Laplace-Domain Analysis of Fluid Line Networks with Applications to Time-Domain Simulation and System Parameter Identification

by

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B.E. (Civil) (Hons), B.Sc.

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Abstract

Networks of closed conduits containing pressurised fluid flow occur in many different instances throughout the natural and man made world. The dynamics of such networks are dependent not only on the complex interactions between the fluid body and the conduit material within each fluid line, but also on the coupling between different lines as they influence each other through their common junctions. The forward modelling (time-domain simulation), and inverse modelling (system parameter identification) of such systems is of great interest to many different research fields. An alternative approach to time-domain descriptions of fluid line networks is the Laplace-domain representation of these systems. A long standing limitation of these methods is that the frameworks for constructing Laplace-domain models have not been suitable for pipeline networks of an arbitrary topology. The objective of this thesis is to fundamentally extend the existing theory for Laplace-domain descriptions of hydraulic networks and explore the applications of this theory to forward and inverse modelling. The extensions are undertaken by the use of graph theory concepts to construct network admittance matrices based on the Laplace-domain solutions of the fundamental pipeline dynamics. This framework is extended to incorporate a very broad class of hydraulic elements. Through the use of the numerical inverse Laplace transform, the proposed theory forms the basis for an accurate and computationally efficient hydraulic network time-domain simulation methodology. The compact analytic nature of the network admittance matrix representation facilitates the development of two successful and statistically based parameter identification methodologies, one based on an oblique filtering approach combined with maximum likelihood estimation, and the other based on the expectation-maximisation algorithm.

Statement of Originality

I Aaron C. Zecchin hereby declare that this work contains no material that has been accepted for the award of any other degree or diploma in any university or other tertiary institution. To the best of my knowledge and belief, it contains no material previously published or written by any other person, except where due reference is made in the text.

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List of Abbreviations

1-D one dimensional

2-D two dimensional

BIBO bounded input/bounded output

DFT discrete Fourier transform

EM expectation-maximisation

FDI fault detection and identification

FDM finite difference method

FFT fast Fourier transform

FT Fourier transform

FVM finite volume method

I/O input/output

IFFT inverse fast Fourier transform

IFT inverse Fourier transform

ILT inverse Laplace transform

ILTM inverse Laplace transform method

IPREM impulse response method

IQR interquartile range

ITM inverse transient method

LSF laminar-steady-friction

LT Laplace transform

LUF laminar-unsteady-friction

MLE maximum likelihood estimation

MOC method of characteristics

NILT numerical inverse Laplace transform

ODE ordinary differential equation

PDE partial differential equation

PDF probability density function

PSO particle swarm optimisation

R-P-V reservoir-pipe-valve

WDS water distribution system

TSF turbulent-steady-friction

TUF turbulent-unsteady-friction

VE viscoelastic

WKB Wentzel-Kramers-Brillouin

List of Symbols

Common symbols used throughout the thesis are listed below. Subscripted versions are only given for cases where the subscripted symbols have a particular meaning as distinct from being an indexed version of the plain symbol.

$\mathbf{0}$	Zero matrix
a	Contour location of integration within the Fourier-Crump numerical inverse Laplace transform
A	Pipe cross-sectional area
A_o	Nominal pipe cross-sectional area
$\mathcal{A}$	Set of nodes for which neither the nodal pressure and flow are known, $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3 \cup \mathcal{A}_4$
$\mathcal{A}_1$	Set of nodes for which both the nodal pressure and flow are measured
$\mathcal{A}_2$	Set of nodes for which only the nodal pressure is measured
$\mathcal{A}_3$	Set of nodes for which only the nodal flow is measured
$\mathcal{A}_4$	Set of nodes for which neither the nodal pressure and flow are measured
$\mathcal{B}$	Set of nodes for which the nodal flow is known, $\mathcal{B} = \mathcal{B}_1 \cup \mathcal{B}_2$
$\mathcal{B}_1$	Set of nodes for which the nodal pressure is measured and the nodal flow is known
$\mathcal{B}_2$	Set of nodes for which the nodal pressure is unmeasured and the nodal flow is known
c	Pipeline wavespeed
c_o	Nominal pipeline wavespeed
$c(t)$	Compliance impulse response function in the compliance operator $\mathcal{C}(\cdot)$
$c(s)$	Laplace transform of $c(t)$
C_0	Compliance operator coefficient
$C(s)$	Laplace transform of pipeline compliance operator
$\mathcal{C}$	Set of compound node functions (Chapter 4), and set of nodes for which the nodal pressure is known, $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2$ (Chapter 6)
$\mathcal{C}_1$	Set of nodes for which the nodal flow is measured and the nodal pressure is known

