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Mineo, H.; Tjon, J. A.; Tsushima, Kazuo; Yang, Shin Nan
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http://link.aps.org/doi/10.1103/PhysRevC.77.055203

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PHYSICAL REVIEW C **77**, 055203 (2008)

Faddeev calculation of the pentaquark Θ^+ in the Nambu–Jona-Lasinio model-based diquark picture

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(Received 30 September 2007; published 7 May 2008)

A Bethe-Salpeter-Faddeev (BSF) calculation is performed for the pentaquark Θ^+ in the diquark picture of Jaffe and Wilczek in which Θ^+ is a diquark-diquark-$\bar{s}$ three-body system. The Nambu–Jona-Lasinio (NJL) model is used to calculate the lowest order diagrams in the two-body scatterings of $\bar{s}D$ and DD. With the use of coupling constants determined from the meson sector, we find that $\bar{s}D$ interaction is attractive in s-wave while the DD interaction is repulsive in the p-wave. With only the lowest three-body channel considered, we do not find a bound $\frac{1}{2}^+$ pentaquark state. Instead, a bound pentaquark Θ^+ with $\frac{1}{2}^-$ is obtained with unphysically strong vector mesonic coupling constants.

DOI: 10.1103/PhysRevC.77.055203 PACS number(s): 24.85.+p, 14.80.−j, 21.45.−v, 12.39.Ki

I. INTRODUCTION

The report of the observation of a very narrow peak in the K^+n invariant mass distribution [1,2] around 1540 MeV in 2003, a pentaquark predicted in a chiral soliton model [3], triggered considerable excitement in the hadronic physics community. It was labeled as Θ^+ and included by the Particle Data Group (PDG) in 2004 [4] under exotic baryons and rated with three stars. Very intensive research efforts, both theoretically and experimentally, ensued. On the experimental side, practically all studies conducted after the first sightings were confirmed by several other groups produced null results, casting doubt on the existence of the five-quark state [5,6]. Subsequently, the PDG in 2006 reduced the rating from three to one stars [4]. More recently, the ZEUS experiment at HERA [7] observed a signal for Θ^+ in a high energy reaction, while H1 [7], SPHINX [8], and CLAS [9] did not see it. This disagreement between the LEPS [1] and other experiments could possibly originate from their differences of experimental setups and kinematical conditions. So the experimental situation is presently not completely settled [10–12]. Many theoretical approaches have been employed, in addition to the chiral soliton model [3], including quark models [13,14], QCD sum rules [15], and lattice QCD [16] to understand the properties and structure of Θ^+. Several interesting ideas were also proposed on the pentaquark production mechanism. Reviews of the theoretical activities in the last couple of years can be found in Refs. [17,18]. One of the most intriguing theoretical ideas suggested for Θ^+ is the diquark picture of Jaffe and Wilczek (JW) [19] in which Θ^+ is considered as a three-body system consisting of two scalar, isoscalar, color $\bar{\mathbf{3}}$ diquarks (D's) and a strange antiquark ($\bar{s}$). It is based, in part, on group theoretical consideration. It would hence be desirable to examine such a scheme from a more dynamical perspective. The idea of the diquark is not new. It is a strongly correlated quark pair and has been advocated by a number of QCD theory groups since the 1960's [20–22]. It is known that a diquark arises naturally from an effective quark theory in the low energy region, the Nambu–Jona-Lasinio (NJL) model [23,24]. The NJL model conveniently incorporates one of the most important features of QCD, namely, chiral symmetry and its spontaneously breaking which dictates the hadronic physics at low energy. Models based on NJL types of Lagrangians have been very successful in describing the low energy meson physics [25,26]. Based on a relativistic Faddeev equation, the NJL model has also been applied to the baryon systems [27,28]. It has been shown that, using the quark-diquark approximation, one can explain the nucleon static properties reasonably well [29,30]. If one further takes the static quark exchange kernel approximation, the Faddeev equation can be solved analytically. The resulting forward parton distribution functions [31] successfully reproduce the qualitative features of the empirical valence quark distribution. The model has also been used to study the generalized parton distributions of the nucleon [32]. Consequently, we will employ an NJL model to describe the dynamics of a diquark-diquark-antiquark system. To describe such a three-particle system, it is necessary to resort to a Faddeev formalism. Since the NJL model is a covariant-field theoretical model, it is important to use relativistic equations to describe both the three-particle and its two-particle subsystems. To this end, we will adopt a Bethe-Salpeter-Faddeev (BSF) equation [33] in our study. For practical purposes, a Blankenbecler-Sugar (BbS) [34] reduction scheme will be followed to reduce the four-dimensional integral equation into three-dimensional ones. In Sec. II, the NJL model in flavor SU(3) will be

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introduced with a focus on the diquark. The NJL model is then used to investigate the antiquark-diquark and diquark-diquark interaction with the Bethe-Salpeter equation in Sec. III. In Sec. IV, we introduce the Bethe-Salpeter-Faddeev equation and solve it for the system of the strange antiquark-diquark-diquark with the interaction obtained in Sec. III. Results and discussions are presented in Sec. V, and we summarize in Sec. VI.

II. SU(3)$_f$ NJL MODEL AND THE DIQUARK

The flavor SU$(3)_f$ NJL Lagrangian takes the form

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \mathcal{L}_I, \tag{1}$$

where $\psi^T = (u, d, s)$ is the SU(3) quark field, and $m = \mathrm{diag}(m_u, m_d, m_s)$ is the current quark mass matrix. $\mathcal{L}_I$ is a chirally symmetric four-fermi contact interaction. By a Fierz transformation, we can rewrite $\mathcal{L}_I$ into a Fierz symmetric form $\mathcal{L}_{I,q\bar{q}} = \frac{1}{2}(\mathcal{L}_I + \mathcal{F}(\mathcal{L}_I))$, where $\mathcal{F}$ stands for the Fierz rearrangement. It has the advantage that the direct and exchange terms give an identical contribution. In the $q\bar{q}$ channel, the chiral invariant $\mathcal{L}_{I,q\bar{q}}$, is given by [35]

$$\begin{aligned}\mathcal{L}_{I,q\bar{q}} &= G_1\big[\big(\bar{\psi}\lambda_f^a\psi\big)^2 - \big(\bar{\psi}\gamma^5\lambda_f^a\psi\big)^2\big]\\ &\quad - G_2\big[\big(\bar{\psi}\gamma^\mu\lambda_f^a\psi\big)^2 + \big(\bar{\psi}\gamma^\mu\gamma^5\lambda_f^a\psi\big)^2\big]\\ &\quad - G_3\big[\big(\bar{\psi}\gamma^\mu\lambda_f^0\psi\big)^2 + \big(\bar{\psi}\gamma^\mu\gamma^5\lambda_f^0\psi\big)^2\big]\\ &\quad - G_4\big[\big(\bar{\psi}\gamma^\mu\lambda_f^0\psi\big)^2 - \big(\bar{\psi}\gamma^\mu\gamma^5\lambda_f^0\psi\big)^2\big] + \cdots,\end{aligned} \tag{2}$$

where $a = 0 \sim 8$, and $\lambda_f^0 = \sqrt{\frac{2}{3}}I$. If we define G_5 by $-G_5(\bar{\psi}_i\gamma^\mu\psi_j)^2 = -(G_2 + G_3 + G_4)(\bar{\psi}_i\gamma^\mu\lambda_f^0\psi_j)^2 - G_2(\bar{\psi}_i\gamma^\mu\lambda_f^8\psi_j)^2$ where $i, j = u, d$, then G_3, G_4, G_5 are related by $G_5 = G_2 + \frac{2}{3}G_v$, with $G_v \equiv G_3 + G_4$. In passing, we mention that the conventionally used G_ω and G_ρ are related to G_5, G_v by $G_\omega = 2G_5$ and $G_\rho = 2G_5 - \frac{4}{3}G_v$. For the diquark channel we rewrite $\mathcal{L}_I$ into an form $(\bar{\psi}A\bar{\psi}^T)(\psi^T B\psi)$, where A and B are totally antisymmetric matrices in Dirac, isospin, and color indices. We will restrict ourselves to scalar-isoscalar diquark with color and flavor in $\bar{\mathbf{3}}$ as considered in the JW model. The interaction Lagrangian for the scalar-isoscalar diquark channel [36,37] is given by

$$\mathcal{L}_{I,s} = G_s\big[\bar{\psi}(\gamma^5 C)\lambda_f^2\beta_c^A\bar{\psi}^T\big]\big[\psi^T(C^{-1}\gamma^5)\lambda_f^2\beta_c^A\psi\big], \tag{3}$$

where $\beta_c^A = \sqrt{\frac{3}{2}}\lambda^A (A = 2, 5, 7)$ corresponds to one of the color $\bar{3}_c$ states. $C = i\gamma^0\gamma^2$ is the charge conjugation operator, and $\lambda's$ are the Gell-Mann matrices. The Bethe-Salpeter (BS) equation for the scalar diquark channel [36,37] is given by

$$\begin{aligned}\tau_s(q) = 4iG_s - 2iG_s\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}\big[\big(C^{-1}\gamma^5\tau_f^2\beta^A\big)\\ \times S(k+q)\big(\gamma^5 C\tau_f^2\beta^A\big)S^T(-q)\big]\tau_s(q),\end{aligned} \tag{4}$$

where the factors 4 and 2 arise from Wick contractions. $S(k) = (k\!\!\!/ - M + i\epsilon)^{-1}$ with $M \equiv M_u = M_d$, the constituent quark mass of u and d quarks, generated by solving the gap equation. $\tau_s(q)$ is the reduced t-matrix which is related to the t-matrix by $t_s(q) = (\gamma^5 C\tau_f^2\beta_c^A)\tau_s(q)(C^{-1}\gamma^5\tau_f^2\beta_c^A)$. The solution to Eq. (4) is

$$\tau_s(q) = \frac{4iG_s}{1 + 2G_s\Pi_s(q^2)}, \tag{5}$$

with

$$\Pi_s(q^2) = 6i\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}_D[\gamma^5 S(q)\gamma^5 S(k+q)]. \tag{6}$$

The gap equation for u, d, and s quarks are given by

$$M_i = m_i - 8G_1\langle\bar{q}_i q_i\rangle, \tag{7}$$

with

$$\langle\bar{q}_i q_i\rangle \equiv -iN_c\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}_D(S(k)), \tag{8}$$

where $i = u, d, s$. The loop integrals in Eqs. (6) and (8) diverge and we need to regularize the four-momentum integral by adopting some cut-off scheme. With regularization, we can solve the gap equation and t-matrix of the diquark in Eqs. (5) and (8) to determine the constituent quark and diquark masses. However, since our purpose in this work is not an exact quantitative analysis but rather a qualitatively study of the interactions inside Θ^+, we will not adopt any regularization scheme and simply use the empirical values of the constituent quark masses $M = M_{u,d} = 400$ MeV, $M_s = 600$ MeV, and the diquark mass $M_D = 600$ MeV as obtained in the study of the nucleon properties [27–29,31,32].

