



# Guiding Light in Low-Index Media via Multilayer Waveguides

by

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in the  
Faculty of Sciences  
School of Chemistry & Physics

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# Declaration of Authorship

I, Kristopher John Rowland, declare that this thesis titled ‘Guiding Light in Low-Index Media via Multilayer Waveguides’ and the work presented in it are my own.

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*This work is dedicated to those who find use in its content.*

*I would like to think that, after all, that is its purpose.*

*More sentimentally, and with self-indulgence, from its conception to completion,  
this work is dedicated to those who have supported and suffered me throughout:*

*family, friends, supervisors and colleagues;*

*but above all others, I dedicate this thesis, and the efforts it embodies,  
to the person who has most endured, second hand, the burden of its creation,  
selfless, supportive, and loving all the while.*

*For Christine.*



*... the laws of ... electric and magnetic action including radiation are the simplest of all laws when we confine them to a so-called vacuum, but become more and more complicated when we treat of them in space containing matter.*

Henry A. Rowland—An excerpt from the presidential address to the second annual meeting of the American Physical Society, October 28, 1899.





THE UNIVERSITY OF ADELAIDE

## *Abstract*

Faculty of Sciences  
School of Chemistry & Physics

Doctor of Philosophy

by Kristopher J. Rowland

This work relates to the theoretical and experimental analysis of light guidance within layered waveguides whose guidance region (the *core*) has a refractive index equal to or lower than the lowest of the surrounding material (the *cladding*).

The Thesis has two primary themes:

- The guidance behaviour of binary-layered-cladding waveguides with cores of arbitrary refractive index equal to or less than the lowest of the cladding;  
Chapters 2, 3 and 4.
- The fabrication, analysis and simulation of single-material hollow-core microstructured optical fibres made from soft-glass via an extrusion process;  
Chapters 2 and 5.

While each chapter discusses the theoretical and/or experimental analysis of distinct phenomena, their concepts are deeply related and highlighted as such throughout.



# Summary

This summary briefly covers the content of this thesis, chapter by chapter, and indicates the author's contribution to the work.

This work relates to the theoretical and experimental analysis of light guidance within layered waveguides whose guidance region (the 'core') has a refractive index equal to or lower than the lowest of the surrounding material (the 'cladding'), referred to here as *depressed-core* waveguides. The primary focus is toward multilayer hollow-core microstructured optical fibres made from soft-glass, but much of the analysis is shown to be more general, applicable to similar areas of waveguide optics, fibre or otherwise. While each chapter discusses the theory and/or experiment of distinct phenomena, their fundamental findings are deeply related and highlighted as such throughout.

*Author's contribution to the thesis in general:* All theoretical, numerical, fabrication and experimental results are my own work, except for the few cases indicated below, with guidance by my supervisors (who allowed me significant freedom to follow my own intuition, for which I am very grateful). All content within has been written and, upon regular reviews by my supervisors, edited by myself including the content, typesetting and figure creation. All figures have been created either from numerical and experimental data or, for diagrams, using a digital image program.

These same comments apply to all publications and conference presentations resulting from this work (detailed in the Publications section).

**Chapter 1:** Beginning with the basics of depressed-core step-index slab and cylindrical waveguide theory, the first chapter highlights the importance of the obstacles to coercing the guidance of light in low refractive indices and the guidance mechanisms and waveguides structures identified and demonstrated to date that achieve this goal. This review and discussion sets the conceptual groundwork for the remaining chapters. The 'spider-web' fibre, forming the basis of Chapter 5 is also introduced.

*Author's contribution:* Review of the literature, its interpretation and presentation as shown in the text (including the relationships between all of the discussed guidance phenomena), and the conceptualisation of the spider-web fibre and its potential behaviour.

**Chapter 2:** The second chapter presents a theoretical analysis of a *level-core* Bragg fibre (a multilayer-cladding fibre with a core refractive index equal to the lowest of the cladding layer indices). The existence of novel higher-order, low-loss, bandgaps in such waveguides is demonstrated numerically and explained. The exploitation of such gaps for confinement loss reduction is analysed.

