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Quantitative Methods for Investment Decisions in Communication Networks

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Abstract

Information and communication technology (ICT) is a multi-billion dollar industry [11]. It is therefore paramount to employ the most up to date decision making strategies when making ICT investments. This thesis uses a framework, originally developed for perpetual American call options, to study pertinent issues in this industry.

The models in existing literature often assume that investment costs are fixed, but in the ICT industry we expect the cost to decrease exponentially according to Moore's and Gilders' laws. The models are therefore extended to support decreasing costs. The investment values and stopping times are determined for various decay parameters. For large decay parameters, we find that the investment values are close in geometric Brownian motion (GBM) and multiplicative jump-diffusion process (JDP) models. Typical error scenarios are explored and the models are found to be fairly robust.

Once a network link has been built, its capacity can be increased by upgrading hardware in the associated switches. We initially develop a general strategy for deciding when to make this investment and find an analytical solution for a GBM demand process. A logistic process is then used to model demand with saturation and Kummer's equation is used to find an analytical solution for the increasing capacity model. In the GBM model, there is a unique optimal trigger which is greater than the link's initial capacity. Compared to the GBM model, investments are made later in the demand saturation model and yield lower investment values. Furthermore, in some extreme cases, we find that the optimal trigger does not exist.