III. TWO-BODY INTERACTIONS FOR STRANGE ANTIQUARK-DIQUARK ($\bar{s}$D) AND DIQUARK-DIQUARK (DD) CHANNELS

In the JW model for Θ^+, the two scalar-isoscalar, color $\bar{\mathbf{3}}$ diquarks must be in a color $\mathbf{3}$ in order to combine with $\bar{s}$ into a color singlet. Since $\mathbf{3}$ is the antisymmetric part of $\bar{\mathbf{3}} \times \bar{\mathbf{3}} = \mathbf{3} \oplus \bar{\mathbf{6}}$, the diquark-diquark wave function must be antisymmetric with respect to the rest of its labels. For two identical scalar-isoscalar diquarks $[ud]_0$, only spatial labels remain so that the spatial wave function must be antisymmetric under space exchange and the lowest possible state is p-state. Since in JW's scheme, Θ^+ has the quantum number of $J^P = \frac{1}{2}^+$, $\bar{s}$ would be in a relative s-wave to the DD pair. Accordingly, we will consider only the configurations where $\bar{s}D$ and DD are in relative s- and p-waves, respectively. We will employ the Bethe-Salpeter-Faddeev equation [33] to describe such a three-particle system of $\bar{s}DD$. For consistency, we will use the Bethe-Salpeter equation to describe two-particles subsystems like $\bar{s}D$ and DD, which read as

$$T = B + BG_0T, \tag{9}$$

where B is the sum of all two-body irreducible diagrams and G_0 is the free two-body propagator. In momentum space, the resulting Bethe-Salpeter equation can be written as

$$\begin{aligned}&T(k', k; P)\\ &\quad = B(k', k; P) + \int d^4k'' B(k', k''; P)G_0(k''; P)T(k'', k; P),\end{aligned} \tag{10}$$

where G_0 is the free two-particle propagator in the intermediate states. k and P are, respectively, the relative and total momentum of the system. In practical applications, B is commonly approximated by the lowest order diagrams prescribed by the model Lagrangian and will be denoted as V hereafter. In addition, it is often to further reduce the dimensionality of the integral equation (10) from four to three, while preserving the relativistic two-particle unitarity cut in the physical region. It is well known (for example, Ref. [38]) that such a procedure is rather arbitrary and we will adopt, in this work, the widely employed Blankenbecler-Sugar (BbS) reduction scheme [34] which, for the case of two spinless particles, amounts to replacing G_0 in Eq. (10) by

$$\begin{aligned}
G_0(k, P) &= \frac{1}{(P/2+k)^2 - m_1^2}\,\frac{1}{(P/2-k)^2 - m_2^2} \\
&\to -i(2\pi)^4 \frac{1}{(2\pi)^3}\int \frac{ds'}{s-s'+i\epsilon}\,\delta^{(+)}\left((P'/2+k)^2 - m_1^2\right) \\
&\quad \times \delta^{(+)}\left((P'/2-k)^2 - m_2^2\right) \\
&= -2\pi i\,\delta\left(k_0 - \frac{E_1(|\vec{k}|) - E_2(|\vec{k}|)}{2}\right) G^{\rm BbS}(|\vec{k}|, s), \qquad (11)
\end{aligned}$$

with

$$\begin{aligned}
G^{\rm BbS}(|\vec{k}|, s) &= \frac{E_1(|\vec{k}|) + E_2(|\vec{k}|)}{2E_1(|\vec{k}|)E_2(|\vec{k}|)}\,\frac{1}{s - (E_1(|\vec{k}|) + E_2(|\vec{k}|))^2 + i\epsilon}, \qquad (12)
\end{aligned}$$

where $s = P^2$ and $P' = \sqrt{s'/s}\,P$. The superscript $(+)$ associated with the delta functions mean that only the positive energy part is kept in the propagator, and $E_{1,2}(|\vec{k}|) \equiv \sqrt{\vec{k}^2 + m_{1,2}^2}$.

A. $\bar{s}$D potential and the t-matrix

In Fig. 1 we show the lowest order diagram, i.e., the first order in $\mathcal{L}_{I,q\bar{q}}$ in $\bar{s}D$ scattering. Due to the trace properties for Dirac matrices, only the scalar-isovector $(\bar{\psi}\lambda_f^a\psi)^2$, the vector-isoscalar $(\bar{\psi}\gamma^\mu\lambda_f^0\psi)^2$, and the vector-isovector $(\bar{\psi}\gamma^\mu\lambda_f^a\psi)^2$ will contribute to the vertex Γ.

Furthermore, the isovector vertex $(\bar{\psi}\Gamma\lambda_f^a\psi)^2$ will not contribute since the trace in flavor space vanishes, $\sum_{a=0}^{8}(\lambda_f^a)_{33}\mathrm{tr}_f(\lambda_f^2\lambda_f^a\lambda_f^2) = 0$. Thus only the vector-isoscalar term, $(\bar{\psi}\gamma^\mu\lambda_f^0\psi)^2$, remains.

For the on-shell diquarks, the lower part of Fig. 1 which corresponds to the scalar diquark form factor, can be calculated as

$$\begin{aligned}
&(p_{Di} + p_{Df})^\mu F_v(q^2) \\
&= i\int \frac{d^4k}{(2\pi)^4}\mathrm{tr}\big[\big(g_D C^{-1}\gamma^5\lambda_f^2\beta_c^A\big)S(k+q)\gamma^\mu \\
&\quad \times S(k)\big(g_D\gamma^5 C\lambda_f^2\beta_c^A\big)S^T(k-p_{Di})\big] \\
&= 6ig_D^2\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}[S(k+q)\gamma^\mu S(k)S(p_{Di}-k)], \qquad (13)
\end{aligned}$$

where we have made use of the relations $C^{-1}(\gamma^\mu)^T C = -\gamma^\mu$, $\mathrm{tr}_c[\beta_c^A\beta_c^A] = 3$. g_D is defined by

$$g_D^{-2} = -\left.\frac{\partial\Pi_D(p^2)}{\partial p^2}\right|_{p^2=M_D^2}, \qquad (14)$$

with

$$\Pi_D(p^2) \equiv 6i\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}[S(k)S(p-k)], \qquad (15)$$

and M_D is the diquark mass. $F_v(0)$ is normalized as $2p^\mu F_v(0) = -g_D^2\frac{\partial\Pi_D(p^2)}{\partial p_\mu}$, such that $F_v(0) = 1$.[1]

Then the matrix element of the potential $V_{\bar{s}D}$ can be expressed as

$$\begin{aligned}
&\langle\bar{s}_f D_f|V|\bar{s}_i D_i\rangle \\
&= (-\bar{v}(p_{\bar{s}i}))(-iV_{\bar{s}D})(p_{Di}, p_{Df})v(p_{\bar{s}f}) \\
&= (+16i)(-G_v)(-\bar{v}(p_{\bar{s}i}))\gamma_\mu v(p_{\bar{s}f})\big[(\lambda_f^0)_{33}\cdot\mathrm{tr}_f\big(\lambda_f^0(\lambda_f^2)^2\big)\big] \\
&\quad \times (p_{Di} + p_{Df})^\mu\frac{F_v(q^2)}{\mathrm{tr}_f\big((\lambda_f^2)^2\big)}, \qquad (16)
\end{aligned}$$

i.e.,

$$V_{\bar{s}D} = \tfrac{64}{3}G_v F_v(q^2)\tilde{V}_{\bar{s}D}(p_{Di}, p_{Df}), \qquad (17)$$

with

$$\tilde{V}_{\bar{s}}D(p_{Di}, p_{Df}) = (\not{p}_{Di} + \not{p}_{Df})/2. \qquad (18)$$

Here the factor $+16i$ in Eq. (16) arises from the Wick contractions, and the factor $\mathrm{tr}_f((\lambda_f^2)^2)$ in Eq. (16) is introduced to divide $F_v(q^2)$, since the factor $\mathrm{tr}_f((\lambda_f^2)^2)$ is already included in the expression of $F_v(q^2)$ by a trace in flavor $\mathrm{SU}(3)_f$ space.

The three-dimensional scattering equation for the $\bar{s}D$ system shown in Fig. 2 is now given by

$$\begin{aligned}
&t_{\bar{s}D}(p_{Di}, p_{Df}) \\
&= V_{\bar{s}D}(p_{Di}, p_{Df}) + 4\pi\int\frac{d|\vec{p}_D'||\vec{p}_D'|^2}{(2\pi)^3}\frac{1}{2} \\
&\quad \times \int_{-1}^{1}dx_i G_{\bar{s}D}^{\rm BbS}(|\vec{p}_D'|, s_2)K_{\bar{s}D}(|\vec{p}_{Di}|, |\vec{p}_D'|, x_i) \\
&\quad \times t_{\bar{s}D}(\vec{p}_D', p_{Df}), \qquad (19)
\end{aligned}$$

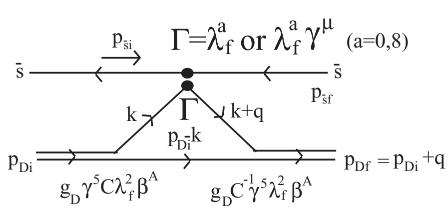

FIG. 1. $\bar{s}$D potential of the lowest order in $\mathcal{L}_{I,q\bar{q}}$.

[1]In the actual calculation we use the dipole form factor, $F_v(q^2) \equiv (1 - q^2/\Lambda^2)^{-2}$ with $\Lambda = 0.84$ GeV since the q^2 dependence for $F_v(q^2)$ in the NJL model is not well reproduced.