*Author's contribution:* As per the general comment above, save for the implementation of the cTMM model which was based upon code written by Mr Michael Oermann.

**Chapter 3:** The third chapter discusses the relationship between bandgap and anti-resonance waveguidance phenomena in depressed-core 1-D layered-cladding waveguides. A new model, coined the *Stratified Planar Anti-Resonant Reflecting Optical Waveguide* (SPARROW) model, is presented. It is demonstrated how the analyticity of the model can be exploited to produce analytic expressions for various nontrivial bandgap properties and phenomena, while also giving an intuitive explanation of the physical mechanisms responsible for various guidance properties in such waveguides. Various theoretical tools are used in the resulting analysis.

*Author's contribution:* As per the general comment above.

**Chapter 4:** The fourth chapter details the experimental investigation into a phenomenon discussed in the previous chapter, namely the shifting of bandgaps as the core refractive index is varied. An experimental demonstration of this effect is presented where the transmission spectrum of a hollow-core Bragg fibre is examined after being systematically filled with liquids of various refractive indices. The effect is sufficiently strong enough such that, for refractive index changes of up to about 50%, the transmission spectrum is shifted across almost the entire visible spectrum.

*Author's contribution:* As per the general comment above, save for the supply of the Bragg fibre and material ellipsometric results from the Photonic Bandgap Fibers and Devices Group, Massachusetts Institute of Technology, Cambridge, Massachusetts, US; in particular Prof. Yoel Fink and Mr Alexander Stolyarov.

**Chapter 5:** The fifth chapter documents what was initially the primary motivation for this body of work: the design and fabrication of a soft-glass hollow-core microstructured fibre via the extrusion method. In step with the work's earlier chapters, the fibre design is based on a layered (annular) structure: concentric glass rings supported by connective radial struts, coined a *spider-web fibre* (a somewhat generalised air-Bragg fibre). The fabrication trials are discussed, beginning from the initial step of preform extrusion through to the transformation of the preform into fibre via the fibre drawing process. Experimental observation of guidance within these fibres is experimentally demonstrated and later compared to theoretical models, from simple anti-resonance or bandgap models

to full-vectorial finite-element solutions to Maxwell's equations evaluated over waveguide geometries derived from scanning electron micrographs of the fabricated fibre.

*Author's contribution:* As per the general comment above, save for the significant guidance and education with respect to the fabrication facilities from Dr Heike Ebendorff-Heidepriem. All fabrication processes (preparation and execution) were conducted by myself, save for the fibre drawing which was conducted by Mr Roger Moore (with my assistance for by-the-minute microscope fibre structure diagnosis).

**Chapter 6:** The final chapter concludes the thesis, providing some general discussion and suggesting possible promising future work.



## *Publications*

Journal publications and conference presentations based on the research for, and written within the duration of, this Thesis:

### *Journal papers*

- K. J. Rowland, S. Afshar V. and T. M. Monro, ‘Bandgaps and antiresonances in integrated-ARROWS and Bragg fibers; a simple model’, *Optics Express*, Vol. 16, Issue 22, pp. 17,935-17,951 (16 pages in total), published October 21, 2008. Co-published in *The Virtual Journal for Biomedical Optics*, Vol. 3, Issue 12, December 1, 2008.
- K. J. Rowland, S. Afshar V. and T. M. Monro, ‘Novel low-loss bandgaps in all-silica Bragg fibers’, *Journal of Lightwave Technology*, Vol. 26, Issue 1, pp. 43-51 (8 pages in total), 2008. Invited, Special Issue (post-deadline sessions—OFC 2007). The paper was an elaboration of the below OFC 2007 post-deadline conference presentation.

### *Conference papers/presentations*

- K. J. Rowland, H. Ebendorff-Heidepriem, S. Afshar V. and T. M. Monro, ‘Extruded soft-glass hollow-core microstructured optical fibres; Fabrication and light guidance properties’, The Australian Conference on Optical Fibre Technology (ACOFT) 2009, Adelaide, Australia. Oral Presentation.