$\mathcal{C}_2$	Set of nodes for which the nodal flow is unmeasured and the nodal pressure is known
$\mathcal{C}(\cdot)$	$\mathscr{L}$-class compliance operator
$\mathbb{C}$	The set of complex numbers
$\mathbb{C}_+$	The set of complex numbers in the closed right hand plane
D	Pipe diameter
D_o	Nominal pipe diameter
e	Pipe wall thickness
E	Young's modulus
f	Darcy-Weisbach friction factor
f_o	Nominal Darcy-Weisbach friction factor
g	Gravitational acceleration
$\boldsymbol{G}(s)$	Transfer matrix of the decoupled measured-state system
$\boldsymbol{G}_m(s)$	Transfer matrix acting on the measured states
$\boldsymbol{G}_u(s)$	Transfer matrix acting on the unmeasured states
$\mathcal{G}(\mathcal{N}, \Lambda)$	Graph comprised of nodes $\mathcal{N}$ and links Λ
$\mathcal{G}(\mathcal{N}, \Xi)$	Multi-graph comprised of nodes $\mathcal{N}$ and multi-links Ξ
$\boldsymbol{h}(t)$	Impulse response matrix for the input/output network model
$\boldsymbol{H}(s)$	Transfer function matrix for the input/output network model
i	Imaginary unit
$\boldsymbol{I}$	Identity matrix
$\mathbb{Im}\{\cdot\}$	Imaginary number function
J	Viscoelastic material compliance function
J_i	Viscoelastic material compliance coefficient for i^{th} Kelvin-Voigt element
l	Pipeline length
$\boldsymbol{L}(s)$	Network decoupling filter
$\boldsymbol{L}_X(s)$	Network decoupling filter acting on the nodal set $X = \mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$, $\mathcal{A}_4$, $\mathcal{B}_1$, $\mathcal{B}_2$, $\mathcal{C}_1$ and $\mathcal{C}_2$
$\mathcal{L}$	Laplace transform operator
$\mathcal{L}^{-1}$	Inverse Laplace transform operator
$\mathbb{M}$	Mach number
$\mathcal{M}$	Set of $\mathscr{M}$-element functions
$\mathbb{M}\mathfrak{R}$	Dissipation number for dimensionless fluid line
n_c	Number of compound nodes
n_d	Number of demand (controlled flow injection) nodes
n_D	Number of nodes for which pressure is the free nodal variable
n_r	Number of reservoir (pressure controlled) nodes
n_{si}	Number of simple connections for compound node i