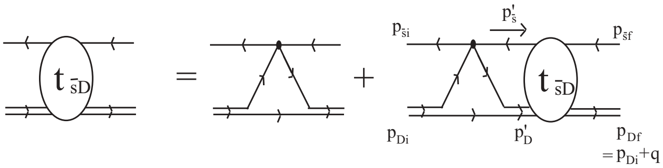

FIG. 2. The BS equation for $\bar{s}D$.

where $x_i \equiv \hat{p}_{Di} \cdot \hat{p}^{'}_D$, $\hat{p} \equiv \vec{p}/|p|$, $s_2 = (p_{Di} + p_{\bar{s}i})^2 = (p_{Df} + p_{\bar{s}f})^2$, $p^0_{Di} = \sqrt{\vec{p}^{\,2}_{Di} + M^2_D}$, $p^0_{Df} = \sqrt{\vec{p}^{\,2}_{Df} + M^2_D}$ and

$$K_{\bar{s}D}(|\vec{p}_{Di}|, |\vec{p}^{'}_D|, x_i) \equiv \tfrac{64}{3} G_v F_v((p'_D - p_{Di})^2) \times \tilde{K}_{\bar{s}D}(p_{Di}, p'_D)|_{p'^0_D = \sqrt{\vec{p}'^2_D + M^2_D}},$$

$$\tilde{K}_{\bar{s}D}(p_{Di}, p'_D) = (\not{p}_{Di} + \not{p}^{'}_D)(-\not{p}^{'}_{\bar{s}} + M_s)/2,$$

with M_s being the constituent quark mass of $\bar{s}$ and s.

We also present the results for the interactions between diquark and $\bar{u}$ or $\bar{d}$, which would be of interest when we study nonstrange pentaquarks. One can just repeat the derivations we describe in the above and easily obtain

$$V_{\bar{u}D} = V_{\bar{d}D} = -16 G_1 F_s(q^2) + 32 G_5 F_v(q^2) \tilde{V}_{\bar{s}D}(p_{Di}, p_{Df}), \tag{20}$$

in analogous to Eqs. (17) and (18).

We add in passing that the sign of the potential for sD is opposite to that of $V_{\bar{s}D}$ due to charge conjugation, i.e.,

$$V_{sD}(p_{Df}, p_{Di}) = -V_{\bar{s}D}(p_{Di}, p_{Df}), \tag{21}$$

which only holds at tree level.

We can immediately write down the scattering equation for the sD as

$$\begin{aligned} t_{sD}(p_{Df}, p_{Di}) &= V_{sD}(p_{Df}, p_{Di}) + 4\pi \int \frac{d|\vec{p}^{'}_D||\vec{p}^{'}_D|^2}{(2\pi)^3} \\ &\times \frac{1}{2}\int_{-1}^{1} dx_f G^{\rm BbS}_{sD}(|\vec{p}^{'}_D|, s_2) K_{sD}(|\vec{p}_{Df}|, |\vec{p}^{'}_D|, x_f) \\ &\times t_{sD}(\vec{p}^{'}_D, p_{Di}), \end{aligned} \tag{22}$$

where $x_f \equiv \hat{p}_{Df} \cdot \hat{p}^{'}_D$, $G^{\rm BbS}_{sD}(|\vec{p}^{'}_D|, s_2) = G^{\rm BbS}_{\bar{s}D}(|\vec{p}^{'}_D|, s_2)$, and

$$\begin{aligned} &K_{sD}(|\vec{p}_{Df}|, |\vec{p}^{'}_D|, x_f) \\ &\quad \equiv \tfrac{64}{3} G_v F_v((p'_D - p_{Df})^2) \tilde{K}_{sD}(p_{Df}, p'_D)|_{p'^0_D = \sqrt{\vec{p}'^2_D + M^2_D}}, \\ &\tilde{K}_{sD}(p_{Df}, p'_D) \\ &\quad = -(\not{p}_{Df} + \not{p}^{'}_D)(\not{p}^{'}_s + M_s)/2, \end{aligned} \tag{23}$$

with $p'_s = p'_{\bar{s}}$.

B. Representation in ρ-spin notation

In the $\bar{s}D$ (or sD) center of mass system the wave function which describes the relative motion in $J = \frac{1}{2}$, is given by the Dirac spinor of the following form (see [39,40]):

$$\Psi_{sD,m_s}(p^0_s, \vec{p}_s) = \begin{pmatrix} \phi_{s1}(p^0_s, |\vec{p}_s|) \\ \vec{\sigma} \cdot \hat{p}_s\, \phi_{s2}(p^0_s, |\vec{p}_s|) \end{pmatrix} \chi_{m_s},$$

$$\Psi_{\bar{s}D,m_s}(p^0_{\bar{s}}, \vec{p}_{\bar{s}}) = \begin{pmatrix} \vec{\sigma} \cdot \hat{p}_{\bar{s}}\, \phi_{\bar{s}2}(p^0_{\bar{s}}, |\vec{p}_{\bar{s}}|) \\ \phi_{\bar{s}1}(p^0_{\bar{s}}, |\vec{p}_{\bar{s}}|) \end{pmatrix} \chi_{m_s}, \tag{24}$$

$$= \gamma^5 \begin{pmatrix} \phi_{\bar{s}1}(p^0_{\bar{s}}, |\vec{p}_{\bar{s}}|) \\ \vec{\sigma} \cdot \hat{p}_{\bar{s}}\, \phi_{\bar{s}2}(p^0_{\bar{s}}, |\vec{p}_{\bar{s}}|) \end{pmatrix} \chi_{m_s}, \tag{25}$$

$$\bar{\Psi}_{sD}(p^0_s, \vec{p}_s) \equiv \Psi^{\dagger}_{sD}(p^0_s, \vec{p}_s)\gamma^0, \tag{26}$$

$$\bar{\Psi}_{\bar{s}D}(p^0_{\bar{s}}, \vec{p}_{\bar{s}}) \equiv \Psi^{\dagger}_{\bar{s}D}(p^0_{\bar{s}}, \vec{p}_{\bar{s}})\gamma^0, \tag{27}$$

where $\vec{p}_D = -\vec{p}_s = -\vec{p}_{\bar{s}}$, i.e., $\Psi_{sD}(p^0_s, \vec{p}_s) = \Psi_{sD}(p^0_s, -\vec{p}_D)$ and $\Psi_{\bar{s}D}(p^0_{\bar{s}}, \vec{p}_{\bar{s}}) = \Psi_{\bar{s}D}(p^0_{\bar{s}}, -\vec{p}_D)$. In the following we simply write $p'_Q = |\vec{p}^{'}_Q|$, $p'_{Qi(f)} = |\vec{p}^{'}_{Qi(f)}|$, $Q = s, \bar{s}$ or D. Note that the index 1 (2) corresponds to large (small) components for both $\bar{s}$ and s quark spinors.

For a discretization in spinor space, we define the complete set of ρ-spin notation ([39,41]) for the operators $\mathcal{O}_{sD} = V_{sD}, t_{sD}, \tilde{V}_{sD}$ and $\mathcal{K}_{sD} = K_{sD}, \tilde{K}_{sD}$ of sD:

$$\begin{aligned} &\mathcal{O}_{sD,nm}(p_{Df}, p_{Di}) \\ &\quad \equiv {\rm tr}[\Omega^{\dagger}_n(p_{sf})\mathcal{O}_{sD}(p_{Df}, p_{Di})\Omega_m(p_{si})], \end{aligned} \tag{28}$$

$$\begin{aligned} &\mathcal{K}_{sD,nm}(p_{Df}, p'_D, x_f) \\ &\quad \equiv {\rm tr}[\Omega^{\dagger}_n(p_{sf})\mathcal{K}_{sD}(p_{Df}, p'_D, x_f)\Omega_m(p'_s)], \end{aligned} \tag{29}$$

where $n, m = 1, 2$, $\Omega_1(p) = \frac{\Omega}{\sqrt{2}}$ and $\Omega_2(p) = \vec{\gamma} \cdot \hat{p}\frac{\Omega}{\sqrt{2}}$, $\Omega = \frac{1+\gamma_0}{2}$. $\Omega_1(p)$ and $\Omega_2(p)$ satisfy ${\rm tr}[\Omega^{\dagger}_n(p)\Omega_m(p')] = \delta_{n1}\delta_{m1} + \hat{p} \cdot \hat{p}^{'}\delta_{n2}\delta_{m2}$.

Concerning the $\bar{s}D$ spinor, the large and small components can be reversed by γ^5, with the minus sign which comes from the definitions Eqs. (25) and (27): $\bar{\Psi}_{\bar{s}D}\mathcal{O}\Psi_{\bar{s}D} = -\bar{\Psi}_{sD}\gamma^5\mathcal{O}\gamma^5\Psi_{sD}$. Then we can define the ρ-spin notation for $\bar{s}D$, i.e., $\mathcal{O}_{\bar{s}D} = V_{\bar{s}D}, t_{\bar{s}D}, \tilde{V}_{\bar{s}D}$ and $\mathcal{K}_{\bar{s}D} = K_{\bar{s}D}, \tilde{K}_{\bar{s}D}$,

$$\begin{aligned} &\mathcal{O}_{\bar{s}D,nm}(p_{Di}, p_{Df}) \\ &\quad \equiv -{\rm tr}[\Omega^{\dagger}_n(p_{\bar{s}i})\gamma^5\mathcal{O}_{\bar{s}D}(p_{Di}, p_{Df})\gamma^5\Omega_m(p_{\bar{s}f})], \end{aligned} \tag{30}$$

$$\begin{aligned} &\mathcal{K}_{\bar{s}D,nm}(p_{Di}, p'_D, x_i) \\ &\quad \equiv -{\rm tr}[\Omega^{\dagger}_n(p_{\bar{s}i})\gamma^5\mathcal{K}_{\bar{s}D}(p_{Di}, p'_D, x_i)\gamma^5\Omega_m(p'_{\bar{s}})]. \end{aligned} \tag{31}$$

From Eqs. (19), (22), (28)–(31), each component $n(n = 1, 2)$ of spinors for the $\bar{s}D$ satisfy the following quadratic equation:

$$\begin{aligned} &\phi^{\dagger}_{\bar{s}n}(p_{\bar{s}i}) t_{\bar{s}D,nm}(p_{Di}, p_{Df}) \phi_{\bar{s}m}(p_{\bar{s}f}) \\ &\quad = \phi^{\dagger}_{\bar{s}n}(p_{\bar{s}i}) \Bigg[V_{\bar{s}D,nm}(p_{Di}, p_{Df}) + 4\pi \sum_{l=1}^{2} \int \frac{dp'_D}{(2\pi)^3} p'^2_D \frac{1}{2} \\ &\quad \times \int_{-1}^{1} dx_i G^{BbS}_{\bar{s}D}(p'_D, s_2) K_{\bar{s}D,nl}(p_{Di}, p'_D, x_i) \\ &\quad \times t_{\bar{s}D,lm}(p'_D, p_{Df}) \Bigg] \phi_{\bar{s}m}(p_{\bar{s}f}). \end{aligned} \tag{32}$$

A similar equation can be obtained for the sD by exchanging $i \leftrightarrow f$ and $s \leftrightarrow \bar{s}$ in Eq. (32).

