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A STOCHASTIC BUCKLEY-LEVERETT MODEL

Simon. J. Carter

Supervisor: Dr John van der Hoek

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To Sunrui

Abstract

Even while numerical simulation methods dominate reservoir modeling, the Buckley-Leverett equation provides important insight into the physical processes behind enhanced oil recovery. The interest in a stochastic Buckley-Leverett equation, the subject of this thesis, arises because uncertainty is at the heart of petroleum engineering. Stochastic differential equations, where one modifies a deterministic equation with a stochastic perturbation or where there are stochastic initial conditions, offers one possible way of accounting for this uncertainty. The benefit of examining a stochastic differential equation is that mathematically rigorous results can be obtained concerning the behavior of the solution.

However, the Buckley-Leverett equation belongs to a class of partial differential equations called first order conservation equations. These equations are notoriously difficult to solve because they are non-linear and the solutions frequently involve discontinuities. The fact that the equation is being considered within a stochastic setting adds a further level of complexity. A problem that is already particularly difficult to solve is made even more difficult by introducing a non-deterministic term.

The results of this thesis were obtained by making the fractional flow curve the focus of attention, rather than the relative permeability curves. Reservoir conditions enter the Buckley-Leverett model through the fractional flow function. In order to derive closed form solutions, an analytical expression

for fractional flow is required. In this thesis, emphasis in placed on modeling fractional flow in such a way that most experimental curves can readily be approximated in a straightforward manner, while keeping the problem tractable. Taking this approach, a range of distributional results are obtained concerning the shock front saturation and position over time, breakthrough time, and even recovery efficiency.

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Statement of Originality

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. The work can be divided broadly into two parts. The first, comprising Chapter One to Chapter Four, is largely background information in which the work of others is explained and accounted for. Chapter Five onwards, which details the stochastic approach, is for the most part original material, with the exception of the derivation of the equations for recovery in Chapter Seven and Annex B (which is appropriately attributed to Welge [169]).

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Preface

One of the most distinguished contributers to reservoir engineering, Laurie Dake [44], has made a call: *for the revival of fractional flow of water which seems to have 'gone missing' since the advent of simulation and, it is argued, is the key to understanding any form of displacement process.*[1] This thesis firmly belongs to the "Dakian" school of thought. From the first line until the last page, the emphasis will be on closed form, analytical solutions. The reader will appreciate that solutions of this kind are significantly more difficult to obtain than numerical ones. This approach is taken in the belief that increased insight is thereby gained into the physical processes behind the model.

[1] Dake [43] page 311.

Contents

Nomenclature

S saturation (without subscripts it refers to the displacing fluid), fraction of pore volume
S_L displacing saturation at end of core/reservoir
S_c connate displacing fluid saturation, fraction of pore volume
$\bar{S}$ average saturation, fraction of pore volume
S^* shock front saturation, fraction of pore volume
k permeability, darcy
k_r relative permeability, fraction
f fractional flow (without subscripts it refers to the displacing fluid), fraction
ϕ porosity, fraction
μ viscosity, cp
ρ density, kg/m^3
p pressure, Pa
P oil production, pore volumes
q_{total} total flow rate (oil + water), m^3/day
q_i water injection rate, m^3/day
Q_i cumulative injected water volume, m^3/day
Q_{dim} dimensionless cumulative injected water, in pore volumes

Subscriptes may be added to specify a number of quantities.

1, 2 fluid 1 or 2
d, nd displacing or non-displacing fluid
w, nw wetting or non-wetting fluid
w, o, g water, oil or gas (it is clear by context whether water or wetting fluid is intended)

c connate or critical (depending on the phase)
ir irreducible
dim dimensionless

Note on Subscripts and Connate Saturation

It would simplify matters greatly if the entire thesis could be written in terms of a single set of subscripts, but this is impractical. Generally, fluid 1 versus fluid 2 is used to present the flow equations, wetting fluid versus non-wetting is used when discussing relative permeability, and displacing fluid versus non-displacing is used for Buckley-Leverett. Other subscripts are used to be consistent with original sources, or when formulas only apply to specific phases. Throughout the examination of Buckley-Leverett, the initial saturation corresponds to the connate saturation.