

Impact of Dynamical Fermions on the Vacuum of Quantum Chromodynamics

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Abstract

The structure of the Quantum Chromodynamics (QCD) vacuum is studied using the formalism of Lattice QCD. To this end, a new smearing algorithm is designed to reveal the long-distance non-perturbative structure of the vacuum, over-improved stout-link smearing. After showing how to quantitatively select the value of the over-improvement parameter ϵ , we demonstrate the topological stability provided by the new algorithm.

A comparison is made between the topological charge density from over-improved stout-link smearing and the topological density from the massless overlap Dirac operator. A good correspondence is observed. We also demonstrate an excellent correlation between the location of Dirac zeromodes and the gluonic topological charge density from smearing. The dependence of the overlap operator on the input negative Wilson-mass regulator parameter is also analysed through calculations of the topological charge density.

Following this, we examine the structure of the QCD vacuum at different energy scales. At short distances an oscillating sheet-like structure is seen, whilst at longer scales the more established instanton dominated vacuum appears. The connection between these two pictures is discussed.

Of particular interest is how dynamical sea quarks affect the vacuum structure. We show how these extra degrees of freedom lead to “rougher” gauge fields containing a greater number of non-trivial topological structures, and explain the physics behind this observation.

The use of smearing as a preconditioner for Maximal Centre Gauge fixing is also examined through an analysis of the centre vortex content of the vacuum. We conclude our analysis of QCD vacuum structure by considering the effects of smearing on the overlap quark propagator. At heavy quark masses, the mass and wave renormalisation functions of the quark propagator are greatly altered, whilst for lighter quark masses the effects are not as strong. Of particular note is the suppression of localised topological structures, giving rise to dynamical chiral symmetry breaking.

Lastly, the non-perturbative effects of dynamical sea-quarks are examined through calculations of the proton and Δ^+ charge radii and magnetic moments using a non-perturbative clover-improved fermion action. The results are compared with earlier quenched calculations. The chiral curvature of the μ_{Δ^+}/μ_p ratio at light quark masses, observed in the previous calculations, is suppressed in our dynamical calculation.

Statement of Originality

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