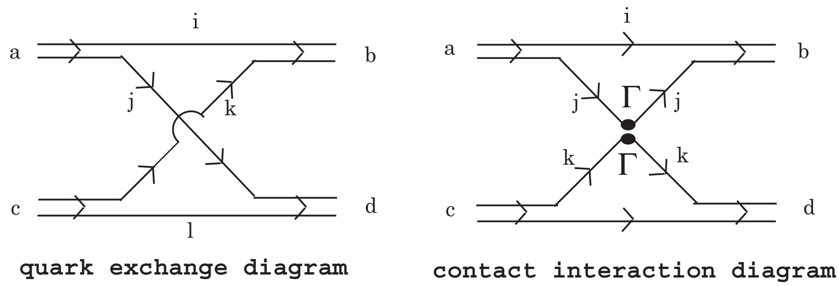

FIG. 3. Lowest order diagrams in DD scattering.

The explicit expressions of the ρ-spin notation for $\tilde{V}_{\bar{s}(s)D}$ and $\tilde{K}_{\bar{s}(s)D}$ are given in Appendix B. We note that there are important relations:

$$\begin{aligned} V_{\bar{s}D,nm}(p,q) &= -V_{sD,nm}(p,q),\\ V_{\bar{s}D}(p,q) &= -V_{sD}(p,q),\\ K_{\bar{s}D,nm}(|\vec{p}\,|,|\vec{q}\,|,x_{pq}) &= -K_{sD,nm}(|\vec{p}\,|,|\vec{q}\,|,x_{pq}),\\ K_{\bar{s}D}(|\vec{p}\,|,|\vec{q}\,|,x_{pq}) &= -K_{sD}(|\vec{p}\,|,|\vec{q}\,|,x_{pq}). \end{aligned}$$

By the partial wave expansion in Eq. (A1) in Appendix A, the BS equation for $t_{\bar{s}D,nm}$ in Eq. (32) for s-wave can be written as

$$\begin{aligned} & t^{l_{\bar{s}D}=0}_{\bar{s}D,nm}(p_{Di},p_{Df})\\ &\quad = V^{l_{\bar{s}D}=0}_{\bar{s}D,nm}(p_{Di},p_{Df}) + 4\pi\int\frac{dp'_D}{(2\pi)^3}p'^2_D\sum_{l=1}^{2}G^{BbS}_{\bar{s}D}(p'_D,s_2)\\ &\quad\quad\times K^{l_{\bar{s}D}=0}_{\bar{s}D,nl}(p_{Di},p'_D)t^{l_{\bar{s}D}=0}_{\bar{s}D,lm}(p'_D,p_{Df}). \end{aligned} \tag{33}$$

C. DD potential and t-matrix

In the case of DD interaction, the lowest order diagrams are depicted in Figs. 3(a) and (b), with (a) the quark rearrangement diagram and (b) of the first order in $\mathcal{L}_{I,q\bar{q}}$, respectively.

We first show that the quark exchange diagram in Fig. 3(a) does not contribute due to its color structure, where $a \sim d$ and $i \sim l$ denote the color indices of the diquarks and quarks, respectively. Since each diquark is in the color $\bar{\mathbf{3}}$ [19,36], the color factor for the qqD vertex is proportional to ϵ_{aij}. Hence the color factor of the quark exchange diagram is given by

$$\epsilon_{aij}\epsilon_{bik}\epsilon_{clk}\epsilon_{dlj} = \delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc}. \tag{34}$$

As we discussed earlier, the color of the DD pair inside Θ^+ is of $\mathbf{3}$ in order to combine with $\bar{s}$ to form a color singlet pentaquark. As a color $\mathbf{3}$ state is antisymmetric under the exchange between diquarks in the initial and final states, the matrix element of Eq. (34) vanishes.

For the contact interaction diagram, Fig. 3(b), only the direct term is shown since the exchange term does not contribute as it has the same color structure as the quark rearrangement diagram of Fig. 3(a). It is easy to see that the color structure of Fig. 3(b) is proportional to $\delta_{ab}\delta_{cd}$. Then the terms in the interaction Lagrangian in Eq. (2) that can give rise to nonvanishing contributions are

$$G_1\big(\bar{\psi}\lambda^a_f\psi\big)^2,\ -G_2\big(\bar{\psi}\gamma^\mu\lambda^a_f\psi\big)^2,\ -G_v\big(\bar{\psi}\gamma^\mu\lambda^0_f\psi\big)^2, \tag{35}$$

with $a = 0 \sim 8$.

We next calculate the form factors, which diagrammatically correspond to the lower part of the diagram in Fig. 1. For $\Gamma = \gamma^\mu\lambda^a_f$, we obtain

$$\begin{aligned} & \mathrm{tr}_f\big(\lambda^a_f(\lambda^2_f)^2\big)(p_{Di}+p_{Df})^\mu\frac{F_v(q^2)}{\mathrm{tr}_f\big((\lambda^2_f)^2\big)}\\ &\quad = \left(\sqrt{\frac{2}{3}}\delta_{a0}+\sqrt{\frac{1}{3}}\delta_{a8}\right)(p_{Di}+p_{Df})^\mu F_v(q^2), \end{aligned} \tag{36}$$

and for $\Gamma = \lambda^a_f$, we get

$$\begin{aligned} & \mathrm{tr}_f\big(\lambda^a_f(\lambda^2_f)^2\big)\frac{F_s(q^2)}{\mathrm{tr}_f\big((\lambda^2_f)^2\big)}\\ &\quad = \left(\sqrt{\frac{2}{3}}\delta_{a0}+\sqrt{\frac{1}{3}}\delta_{a8}\right)F_s(q^2), \end{aligned} \tag{37}$$

where the factor $\mathrm{tr}_f((\lambda^2_f)^2)$ in Eqs. (36) and (37) is introduced by the same reason for Eq. (16), and we have used $\mathrm{tr}(\lambda^2_f\lambda^a_f\lambda^2_f) = 2(\sqrt{\frac{2}{3}}\delta_{a0}+\sqrt{\frac{1}{3}}\delta_{a8})$.

For the on-shell diquarks, $F_s(q^2)$ is calculated as[2]

$$\begin{aligned} F_s(q^2) &= i\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}\big[\big(g_D C^{-1}\gamma^5\lambda^2_f\beta^A\big)S(k+q)S(k)\\ &\quad\times\big(g_D\gamma^5 C\lambda^2_f\beta^A\big)S^T(k-p_{Di})\big]\\ &= 6ig^2_D\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}[S(k+q)S(k)S(k-p_{Di})]. \end{aligned} \tag{38}$$

With the form factors $F_v(q^2)$ and $F_s(q^2)$ obtained in the above, V_{DD} is given by

$$\begin{aligned} & -iV_{DD}(\vec{p}_{Di},\vec{p}_{Df})\\ &\quad = +128i\big[G_1F^2_s(q^2) - \big(G_2+\tfrac{2}{3}G_v\big)(p_{D1i}+p_{D1f})\\ &\quad\quad\cdot(p_{D2i}+p_{D2f})F^2_v(q^2)\big]\\ &\quad = 128i\big[G_1F^2_s(q^2) - G_5(p_{D1i}+p_{D1f})\cdot(p_{D2i}+p_{D2f})\\ &\quad\quad\times F^2_v(q^2)\big], \end{aligned} \tag{39}$$

where the factor $+128i$ in a first line of Eq. (39) comes from the Wick contractions, and in a second line we have used the relation between coupling constants in meson sectors; $G_5 =$

[2]Same as the case for $\bar{s}D$ potential, we use the dipole form factor, $F_s(q^2) \equiv c_s(1-q^2/\Lambda^2)^{-2}$ with $\Lambda = 0.84$ GeV and c_s is a constant. In the original NJL model calculation with the Pauli-Villars (PV) cutoff, c_s is given by $F_s(0) = c_s = 0.53$ GeV [32].

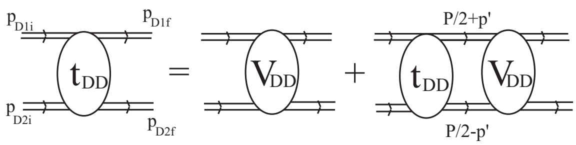

FIG. 4. BS equation for DD.

$G_2 + \frac{2}{3}G_v$ which is explained in Sec. II. The momenta of the diquarks in the initial and final states in Fig. 4 are given by

$$\begin{aligned} p_{D1i(f)} &= (\sqrt{s_2}/2, \vec{p}_{Di(f)}), \\ p_{D2i(f)} &= (\sqrt{s_2}/2, -\vec{p}_{Di(f)}), \end{aligned} \tag{40}$$

with $q = p_{D1f} - p_{D1i} = p_{D2i} - p_{D2f} \cdot s_2 = 4(\vec{p}_{Di}^{\,2} + M_D^2) = 4(\vec{p}_{Df}^{\,2} + M_D^2)$ is the DD center of mass energy squared.

As in the case of $\bar{s}D$ scattering, we use the BbS three-dimensional reduction scheme and the resulting equation for DD scattering reads as

$$\begin{aligned} t_{DD}(\vec{p}_{Df}, \vec{p}_{Di}) &= V_{DD}(\vec{p}_{Df}, \vec{p}_{Di}) + \int \frac{d^3p'}{(2\pi)^3} V_{DD}(\vec{p}_{Df}, \vec{p}^{\,'}) \\ &\quad \times G_{DD}^{BbS}(|\vec{p}^{\,'}|, s_2) t_{DD}(\vec{p}^{\,'}, \vec{p}_{Di}), \end{aligned} \tag{41}$$

with

$$\begin{aligned} G_{DD}^{\rm BbS}(|\vec{p}^{\,'}|, s_2) &= \frac{1}{4E_D(|\vec{p}^{\,'}|)(s_2/4 - E_D(|\vec{p}^{\,'}|)^2 + i\epsilon)} \\ &= \frac{1}{4E_D(|\vec{p}^{\,'}|)\big(\vec{p}_{Df}^{\,2} - \vec{p}^{\,\prime 2} + i\epsilon\big)}, \end{aligned} \tag{42}$$

with $E_D(|\vec{p}^{\,'}|) = \sqrt{\vec{p}^{\,\prime 2} + M_D^2}$.