Winner of the Wanda Henry Prize for ‘Best student presentation’.

- K. J. Rowland, S. Afshar V., A. Stolyarov, Y. Fink and T. M. Monro, ‘Spectral Properties of Liquid-Core Bragg Fibers’, Conference on Lasers and Electro-Optics (CLEO) 2009. Oral presentation.

This work was a collaboration with the Photonic Bandgap Fibers and Devices Group, Massachusetts Institute of Technology, Cambridge, Massachusetts, US.

Recipient of the OSA/Incubic Milton Chang student travel grant.

- K. J. Rowland, S. Afshar V. and T. M. Monro, ‘A novel approach to Bragg fibre bandgap analysis: Stratified planar anti-resonant reflecting optical waveguides’, The Australian Conference on Optical Fibre Technology (ACOFT) 2008, Sydney, Australia. Poster presentation.
- K. J. Rowland, S. Afshar and T. M. Monro, ‘Reducing confinement loss in all-silica Bragg bandgap fibres’, Optical Fibre Communications Conference and Exhibition (OFC) 2007, Anaheim, California, US. Presented in Post-Deadline Session E ‘Fibers and Optical Propagation Effects’, paper PDP41. Oral presentation.

Post-deadline sessions are competitive and reserved for the latest research completed close to the conference proceedings (after the deadline for conventional paper submissions).

*Journal papers (in progress)*

- K. J. Rowland, S. Afshar V., and T. M. Monro, ‘Binary Layered Optical Media; Phase Order Analysis’. An extension of the work of Chapter 3.
- K. J. Rowland, S. Afshar V., A. Stolyarov, Y. Fink and T. M. Monro, ‘Liquid Filled Bragg Waveguides’. Based on the contents of Chapter 4 and an extension of the work of Chapter 3. Almost complete.
- K. J. Rowland, H. Ebendorff-Heidepriem, S. Afshar V., and T. M. Monro, ‘Soft-Glass Hollow-Core Microstructured Optical Fibres via Extrusion’. Based on the contents of Chapter 5.
- K. J. Rowland, S. Afshar V. and T. M. Monro, ‘Guidance Properties of Air-Bragg Hollow-Core Fibres; the Influence of Structural Features’. Partially based on the contents of §§ 2.7 and 5.5.
- K. J. Rowland, S. Afshar V., and T. M. Monro, ‘Resonance and Reflectance Phenomena in Binary Layered Optical Media’. An extension of the work of Chapter 3.
- K. J. Rowland, S. Afshar V., and T. M. Monro, ‘Low-Index Guiding Fibres and Waveguides’. A review of the current state-of-the-art in low-index guiding (hollow-, filled- and solid-core) fibres and waveguides, based on Chapter 1.







## *Acknowledgements*

Unerringly, and without intention of platitude, I give most professional (and some personal, I'm pleased to say) acknowledgment to my supervisors, Prof. Tanya Monro and Dr Shahraam Afshar Vahid, without whom this thesis would not have been possible.

Professor Monro's arrival at the University of Adelaide coincided with the time at which I was to choose a supervisor for my honours year. Her infectious enthusiasm made my choice, when the opportunity presented itself, an easy one. I was one of the first two of Tanya's student as she established the Centre of Expertise in Photonics (CoEP), which has since essentially evolved into the Institute for Photonics and Advanced Sensing (IPAS). To have been one of the first members of a fledgling research group, when I was just a fledgling researcher myself, provided me with experience, skills and abilities I am certain I would not have otherwise obtained. For this experience, and for her ever present sharp mind and incisive comments and guidance, I give Tanya my sincerest appreciation. I must also thank her for the opportunity of employment as a post-doctoral researcher within the CoEP/IPAS during this last year; while working a 'real job' with the constancy of an unfinished thesis in wait has rarely been trivial, the experience has undoubtedly helped my growth as a research scientist. Given this, I must most of all give Tanya my appreciation for the patience, understanding, guidance and support she has given me throughout my PhD candidature, particularly this year just past, as my efforts have been variably focused on work, study and personal life.

