., (2004), Reliability-based approach to MCDA for water resources, Journal of Water Resources Planning and Management, Vol 130, Iss 6, pp 429-438.

Hyde, K.M., Maier, H.R., Colby, C.B., (2003), Incorporating Uncertainty in the PROMETHEE MCDA Method, Journal of Multi-Criteria Decision Analysis, Vol 12, Iss 4-5, pp 245-259.

Conference Papers:

Hyde, K.M., Maier, H.R., (2004), "Distance Based Uncertainty Analysis for Multi-Criteria Decision Analysis in Excel using Visual Basic for Applications", Mini Euro Conference 2004 – Managing Uncertainty in Decision Support Models, Coimbra, Portugal, 22 – 24 September.

Hyde, K.M., Maier, H.R., (2004), "Incorporating a Distance-based Uncertainty Analysis Approach to PROMETHEE", MCDM 2004 – New Paradigms for New Decisions, Whistler, Canada, 6 – 11 August.

Hyde, K.M., Maier, H.R., Colby, C.B., (2003), "The Applicability of Robustness Measures to Water Resources Decision-making", MODSIM Conference Proceedings, International Congress on Modelling and Simulation, Integrative Modelling of Biophysical, Social and Economic Systems for Resource Management Solutions, Townsville, Australia, July 14 – 17.

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Acronyms

AHP	Analytic Hierarchy Process
ANN	Artificial Neural Network
BCA	Benefit Cost Analysis
CAM	Conflict Analysis Model
CGT	Cooperative Game Theory
CP	Compromise Programming
CTP	Composite Programming
CWs	Criteria Weights
DEA	Data Envelope Analysis
DISID	Displaced Ideal
DIVAPIME	Determination d'Intervalles de Variation pour les Parametres d'Importance des Methodes Electre
DM	Decision Maker
DSS	Decision Support System
DST	Dempster-Shafer Theory
EF	Ecological Footprint
EIA	Environmental Impact Assessment
EIS	Environmental Impact Statement
ELECTRE	Elimination and Choice Translating Reality (Elimination Et Choix Tradusiant la Réalité)
ESAP	Evaluation and Sensitivity Analysis Program
EVI	Expected Value of Information
EVPI	Expected Value of Perfect Information

GA	Genetic Algorithm
GAIA	Graphical Analysis for Interactive Assistance
GIS	Geographical Information System
GP	Goal Programming
GRAPA	Graphical Point Allocation
GRG2	Generalised Reduced Gradient Nonlinear Optimisation Method
GRS	Graphical Rating Scale
HDT	Hasse Diagram Technique
HIPRE	Hierarchical Preference Analysis Software
IMGP	Interactive Multiple Goal Programming
IOC	Importance Order of Criteria
JAS	Judgmental Analysis System
LCA	Life Cycle Assessment
LHS	Latin Hypercube Sampling
MACBETH	Measuring Attractiveness by a Categorical Based Evaluation Technique
MAS	Multi-Agent Systems
MADM	Multiple Attribute Decision Making or Modelling
MAUT	Multi-Attribute Utility Theory
MAVF	Multi-Attribute Value Function
MAVT	Multi-Attribute Value Theory
MCA	Multiple Criteria Analysis
MCDA	Multi-Criteria Decision Analysis
MCE	Multi-Criteria Evaluation
MCQA	Multi-Criterion Q Analysis
MCS	Monte Carlo Simulation
MDI	Minimum Discrimination Information
MESA	Matrix for the Evaluation of Sustainability Achievement

MEW	Multiplicative Exponent Weighting
MODM	Multi-Objective Decision Making
MODS	Multi-Objective Decision Support
NA	Not Applicable
NAIADE	Novel Approach for Imprecise Assessment and Decision Evaluations
NPV	Net Present Value
PC	Preference Cones
PROBE	Preference Robustness Evaluation
PROMETHEE	Preference Ranking Organisation METHod for Enrichment Evaluations
PROTR	Probabilistic Trade-off Development Method
PVs	Performance Values
PW	Present Worth
SAW	Simple Additive Weighting
SMAA	Stochastic Multiobjective Acceptability Analysis
SMAA-O	Stochastic Multicriteria Acceptability Analysis with Ordinal Criteria
SMART	Simple Multi-Attribute Rating Technique
SMARTER	SMART Exploiting Ranks
STEM	Step Method
SWT	Surrogate Worth Trade-Off
TBL	Triple Bottom Line
TOPSIS	Technique for Order Preference by Similarity to an Ideal Solution
UNK	Unknown
UTA	Utility Additive

VAS	Visual Analogue Scale
VBA	Visual Basic for Applications
VIP	Variable Interdependent Parameters

WA	Weighted Average
WLAM	Weighted Linear Assignment Method
WPM	Weighted Product Method
WSM	Weighted Sum Method

ZAPROS	Closed Procedures Near Reference Situations (abbreviation of Russian words)
Z-W	Zionts-Wallenius

Notation

d_e *or* L_2	Euclidean distance
d_m or L_1	Manhattan distance
d_k	Kullback-Leibler distance
LL_{xl} and UL_{xl}	lower and upper limits, respectively, of the PVs of each criterion for the initially lower ranked alternative
LL_{xh} and UL_{xh}	lower and upper limits, respectively, of the PVs of each criterion for the initially higher ranked alternative
LL_w and UL_w	lower and upper limits, respectively, of each of the CWs
M	total number of criteria
p	preference threshold
q	indifference threshold
$V(a_y)opt$	modified total value of the initially lower ranked alternative obtained using the optimised parameters
$V(a_x)opt$	modified total value of the initially higher ranked alternative obtained using the optimised parameters
w_{mi}	initial CW of criterion *m*
w_{mo}	optimised CW of criterion *m*

x_{mnli}	initial PV of criterion *m* of initially lower ranked alternative *n*
x_{mnlo}	optimised PV of criterion *m* of initially lower ranked alternative *n*
x_{mnhi}	initial PV of criterion *m* of initially higher ranked alternative *n*
x_{mnho}	optimised PV of criterion *m* of initially higher ranked alternative *n*
$\Pi(a,b)$	outranking degree of every alternative *a* over alternative *b*
$\phi+$	leaving flow
$\phi-$	entering flow