In the JW model for Θ^+, the diquark-diaquark spatial wave function must be antisymmetric and we will consider here only the lowest configuration, namely, DD are in a relative p-wave. The partial wave expansion of Eq. (A1) then gives

$$\begin{aligned} t_{DD}^{l=1}(p_f, p_i) &= V_{DD}^{l=1}(p_f, p_i) + 4\pi \int \frac{dp'}{(2\pi)^3} p'^2 G_{DD}^{\rm BbS}(p', s_2) \\ &\quad \times V_{DD}^{l=1}(p_f, p') t_{DD}^{l=1}(p', p_i), \end{aligned} \tag{43}$$

with $p_{i(f)} \equiv |\vec{p}_{Di(f)}|$, $p' \equiv |\vec{p}^{\,'}|$.

IV. RELATIVISTIC FADDEEV EQUATION

A. Three-body Lippmann-Schwinger equation

For a system of three particles with momenta $\vec{k}_i's (i = 1, 2, 3)$, we introduce the Jacobi momenta with particle 3 as a special choice:

$$\begin{aligned} \vec{k}_1 &= \mu_1 \vec{P} + \vec{\tilde{p}} + \alpha_1 \vec{\tilde{q}}_3, \\ \vec{k}_2 &= \mu_2 \vec{P} - \vec{\tilde{p}} + \alpha_2 \vec{\tilde{q}}_3, \\ \vec{k}_3 &= \mu_3 \vec{P} + \alpha_3 \vec{\tilde{q}}_3, \end{aligned} \tag{44}$$

with $\sum \mu_n = 1$ and $\alpha_3 = -\alpha_1 - \alpha_2$. For the coefficients we find $\mu_n = m_n/M$, $M = m_1 + m_2 + m_3$, and $\alpha_1 = m_1/m_{12}$, $\alpha_2 = m_2/m_{12}$, $\alpha_3 = -1$, where $m_{ij} = m_i + m_j$ $(i \neq j)$. In terms of the Jacobi momenta the total kinetic energy is given by

$$K_{\rm tot} = \frac{P^2}{2M} + \frac{\tilde{p}^{\,2}}{2m_{12}} + \frac{\tilde{q}_3^2}{2m_{(12)3}}, \tag{45}$$

where $m_{(ij)k} = m_k m_{ij}/M$.

New integration variables are chosen to be $\tilde{p} = f_{p3}\, p$ with $f_{p3} = \sqrt{2m_{12}}$ and $\tilde{q}_3 = f_{q3}\, q$ with $f_{q3} = \sqrt{2m_{(12)3}}$, and in general for cyclic (ijk), $f_{pi} = \sqrt{2m_{jk}}$ and $f_{qi} = \sqrt{2m_{(jk)i}}$. In terms of the new integration variables we have

$$K_{\rm tot} = \frac{P^2}{2M} + p^2 + q^2, \tag{46}$$

and the three-body Lippmann-Schwinger equation for the T-matrix becomes

$$\begin{aligned} T(\vec{p}, \vec{q}) &= V + {f_{p3}}^3 {f_{q3}}^3 \int \frac{d^3p'}{(2\pi)^3} \int \frac{d^3q'}{(2\pi)^3} V\, G_3(p', q') \\ &\quad \times T(\vec{p}^{\,\prime}, \vec{q}^{\,\prime}), \end{aligned} \tag{47}$$

with $G_3(p, q) = 1/(z - K_{\rm tot})$. The parameter z is implicit in the arguments of T and G_3 in Eq. (47), a convention to be followed hereafter.

Similarly we define the Jacobi momenta $\vec{p}_i, \vec{q}_i$ with particle i as the special choice. The momenta are related to each other as

$$\vec{p}_i = a_{ij}\vec{p}_j + b_{ij}\vec{q}_j, \quad \vec{q}_i = c_{ij}\vec{p}_j + d_{ij}\vec{q}_j, \tag{48}$$

where (ijk) are cyclic, and $a_{ij} = -[m_i m_j/(m_i + m_k)(m_j + m_k)]^{1/2}$, $b_{ij} = \sqrt{1 - a_{ij}^2} = -b_{ji}$, $c_{ij} = -b_{ij}$ and $d_{ij} = a_{ij}$.

It can be shown that the total angular momentum is related to the angular momentum $\vec{l}_{pi}$ and $\vec{l}_{qi}$ by

$$\vec{L} = \sum_{i=1}^{3} (\vec{r}_i \times \vec{k}_i) = \sum_{i=1}^{3} (\vec{l}_{pi} + \vec{l}_{qi}) + \vec{l}_c. \tag{49}$$

With these three choices of Jacobi momenta we may introduce corresponding three-particle states $|\,\rangle_n$ where particle n plays a special role. For the three-particle T-matrix we have

$$\langle \vec{k}_1, \vec{k}_2, \vec{k}_3 | T | \alpha \rangle =_n \langle \vec{p}_n, \vec{q}_n | T | \alpha \rangle, \tag{50}$$

or in terms of the Faddeev amplitudes T_n,

$$\langle \vec{k}_1, \vec{k}_2, \vec{k}_3 | T | \alpha \rangle = T_1(\vec{p}_1, \vec{q}_1) + T_2(\vec{p}_2, \vec{q}_2) + T_3(\vec{p}_3, \vec{q}_3), \tag{51}$$

with $T_n(\vec{p}_n, \vec{q}_n) =_n \langle \vec{p}_n, \vec{q}_n | T_n | \alpha \rangle$.

For the pentaquark system we now chose particles 1 and 3 as the diquark and particle 2 to be the $\bar{s}$. The Faddeev equations for $T = T_1 + T_2 + T_3$ with $T_i = t_i + \sum_{j\neq i} t_i G_2(s) T_j$ $(i = 1, 2, 3)$, with t_i denoting the two-body t-matrix between particle pair (jk), become

$$\begin{aligned} T_1(\vec{p}_1, \vec{q}_1) &= {f_{p3}}^3 {f_{q3}}^3 \int \frac{d^3p_3'}{(2\pi)^3} \\ &\quad \times \int \frac{d^3q_3'}{(2\pi)^3} K_{13}\, G_3(p_3', q_3') T_3(\vec{p}_3^{\,\prime}, \vec{q}_3^{\,\prime}) \\ &\quad + {f_{p2}}^3 {f_{q2}}^3 \int \frac{d^3p_2'}{(2\pi)^3} \int \frac{d^3q_2'}{(2\pi)^3} K_{12} \\ &\quad \times G_3(p_2', q_2')\, T_2(\vec{p}_2^{\,\prime}, \vec{q}_2^{\,\prime}), \end{aligned} \tag{52}$$

where the channels 1 and 3 correspond to $D(\bar{s}D)$ states and channel 2 to the $\bar{s}(DD)$ states. Since diquarks obey Bose-Einstein statistics, we have $T_3(\vec{p}_3,\vec{q}_3)=T_1(-\vec{p}_3,\vec{q}_3)$ and $T_3(\vec{p}_3,\vec{q}_3)=T_1(-\vec{p}_1,\vec{q}_1)$. We note that the symmetry property which requires the amplitude T be antisymmetric with respect to interchange of the two diquarks is automatically satisfied by the angular momentum content $L=l_{q1}=l_{p2}=1, l_{p1}=l_{q2}=0$.

The $\bar{s}(DD)$ T-matrix T_2 satisfies

$$T_2(\vec{p}_2,\vec{q}_2)=2fp_1^3fq_1^3\int\frac{d^3p_1'}{(2\pi)^3}\int\frac{d^3q_1'}{(2\pi)^3}K_{21}\;G_3(p_1',q_1')\times T_1(\vec{p}_1',\vec{q}_1'). \tag{53}$$

The kernels K_{13} and K_{12} are expressed in terms of the $\bar{s}D$ t-matrix

$$K_{13}=K_{12}=t_{\bar{s}D}\big(\vec{p}_1,\vec{p}_1';z-q_1^2\big)\frac{(2\pi)^3}{{f_{q_1}}^3}\delta^{(3)}[\vec{q}_1-\vec{q}_1']. \tag{54}$$

Similarly the kernel K_{21} is given by

$$K_{21}=t_{DD}\big(\vec{p}_2,\vec{p}_2';z-q_2^2\big)\frac{(2\pi)^3}{{f_{q_2}}^3}\delta^{(3)}[\vec{q}_2-\vec{q}_2']. \tag{55}$$

The term with K_{13} can be worked out by making use of the δ-function relation

$$\delta^{(3)}[\vec{q}_1-\vec{q}_1']=\frac{2}{q_1}\delta\big(q_1^2-{q_1'}^2\big)\delta\big(\cos\theta_{q_3}-\cos\theta_{q_3'}\big)\times\delta\big(\phi_{q_3'}-\phi_{q_3}\big), \tag{56}$$

and the linear relation $\vec{q}_1'=c_{13}\vec{p}_3'+d_{13}\vec{q}_3'$, which lead to

$$\delta^{(3)}\left[\vec{q}_1-\vec{q}_1'\right]=\frac{1}{q_1c_{13}d_{13}p_3'q_3'}\delta\left(\cos\theta_{p_3'q_3'}-\frac{q_1'^2-c_{13}'^2p_3'^2-d_{13}^2q_3'^2}{2c_{13}d_{13}p_3'q_3'}\right)\times\delta\big(\cos\theta_{q_3}-\cos\theta_{q_3'}\big)\delta\big(\phi_{q_3'}-\phi_{q_3}\big). \tag{57}$$

We mention that similar expression for a delta function in the term K_{12} can also be obtained by replacing $3\to 2$.

