

# Drought Predictions: Applications in Australia

Geraldine H. Wong

*Thesis submitted for the degree of*

*Doctor of Philosophy*

*in*

*Statistics*

*at*

*The University of Adelaide*

Discipline of Statistics, School of Mathematical Sciences,  
Faculty of Engineering, Computer and Mathematical Sciences



January 9, 2010



# Abstract

Drought is a global and recurrent natural phenomenon, the inevitable consequence of meteorological variability. This natural hazard brings about devastating effects because water is one of the most fundamental commodities for human survival, and a lack of water can result in varying consequences, from mere inconvenience to life threatening instances. Drought cannot be prevented but its effects can be mitigated through the design of appropriate water resource infrastructure and management strategies. The goal of this thesis is to model the spatial and temporal characteristics of drought occurrence in Australia, the driest continent. In doing so predictions can be made, levels of risk can be evaluated and conditional estimates of drought can be based on climatic state variables.

For insight into the nature of drought in Australia, multivariate models of drought characteristics are developed. Preliminary analysis demonstrates high correlations between several drought characteristics, these are the drought severity, intensity and duration. This thesis applies the copula concept, which is a versatile means of modelling their dependence structure. Copulas are multivariate uniform distributions, which allow the joint behaviour of variables to be modelled independently from their marginal distributions. This research extends the application of copulas by investigating the effect of climate variability on copula models and subsequent drought characteristics. Two different copula families are fitted to the drought characteristics to demonstrate the importance of tail dependence when modelling extreme climatic events. An important application of these models is the calculation of return periods of extreme drought events exceeding certain thresholds, taking account of variability in climatic indices.

A second objective is to forecast drought at various spatial resolutions. The most straightforward method are regression and ARMA models that incorporate global climatic indicators. The effect of climatic variation on Australia's precipitation is examined by investigating the association between climatic indices and the multivariate distribution of drought at numerous sites across Australia. Two classification strategies for forecasting rainfall are compared using significance testing based on multiple comparison techniques. Further to this, rainfall forecasting relationships are explored using global sea-surface temperature anomalies.

The versatility of copula models is demonstrated through short-term rainfall predictions for neighbouring rainfall districts, using separate copulas conditioned on antecedent climate conditions. This technique is shown to improve rainfall predictions in neighbouring districts and improve estimates of drought probability.



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# Publications Arising from this Thesis

G. Wong, A. Osti, M. F. Lambert and A. V. Metcalfe. Drought Forecasting using adaptive stochastic models in New South Wales. In *Proceedings of the 30th Hydrology and Water Resources Symposium*, Hobart, Tasmania, 2006.

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G. Wong, A. V. Metcalfe and M. F. Lambert. The Role of ENSO in the Characterization of Drought in Australia. In *Proceedings of HydroPredict, Czech Association of Hydrogeologists*, pages 281-284, Prague, Czech Republic, 2008.

G. Wong, M. Leonard, A. V. Metcalfe and M. F. Lambert. Drought analysis using trivariate copulas conditional on climatic states. *Journal of Hydrologic Engineering (Accepted for Publication)*, 2009.



# Declaration

I hereby declare this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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# Acknowledgments

*“Trust in the LORD with all your heart and lean not on your own understanding; in all your ways acknowledge Him, and He will make your paths straight.”*

Proverbs 3:5-6

This three-year journey has finally reached its ‘destination’ and I would like to give God all the glory and thanks, for His faithfulness and providence during this time.

The result of this dissertation would not have been possible without the guidance and support of the following people. Firstly, my heartfelt thanks to my supervisors, Associate Professor Andrew Metcalfe and Professor Martin Lambert for their advice, patience and support throughout this entire project and for teaching me to be confident in taking charge of the direction of this project. I am also grateful to my mentor, fellow colleague and friend, Michael Leonard for his generosity in imparting his knowledge and his patience.

This journey has been made more enjoyable with great friends that I’ve made over this 3 years. I would like to thank the ‘geeky’ postgraduate bunch for your company over coffees, suppers and games, the OCF group who keeps me in constant prayer, the ACJC SB6 group for taking time out to catch up over dinners, suppers and chill-out times whenever I’m home and to all the other awesome friends that I’ve met along the way.

Finally, I am grateful to Mum and Pa for their financial support, confidence and trust in me to do whatever I’m interested in pursuing. Thanks Pa for being my personal courier service and Mum for her constant phone calls to check on my well-being. Thanks also to my awesome and cool sister, Gwendo, for being a listening ear over MSN and my fashion consultant. And lastly to Steven, for all the intellectual discussions we have and just being you.



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# Glossary of Technical Terms and Abbreviations

## Climate terminology

ENSO	El-Niño Southern Oscillation
IOD	Indian Ocean Dipole
IPO	Inter-decadal Pacific Oscillation
MEI	Multivariate ENSO Index
PDO	Pacific Decadal Oscillation
SLP	Sea Level Pressure
SST	Sea-surface temperature
SOI	Southern Oscillation Index

## Drought terminology

PDHI	Palmer Drought Hydrological Index
PDSI	Palmer Drought Severity Index
SPI	Standardized Precipitation Index
$S_t$	SPI at time $t$

## Statistical terminology

AIC	Akaike's Information Criterion
AEP	Annual Exceedance Probability
ARMA	Auto-regressive Moving Average

<b>ARIMA</b>	Auto-regressive Integrated Moving Average
$R^2$	Coefficient of determination
$\varphi(t)$	Copula generator
$\theta$	Copula parameter
<b><i>cdf</i></b>	Cumulative Distribution Function
<b>IFM</b>	Inference Function of Margins
$\lambda_L$	Lower tail dependence
<b>MLE</b>	Maximum Likelihood Estimation
$C(u_1, \dots, u_n)$	$n$ -dimensional copula
<b><i>pdf</i></b>	Probability Density Function
$\lambda_U$	Upper tail dependence