n_λ	Number of links
N_H	Number of harmonics included within the numerical inverse Laplace transform
N_Δ	Discretisation of harmonics for the numerical inverse Laplace transform
$\boldsymbol{N_d}$	Incidence matrix for downstream nodes
$\boldsymbol{N_u}$	Incidence matrix for upstream nodes
$\boldsymbol{N_{di}}$	Compound node incidence matrix for downstream nodes
$\boldsymbol{N_{ui}}$	Incidence matrix for upstream nodes
$\mathcal{N}$	Set of nodes
$\mathcal{N}_c$	Set of compound nodes
$\mathcal{N}_c(\cdot,\cdot)$	Complex Gaussian distribution
$\mathcal{N}_d$	Set of demand (controlled flow injection) nodes
$\mathcal{N}_D$	Set of nodes for which pressure is the free nodal variable
$\mathcal{N}_i$	Set of connections for compound node i
$\mathcal{N}_J$	Set of junctions
$\mathcal{N}_o$	Set of nodes for the simple node expanded network
$\mathcal{N}_r$	Set of reservoir (pressure controlled) nodes
$\mathcal{N}_s$	Set of simple nodes
$\mathbb{N}$	The set of natural numbers
$p(x,t)$	Fluid line pressure (taken throughout the majority of the thesis as transient pressure fluctuations about an initial state)
$\boldsymbol{p}(\boldsymbol{x},t)$	Vector of fluid line pressures
$P(x,s)$	Laplace transform of fluid line pressure
$\boldsymbol{P}(\boldsymbol{x},s)$	Vector of Laplace transform of fluid line pressures
$\boldsymbol{P_{ci}}(s)$	Vector of Laplace transform of connection pressures for compound node i
$\boldsymbol{P_d}(s)$	Vector of pipeline pressures at upstream point
$\boldsymbol{P_u}(s)$	Vector of pipeline pressures at downstream point
$\mathcal{P}$	Set of pipeline functions
$q(x,t)$	Fluid line axial flow rate (taken throughout the majority of the thesis as transient flow fluctuations about an initial state)
$\boldsymbol{q}(\boldsymbol{x},t)$	Vector of fluid line axial flow rates
$Q(x,s)$	Laplace transform of fluid line axial flow rate
$\boldsymbol{Q}(\boldsymbol{x},s)$	Vector of Laplace transform of fluid line axial flow rates
$\boldsymbol{Q_{ci}}(s)$	Vector of Laplace transform of connection flows for compound node i
$\boldsymbol{Q_d}(s)$	Vector of pipeline axial flow rates at upstream point
$\boldsymbol{Q_u}(s)$	Vector of pipeline axial flow rates at downstream point
r	Radius

r_0	Steady-state resistive coefficient
$r(t)$	Resistive impulse response function for unsteady friction component of pipeline resistive operator
$r(s)$	Laplace transform of $r(t)$
R_0	Coefficient for resistive operator
$\mathcal{R}$	$\mathscr{L}$-class resistive operator
$\mathbb{R}$	The set of real numbers
$\mathbb{R}_+$	The set of positive real numbers
$\mathbb{R}\mathrm{e}\{\cdot\}$	Real number function
$\mathbb{R}_\mathrm{e}$	Reynolds number
s	Laplace variable
t	Time
$\boldsymbol{u}_i(t)$	Controlled internal nodal states for compound node i
$\tilde{\boldsymbol{u}}_i(t)$	Response internal nodal states for compound node i
$\boldsymbol{U}_i(s)$	Laplace transform of $\boldsymbol{u}_i(t)$
$\widetilde{\boldsymbol{U}}_i(s)$	Laplace transform of $\tilde{\boldsymbol{u}}_i(t)$
v	Axial velocity
x	Axial spatial coordinate of fluid line
$\boldsymbol{x}$	Vector of axial spatial coordinates
$\boldsymbol{Y}(s)$	Network admittance transfer matrix
$\boldsymbol{Y}_j(s)$	Admittance transfer matrix for hydraulic element j
$\boldsymbol{Y}_{\boldsymbol{c}i}(s)$	Admittance matrix operating on the compound node connection pressures for canonical form of compound node dynamics
$\boldsymbol{Y}_{\boldsymbol{u}i}(s)$	Admittance matrix operating on the compound node controlled states for canonical form of compound node dynamics
$\boldsymbol{Y}_o(s)$	Network admittance matrix for simple node expanded network
$\boldsymbol{Y}_{XY}(s)$	Partition of network admittance transfer matrix for nodal set X to nodal set Y
$\boldsymbol{\mathcal{Y}}(t)$	Network admittance matrix impulse response function
$\boldsymbol{\mathcal{Y}}_j(t)$	Admittance impulse response function for hydraulic element j
$Z_c(s)$	Fluid line series impedance
α	Pipe axial restraint coefficient
$\Gamma(s)$	Fluid line propagation operator
$\boldsymbol{\Gamma}(s)$	Fluid line propagation operator matrix for hydraulic network
ϵ_r	Retarded circumferential strain
$\theta(t)$	Nodal flows (flow injections)
$\theta_d(t)$	Nodal flows at demand nodes
$\boldsymbol{\theta}(s)$	Vector of Laplace transformed network nodal flows
$\boldsymbol{\theta}_d(s)$	Vector of Laplace transformed network demand node flows