Performing a partial wave expansion for the $D(\bar{s}D)$ amplitude

$$T_1(\vec{p}_1,\vec{q}_1)=4\pi Y^*_{l_{p_1}0}(\Omega_{p_1})Y_{l_{q_1}0}(\Omega_{q_1})T_1^L(p_1,q_1), \tag{58}$$

and for the $\bar{s}D$ t-matrix $t_{\bar{s}D}(\vec{p}_1,\vec{p}_1';z-q_1^2)$,

$$t_{\bar{s}D}\big(\vec{p}_1,\vec{p}_1';z-q_1^2\big)=4\pi Y^*_{l_{p_1}0}\big(\Omega_{p_1}\big)Y_{l_{p_1}0}\big(\Omega_{p_1'}\big)t_{\bar{s}D}^{(l_{p_1})}\big(p_1,p_1';z-q_1^2\big), \tag{59}$$

yield

$$\begin{aligned}T_1^L(p_1,q_1)&=c_3\int_0^\infty{q_3'}^2dq_3'\int_{A_{13}}^{B_{13}}{p_3'}^2dp_3'\,t_{\bar{s}D}^{(l_{p_1})}\big(p_1,p_1';z-q_1^2\big)X_{13}\\&\times\frac{1}{c_{13}\,d_{13}\,q_1\,p_3'\,q_3'}\;G_3(p_3',q_3')\,T_3^L(p_3',q_3')\\&+c_2\int_0^\infty{q_2'}^2dq_2'\int_{A_{12}}^{B_{12}}{p_2'}^2dp_2'\,t_{\bar{s}D}^{(l_{p_1})}\big(p_1,p_1';z-q_1^2\big)\\&\times X_{12}\frac{1}{c_{12}\,d_{12}\,q_1\,p_2'\,q_2'}\;G_3(p_2',q_2')\,T_2^L(p_2',q_2'),\end{aligned} \tag{60}$$

with

$$c_3=\frac{2}{\sqrt{\pi}}(f_{p3}f_{q3}/f_{q1})^3,\quad c_2=\frac{2}{\sqrt{\pi}}(f_{p2}f_{q2}/f_{q1})^3, \tag{61}$$

and where the boundaries A, B for the p' integration can easily be found from the condition $q_1^2={q_1'}^2$ in Eq. (57), given by

$$A_{ij}=\left|\frac{c_{ij}q_j'+q_i}{d_{ij}}\right|, \tag{62}$$

$$B_{ij}=\left|\frac{c_{ij}q_j'-q_i}{d_{ij}}\right|. \tag{63}$$

For the $\bar{s}(DD)$ amplitude T_2, partial wave expansion gives

$$\begin{aligned}T_2^L(p_2,q_2)&=2c_1\int_0^\infty{q_1'}^2dq_1'\int_{A_{21}}^{B_{21}}{p_1'}^2dp_1'\\&\times t_{DD}^{(l_{p_2})}\big(p_2,p_2';z-q_2^2\big)\,X_{21}\;\frac{1}{c_{21}\,d_{21}\,q_2\,p_1'\,q_1'}\\&\times G_3(p_1',q_1')\;T_1^L(p_1',q_1'),\end{aligned} \tag{64}$$

where A_{21} and B_{21} are given by Eq. (63), and

$$c_1=\frac{2}{\sqrt{\pi}}(f_{p1}f_{q1}/f_{q2})^3. \tag{65}$$

In the above equations X_{ij} are angular momentum functions depending on the states we consider. In our case, the $\bar{s}D$ two-body channel is an s-wave, $lp=0$, and the DD channel a p-wave, $lp=1$. Hence, for the three-body channel with total angular momentum $L=1$ we have for the $D(\bar{s}D)$ three-body channel $lp_1=0, lq_1=L$ and $lp_3=0, lq_3=L$, while for $\bar{s}(DD)$ $lp_2=1, lq_2=0$. The obtained X_{ij} have the form

$$\begin{aligned}X_{13}&=\frac{1}{4\pi\sqrt{3}}Y_{lq_30}(\theta_{q_3q_1}),\quad X_{12}=\frac{1}{4\pi\sqrt{3}}Y_{lq_20}(\theta_{q_2q_1}),\\X_{21}&=\frac{1}{4\pi\sqrt{3}}Y_{lp_20}(\theta_{p_2p_1}).\end{aligned} \tag{66}$$

B. Relativistic Faddeev equations

Following Amazadeh and Tjon [42] (see also [33]) we adopt the relativistic quasipotential prescription based on a dispersion relation in the two-particle subsystem. Then the three-body Bethe-Salpeter-Faddeev equations have essentially the same form as the nonrelativistic version. Taking the representation with particle 3 as a special choice we may write down for the three-particle Green function a dispersion relation of the (1,2)-system, i.e.,

$$G_3(p_3,q_3;s_3)=\frac{E_1(k_1)+E_2(k_2)}{E_1(k_1)\,E_2(k_2)}\;\frac{1}{s_3-q_3^2-(E_1(k_1)+E_2(k_2))^2}, \tag{67}$$

with $E_1(k_1)=\sqrt{k_1^2+m_1^2}$, $E_2(k_2)=\sqrt{k_2^2+m_2^2}$, and $s_3=P^2$ being the invariant three-particle energy square. In the three-particle c.m. system we have $\sqrt{s_3}=M+E_b$. The resulting two-body Green function with invariant two-body energy square s_2 has then the form of the BSLT quasipotential Green function

$$G_2(p_3; s_2) = \frac{E_1(k_1) + E_2(k_2)}{E_1(k_1)\, E_2(k_2)} \frac{1}{s_2 - (E_1(k_1) + E_2(k_2))^2}. \tag{68}$$

This quasipotential prescription for G_3 has obviously the advantage that the two-body t-matrix in the Faddeev kernel satisfies the same equation as the one in the two-particle Hilbert space with only a shift in the invariant two-body energy. So the structure of the resulting three-body equations are the same as in the nonrelativistic case.

V. RESULTS AND DISCUSSIONS

In the NJL model some cut-off schemes must be adopted since the NJL model is nonrenormalizable. However, in this work we will not use any cut-off scheme but simply employ the dipole form factors for the scalar and vector vertices. Namely, the NJL model is only used to study the Dirac, flavor, and color structure of the $\bar{s}D$ and DD potentials.

For the values of the masses $M_{u,d}$, M_s, and M_D, we use the empirical values $M = M_u = M_d = 400$ MeV and $M_s = M_D = 600$ MeV [32]. We will treat the coupling constants G_i ($i = 1 \sim 5$) in Eq. (2) as free parameters. For the $\bar{s}D$ channel, it depends only on $G_v = G_3 + G_4 = \frac{3}{2}(G_5 - G_2)$ as seen in Eq. (16).

In the NJL model calculation with the Pauli-Villars (PV) cut-off regularization [32], the coupling constants G_π, G_ρ, and G_ω are related with the parameters used in our work by $G_1 = G_\pi/2$, $G_2 = G_\rho/2$, and $G_5 = G_\omega/2$. Thus by using the values of mesonic coupling constants in the NJL model, G_v is determined as $G_v = \frac{3}{2}(G_\omega/2 - G_\rho/2) = \frac{3}{2}(7.34/2 - 8.38/2) = -0.78$ GeV^{-2}. We remark that the sign of G_v is definitely negative since experimentally the omega meson is heavier than the rho meson. Then the interaction between the $\bar{s}$ and diquark in s-wave is attractive, as can be seen from the $\bar{s}D$ s-wave phase shift shown in Fig. 5 with $G_v = -0.78$ GeV^{-2}, while the interaction between s and diquark is repulsive which can be seen in Fig. 6. In both figures we find that the magnitudes of the phase shift is within 10 degrees, that is, $G_v = -0.78$ GeV^{-2} gives a very weak interaction between $\bar{s}$ (s) and the diquark. As we can see in Figs. 5 and 6, generally the phase shift in the s-wave is more sensitive to the three-momentum than that in the p-wave. We note that the $\bar{s}D$ and sD phase shift are not symmetric around the p_E axis, which can be understood from the decompositions of t_{sD} and $t_{\bar{s}D}$ in the spinor space in Appendix B. We further mention that if G_v is determined from the Λ hyperon mass $M_\Lambda = 1116$ MeV within the sD picture, one obtains $G_v = 6.44$ GeV^{-2}, which is different from $G_v = -0.78$ GeV^{-2} determined from the meson sector in the NJL model in sign. In this case the rho meson mass is larger than the omega meson mass, that is, the vector meson masses are not correctly reproduced.

The DD phase shift is plotted in Fig. 7 where we have used the values of coupling constants $G_1 = G_\pi/2 = 5.21$ GeV^{-2} and $G_5 = G_\omega/2 = 3.67$ GeV^{-2} which are determined from meson sectors in the NJL model calculation with the Pauli-Villars cutoff [32]. We can easily see that the phase shift δ_l is definitely negative, i.e., the DD interaction is repulsive, and its dependence on three-momentum p_E is very strong and

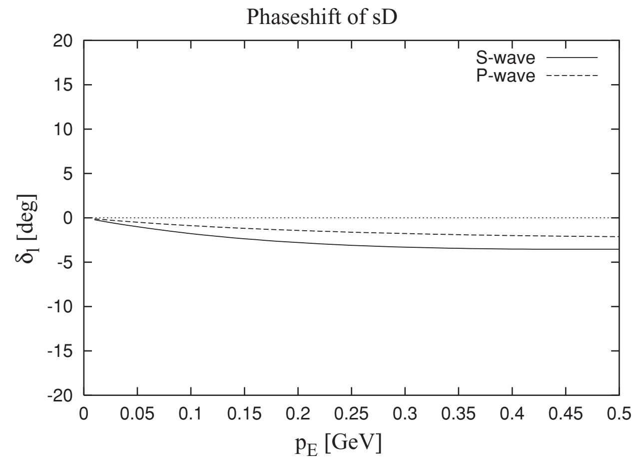

FIG. 6. Three momentum p_E dependence of the phase shift δ_l for the sD interaction with the coupling constant $G_v = -0.78$ GeV^{-2}.

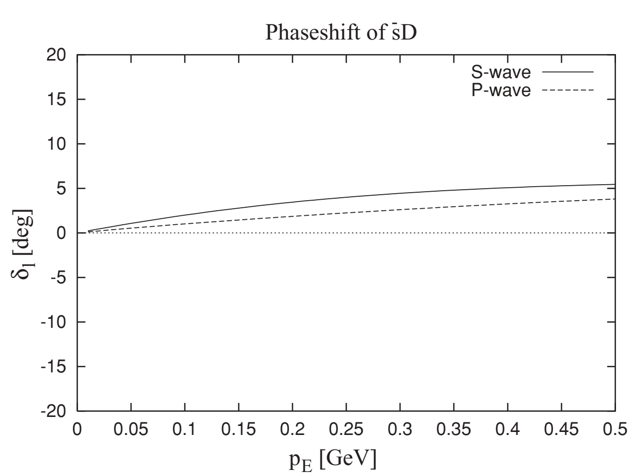

FIG. 5. Three-momentum p_E dependence of the phase shift δ_l for the $\bar{s}D$ interaction with the coupling constant $G_v = -0.78$ GeV^{-2}.

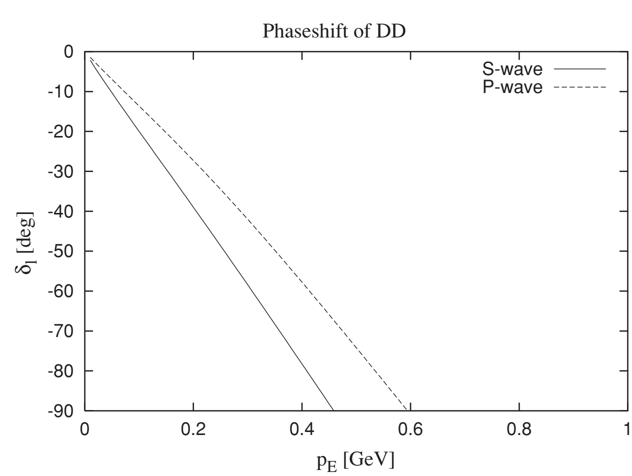

FIG. 7. Three momentum p_E dependence of the phase shift δ_l for the DD interaction.

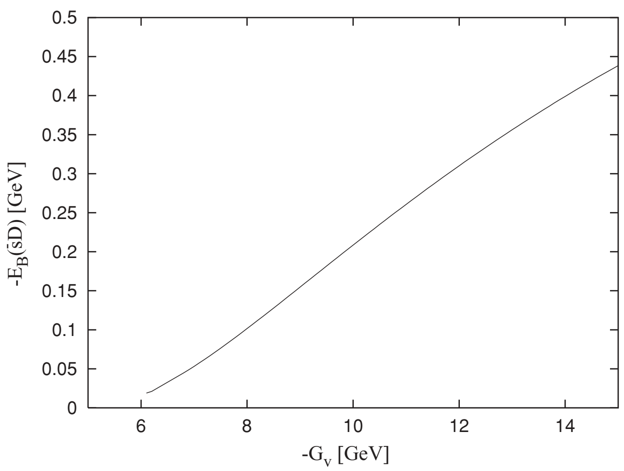

FIG. 8. G_v dependence of the $\bar{s}D$ binding energy.

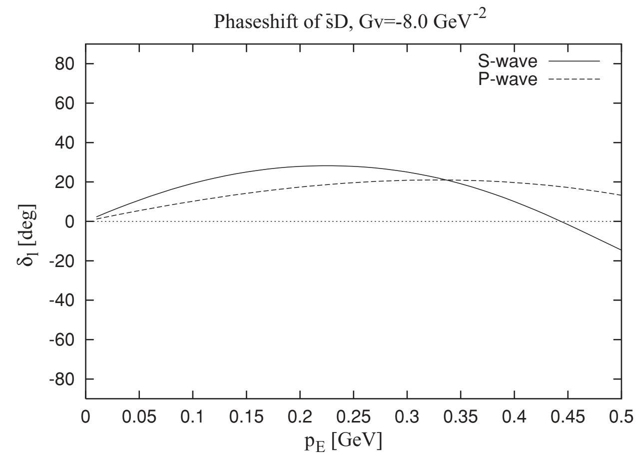

FIG. 9. Three-momentum p_E dependence of the phase shift δ_l for the $\bar{s}D$ interaction with the coupling constant $G_v = -8.0\ \text{GeV}^{-2}$.

almost proportional to p_E both for the s-wave and p-wave. This strong p_E dependence of the phase shift comes from the p_E^2 dependence of a second term $(p_{D1i} + p_{D1f}) \cdot (p_{D2i} + p_{D2f})$ in Eq. (39).

The G_v dependence of the $\bar{s}D$ binding energy, $E_{\bar{s}D}$, is presented in Fig. 8. We find that the $\bar{s}D$ bound state begins to appear around $G_v = -5 \sim -6\ \text{GeV}^{-2}$, and becomes more deeply bound as G_v becomes more negative. It is easily seen that $E_{\bar{s}D}$ is almost proportional to G_v. However even for the case of a weakly bound state with $|E_{\bar{s}D}|$ less than 0.1 GeV, it will require a value of $-G_v = 5 \sim 6\ \text{GeV}^{-2}$ which is about eight times larger than the $-G_v$ determined from the meson sector in the original NJL model with the PV cut-off regularization.

For the calculation of the pentaquark binding energy we use the relativistic three-body Faddeev equation which is introduced in Sec. IV. If the pentaquark state is in the $J^P = \frac{1}{2}^+$ state with which we are concerned in the present paper, the total force is attractive but there is no pentaquark bound state.

On the other hand if the pentaquark state is in the $J^P = \frac{1}{2}^-$ state, a bound pentaquark state begins to appear when G_v becomes more negative than $-8.0\ \text{GeV}^{-2}$, a value inconsistent with what is required to predict a bound Λ hyperon with $M_\Lambda = 1116$ MeV in a quark-diquark model as mentioned in Sec. V. The lowest configuration which would correspond to a $J^P = \frac{1}{2}^-$ state is for the spectator $\bar{s}$ to be in p-wave with respect to a DD pair in the p-wave, or alternatively speaking, the spectator diquark in the relative s-wave to $\bar{s}D$ in the s-wave. Our results for the binding energy of a $J^P = \frac{1}{2}^-$ pentaquark state for the case with and without the DD channel are given in Table I. It is found that although the DD interaction is repulsive, including the DD channel gives an additional binding energy which is leading to the more deeply pentaquark bound state. It is because the coupling to the DD channel is attractive due to the sign of the effective kernel K_{21} in Eqs. (53) and (55). This depends on the recoupling coefficients X_{21}, X_{12} in Eq. (66) and the two-body t-matrices.

TABLE I. The binding energy of the $J^P = \frac{1}{2}^-$ pentaquark state. $E_B^0(5q)$ ($E_B(5q)$) is the binding energy without (including) the DD channel.

G_v [GeV^{-2}]	$E_B^0(5q)$ [MeV]	$E_B(5q)$ [MeV]
−8.0	47	77
−9.0	87	139
−10.0	132	205
−12.0	226	333
−14.0	316	505

In Fig. 9 (Fig. 10) the phase shift of $\bar{s}D$ is plotted, where the coupling constant is fixed at $G_v = -8.0\ \text{GeV}^{-2}$ ($G_v = -14.0\ \text{GeV}^{-2}$). It is easily seen that in Figs. 9 and 10 the phase shift of $\bar{s}D$ in the s-wave is positive for small $p_E < 0.3$ GeV and $p_E < 0.45$ GeV, but it changes the sign around $p_E = 0.3$ and $p_E = 0.45$ GeV, thus the phase shift of $\bar{s}D$ in the s-wave is very sensitive to three-momentum p_E. Whereas the phase shift of $\bar{s}D$ in the p-wave is definitely positive.

In Fig. 11 we plot the phase shift of sD with the coupling constant $G_v = -14.0\ \text{GeV}^{-2}$ which is the same as the one used in Fig. 10. Different from the phase shift of $\bar{s}D$, the phase shifts of sD in the s- and p-waves do not change the sign for

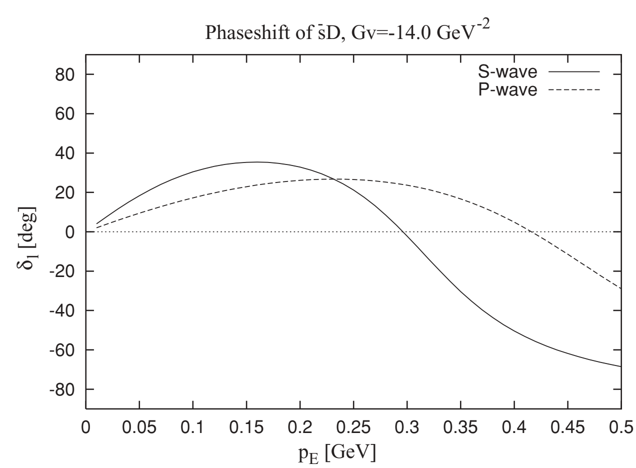

FIG. 10. Three-momentum p_E dependence of the phase shift δ_l for the $\bar{s}D$ interaction with the coupling constant $G_v = -14.0\ \text{GeV}^{-2}$.

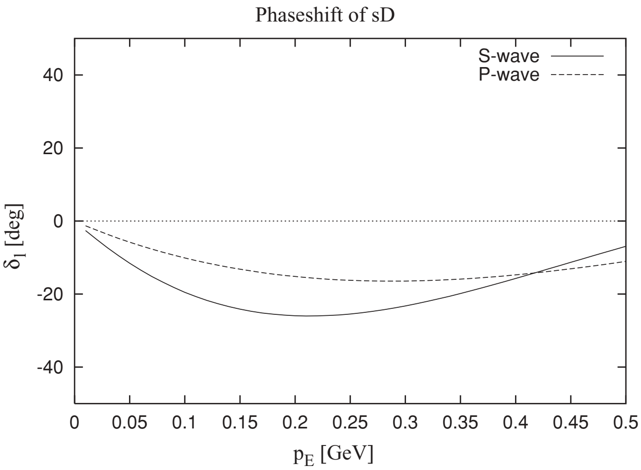

FIG. 11. Three-momentum p dependence of the phase shift δ_l for the sD interaction with the coupling constant $G_v = -14.0\ \mathrm{GeV}^{-2}$.

higher three-momentum p_E, i.e., the sign of the phase shifts are definitely negative.

From the above results we find that even if we use a very strong coupling constant G_v which is unphysical because it gives much larger mass difference of rho and omega mesons than the experimental value, $M_\omega - M_\rho = 13$ MeV, it is impossible to obtain the pentaquark bound state with $J^P = \frac{1}{2}^+$. With only the $J = \frac{1}{2}$ three-body channels considered, we do not find a bound $J^P = \frac{1}{2}^+$ pentaquark state. The $J^P = \frac{1}{2}^-$ channel is more attractive, resulting in a bound pentaquark state in this channel, but for unphysically large values of vector mesonic coupling constants.