$\boldsymbol{\theta}_r(s)$	Vector of Laplace transformed network reservoir flows
$\boldsymbol{\vartheta}$	Set of network parameters requiring estimation
$\widehat{\boldsymbol{\vartheta}}$	Estimate of $\boldsymbol{\vartheta}$ (*e.g.* the maximum likelihood estimate)
$\boldsymbol{\vartheta}_\xi$	Set of network parameters for hydraulic element ξ
λ	Graph or network link
Λ	Set of links within a graph or network
Λ_i	Set of links incident to node i
Λ_{di}	Set of links for which the downstream node is node i
Λ_{ui}	Set of links for which the upstream node is node i
ν	Kinematic viscosity
ν_o	Nominal kinematic viscosity
ξ	Hydraulic $\mathscr{M}$-element
Ξ	Set of $\mathscr{M}$-elements
ρ	Density of fluid
ρ_o	Nominal density of fluid
$\boldsymbol{\Sigma}_m$	Covariance matrix of measured network states
$\boldsymbol{\Sigma}_u$	Covariance matrix of unmeasured network states
τ	Pipe wall shear stress
$\boldsymbol{\Upsilon}$	Network parameter space, *i.e.* $\boldsymbol{\vartheta} \in \boldsymbol{\Upsilon}$
$\boldsymbol{\phi}$	Dynamics for compound node
$\boldsymbol{\Phi}(s)$	Laplace transform of $\boldsymbol{\phi}$
χ	Spatial domain of fluid line
$\boldsymbol{\chi}$	Set of spatial domains of fluid lines within a network
$\psi(t)$	Nodal pressure
$\psi_r(t)$	Nodal pressure at reservoir node
$\boldsymbol{\Psi}(s)$	Vector of Laplace transformed network nodal pressures
$\boldsymbol{\Psi}_d(s)$	Vector of Laplace transformed network demand (flow control) node pressures
$\boldsymbol{\Psi}_r(s)$	Vector of Laplace transformed network reservoir pressures
ω	Radial frequency

List of Publications

The following publications were produced from the research associated with the work presented within this dissertation.

Zecchin, A. C., A. R. Simpson, M. F. Lambert, L. B. White, and J. P. Vítkovský, Transient Modeling of Arbitrary Pipe Networks by a Laplace-Domain Admittance Matrix, *Journal of Engineering Mechanics-ASCE*, *135(6)*, 538-547, 2009.

Zecchin, A. C., A. R. Simpson, M. F. Lambert, and L. B. White, Frequency-domain modelling of transients in pipe networks with compound nodes using a Laplace-domain admittance matrix, *Journal of Hydraulic Engineering-ASCE*, (Accepted for publication), 2009.

Zecchin, A. C., M. F.Lambert, A. R. Simpson, and N. S. Arbon, Numerically efficient transient-state model for a pipeline based on the Fourier-Crump inverse Laplace transform, *33-rd IAHR World Congress*, Vancouver, Canada, 2009.

Zecchin, A. C., M. F. Lambert, and A. R. Simpson, Damage detection of operating transmission mains with measured boundary conditions, *2009 World Environmental and Water Resources Congress*, Kansas City, US, 2009.

Zecchin, A. C., A. R. Simpson, and M. F. Lambert, von Neumann stability analysis of a method of characteristics viscoelastic pipeline model, *Surge Analysis - System Design, Simulation, Monitoring and Control*, edited by S. Hunt, pp. 333-347, Edinburgh, UK, 2008.

Zecchin, A. C., M. F. Lambert, A. R. Simpson, and L. B. White, Laplace-domain comparison of Linear Models of a Reservoir-Pipe-Valve System with a Leak, *8th Annual International Water Distribution Systems Analysis Symposium*, Cincinnati, US, 2006.

Zecchin, A. C., L. B. White, M. F. Lambert, and A. R. Simpson, Frequency-domain hypothesis testing approach to leak detection in a single fluid line, *CCWI 2005, Water Management for the 21st Century*, vol. 2, edited by D. Savic, G. Walters, R. King, S. T. Khu, pp. 149-154, Exeter, UK, 2005.