VI. SUMMARY

In this work, we have presented a Bethe-Salpeter-Faddeev (BSF) calculation for the pentaquark Θ^+ in the diquark picture of Jaffe and Wilczek in which Θ^+ is treated as a diquark-diquark-$\bar{s}$ three-body system. The Blankenbecler-Sugar reduction scheme is used to reduce the four-dimensional integral equation into three-dimensional ones. The two-body diquark-diquark and diquark-$\bar{s}$ interactions are obtained from the lowest order diagrams prescribed by the Nambu–Jona-Lasinio (NJL) model. The coupling constants in the NJL model as determined from the meson sector are used. We find that the $\bar{s}D$ interaction is attractive in the s-wave while the DD interaction is repulsive in the p-wave. Within the truncated configuration where DD and $\bar{s}D$ are restricted to p- and s-waves, respectively, we do not find any bound $\frac{1}{2}^+$ pentaquark state, even if we turn off the repulsive DD interaction. It indicates that the attractive $\bar{s}D$ interaction is not strong enough to support a bound $DD\bar{s}$ system with $J^P = \frac{1}{2}^+$.

However, a bound pentaquark with $J^P = \frac{1}{2}^-$ begins to appear if we change the vector mesonic coupling constant G_v from $-0.78\ \mathrm{GeV}^{-2}$, as determined from the mesonic sector, to around $G_v = -8\ \mathrm{GeV}^{-2}$. And it becomes more deeply bound as G_v becomes more negative.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Council of ROC under grant no. NSC93-2112-M002-004 (H.M. and S.N.Y.). J.A.T. wishes to acknowledge the financial support of NSC for a visiting chair professorship at the Physics Department of NTU and the warm hospitality he received throughout the visit. K.T. acknowledges the support from the Spanish Ministry of Education and Science, Reference no. SAB2005-0059.

APPENDIX A: PARTIAL WAVE EXPANSION

In the two-body center of mass frame the partial wave expansion is defined by

$$t(\vec{p}_f, \vec{p}_i) = \sum_l \frac{2l+1}{4\pi} P_l(\cos\theta_{p_i p_f})\langle p_f l|t|p_i l\rangle$$
$$\equiv \sum_l (2l+1) P_l(\cos\theta_{p_i p_f}) t^l(|\vec{p}_f|, |\vec{p}_i|), \quad \text{(A1)}$$

with $\vec{p}_{i(f)} \equiv \vec{p}_{1i(f)} = -\vec{p}_{2i(f)}$. Then $t^l(|\vec{p}_f|, |\vec{p}_i|)$ in Eq. (A1) is written in terms of $t(\vec{p}_f, \vec{p}_i)$ by

$$t^l(|\vec{p}_f|, |\vec{p}_i|) = \frac{1}{2}\int_{-1}^{1} d\cos\theta_{p_i p_f} P_l(\cos\theta_{p_i p_f}) t(\vec{p}_f, \vec{p}_i). \quad \text{(A2)}$$

The phase shift δ_l is given by

$$t^l(p, p) = -\frac{8\pi\sqrt{s_2}}{p} e^{i\delta_l}\sin\delta_l, \quad \text{(A3)}$$

where $p \equiv |\vec{p}_{1i}| = |\vec{p}_{2i}| = |\vec{p}_{1f}| = |\vec{p}_{2f}|$ and $s_2 = (p_{1i} + p_{2i})^2 = (p_{1f} + p_{2f})^2$.

APPENDIX B: THE RESULTS FOR $\tilde{V}_{\bar{s}(s)D,nm}$ AND $\tilde{K}_{\bar{s}(s)D,nm}$ ($n, m = 1, 2$)

In this appendix we show the results for $\tilde{V}_{\bar{s}(s)D,nm}$ and $\tilde{K}_{\bar{s}(s)D,nm}$ ($n, m = 1, 2$) defined in Eqs. (28)–(31):

$$\tilde{V}_{\bar{s}D,11}(p_{Di}, p_{Df}, x) = \frac{p^0_{Di} + p^0_{Df}}{2},$$
$$\tilde{V}_{\bar{s}D,12}(p_{Di}, p_{Df}, x) = -\frac{p_{Df} + x p_{Di}}{2} = -\frac{p_{\bar{s}f} + x p_{\bar{s}i}}{2},$$
$$\tilde{V}_{\bar{s}D,21}(p_{Di}, p_{Df}, x) = \frac{p_{Di} + x p_{Df}}{2} = \frac{p_{\bar{s}i} + x p_{\bar{s}f}}{2},$$
$$\tilde{V}_{\bar{s}D,22}(p_{Di}, p_{Df}, x) = \frac{x}{2}(p^0_{Di} + p^0_{Df}),$$

and

$$\tilde{K}_{\bar{s}D,11}(p_{Di}, p'_D, x_i)$$
$$= \tfrac{1}{2}\big[(p^0_{Di} + p'^0_D)M_s + (\sqrt{s_2} - p'^0_D)(p'^0_D + p^0_{Di})$$
$$+ p'^2_D + x_i p'_D p_{Di}\big],$$
$$\tilde{K}_{\bar{s}D,12}(p_{Di}, p'_D, x_i)$$
$$= -\tfrac{1}{2}\big[(p'_D + x_i p_{Di})(M_s - \sqrt{s_2} + p'^0_D) - p'_D(p^0_{Di}$$
$$+ p'^0_D)\big],$$

$$\tilde{K}_{\bar{s}D,21}(p_{Di}, p'_D, x_i) = \frac{1}{2}\left[(p_{Di} + x_i p'_D)\left(M_s + \sqrt{s_2} - p'^0_D\right) + x_i p'_D\left(p^0_{Di} + p'^0_D\right)\right],$$

$$\tilde{K}_{\bar{s}D,22}(p_{Di}, p'_D, x_i) = -\frac{1}{2}\left[x_i M_s\left(p^0_{Di} + p'^0_D\right) - \left(p_{Di} p'_D + x_i p'^2_D\right) + x_i\left(p'^0_D - \sqrt{s_2}\right)\left(p^0_{Di} + p'^0_D\right)\right],$$

where $x \equiv \hat{p}_{Di} \cdot \hat{p}_{Df}$, $x_i \equiv \hat{p}_{Di} \cdot \hat{p}'_D$.

$\tilde{V}_{\bar{s}D,nm}$ and $\tilde{K}_{\bar{s}D,nm}$ are related with $\tilde{V}_{sD,nm}$ and $\tilde{K}_{sD,nm}$ by

$$\tilde{V}_{\bar{s}D,nm}(p, q, x_{pq}) = -\tilde{V}_{sD,nm}(p, q, x_{pq}),$$
$$\tilde{K}_{\bar{s}D,nm}(p, q, x_{pq}) = -\tilde{K}_{sD,nm}(p, q, x_{pq}).$$

APPENDIX C: PARAMETRIZATIONS FOR $t_{\bar{s}D}$ AND t_{sD}

$t_{\bar{s}D}$ can be parametrized as

$$t_{\bar{s}D}(p_{Di}, p_{Df}) = \sum_{\rho,\rho'=\pm} \Lambda_\rho\left[F_S^{\rho\rho'} + F_T^{\rho\rho'} i\sigma_{\mu\nu} p^\mu_{Df} p^\nu_{Di}\right]\Lambda_{\rho'}, \tag{C1}$$

where $\Lambda_\pm = \frac{1\pm\gamma_0}{2}$. Components of $t_{\bar{s}D}$ are written as

$$t_{\bar{s}D}(p_{Di}, p_{Df}) = \begin{pmatrix} F_S^{++} + F_T^{++} i\vec{\sigma}\cdot\vec{n} & F_T^{+-}\vec{\sigma}\cdot\vec{v} \\ F_T^{-+}\vec{\sigma}\cdot\vec{v} & F_S^{--} + F_T^{--} i\vec{\sigma}\cdot\vec{n} \end{pmatrix}, \tag{C2}$$

where $\vec{n} = \vec{p}_{Df} \times \vec{p}_{Di}$, $\vec{v} = p^0_{Df}\vec{p}_{Di} - p^0_{Di}\vec{p}_{Df}$, and $\pm$ means upper and lower components in the spinor space, i.e., $(t_{\bar{s}D})_{\rho,\rho'} = \Lambda_\rho t_{\bar{s}D}\Lambda_{\rho'}$.

The decomposition into upper and lower components in Eq. (30) for $t_{\bar{s}D}$ gives

$$t_{\bar{s}D,11}(p_{Di}, p_{Df}) = -F_S^{--},$$
$$t_{\bar{s}D,12}(p_{Di}, p_{Df}) = -F_T^{-+}\left(x p^0_{Df} p_{Di} - p^0_{Di} p_{Df}\right),$$
$$t_{\bar{s}D,21}(p_{Di}, p_{Df}) = -F_T^{+-}\left(p^0_{Df} p_{Di} - x p^0_{Di} p_{Df}\right),$$
$$t_{\bar{s}D,22}(p_{Di}, p_{Df}) = -F_T^{++} p_{Di} p_{Df}(x^2 - 1).$$

We can parametrize t_{sD} in the same way [equal to Eq. (C1)]:

$$t_{sD}(p_{Df}, p_{Di}) = \sum_{\rho,\rho'=\pm} \Lambda_\rho\left[F_S^{\rho\rho'} + F_T^{\rho\rho'} i\sigma_{\mu\nu} p^\mu_{Df} p^\nu_{Di}\right]\Lambda_{\rho'}, \tag{C3}$$

where $\Lambda_\pm = \frac{1\pm\gamma_0}{2}$.

Similar to $t_{\bar{s}D}$, the decomposition into upper and lower components by Eq. (28) gives

$$t_{sD,11}(p_{Df}, p_{Di}) = F_S^{++},$$
$$t_{sD,12}(p_{Df}, p_{Di}) = F_T^{+-}\left(p^0_{Df} p_{Di} - x p^0_{Di} p_{Df}\right),$$
$$t_{sD,21}(p_{Df}, p_{Di}) = F_T^{-+}\left(x p^0_{Df} p_{Di} - p^0_{Di} p_{Df}\right),$$
$$t_{sD,22}(p_{Df}, p_{Di}) = F_T^{--} p_{Di} p_{Df}(x^2 - 1).$$

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