I am afraid my appreciation for Shahraam's supervision and friendship cannot be adequately expressed in such a small space. Shahraam's enthusiasm for his research is rivalled only by his remarkable ability to reduce the complex to the tangible. His humility, humour and patience have been a significant stabilising factor for me throughout; often ready with a Persian parable or life story alongside his considered comments and guidance. The mentorship and friendship I have had in Shahraam has been unique, enjoyable and, in retrospect, cherished, as it is still today and hoped for for the future.

I thank both Tanya and Shahraam not just for their guidance but also for their willingness to often let me follow my own nose, as it were, when it came to research—never letting me stray too far from reality.

So much of my PhD experience was made easier and more enjoyable thanks to one of CoEP/IPAS's most integral members: Dr Heike Ebendorff-Heidepriem; few CoEP members could complete their research without her. Heike's approach to research seems at the same time both tireless and effortless, and always with a smile. Her knowledge and skill in glass chemistry and fibre fabrication has made the fabrication components of this work not just easier than they would otherwise have been, but possible (in particular, the fabrication processes in Chapter 5). Heike's guidance in the particulars of all steps of the fabrication process, from extrusion die design to glass chemistry and optics, as well as her willingness to listen to and comment on ideas and understanding, has been indispensable and is greatly appreciated.

For his technical ability and sometime artistry, Mr Roger Moore is acknowledged for turning my fibre preforms into countless spools of fibre, some of the most successful of which are shown within. If nothing else, working with Roger has been an enjoyable and memorable experience, thanks in no small part to his character.

The Photonic Bandgap Fibers and Devices Group, particularly Prof. Yoel Fink and Mr Alexander Stolyarov, is acknowledged for the supply of their hollow-core Bragg fibre used for the liquid filling experiments of Chapter 4. They are also acknowledged for providing the ellipsometric refractive index data for the material dispersion—based bandgap analysis of the same chapter.

Asahi Glass are acknowledged for providing the Bismuth Oxide glass used for the initial fabrication trials of the spider-web fibre demonstrated in Chapter 5.

My colleagues, past and present (of too great a number to mention here), are acknowledged for both their professional and personal input into my experience over the last few years. In particular, my appreciation goes to Mr Michael Bammann (half of the first couple of CoEP students), for his friendship and humour, as well as professional insights, during the early stages of my Honours year and PhD. Looking back, it was a unique and memorable time for us both.

Finally, my sincerest gratitude goes to my family and friends for their Herculean patience with me as I have focused almost exclusively on work and study for the last year or so in order to complete this thesis while meeting my responsibilities as a (not quite technically) post-doctoral researcher. Working while completing this thesis has not been a trivial affair as far as personal relationships go, but it has definitely been made easier in the knowledge that I have had, and continue to have, the unique support that comes from such unspoken affections.

Most of all, my everlasting gratitude, admiration and love goes to my partner, Christine—the one person to whom I am certain I cannot adequately express my feelings in print.





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# Abbreviations

<b>ARROW</b>	<b>Anti-Resonant Reflecting Optical Waveguide</b>
<b>I-ARROW</b>	<b>Integrated Anti-Resonant Reflecting Optical Waveguide</b>
<b>FEM</b>	<b>Finite Element Method</b>
<b>HCF</b>	<b>Hollow-Core Fibre</b>
<b>(N/M/F)IR</b>	<b>(Near/Mid/Far) Infra-Red</b>
<b>MOF</b>	<b>Microstructured Optical Fibre</b>
<b>HC-MOF</b>	<b>Hollow-Core Microstructured Optical Fibre</b>
<b>SM-HCF</b>	<b>Single-Material Hollow-Core Fibre</b>
<b>OSA</b>	<b>Optical Spectrum Analyser</b>
<b>PCF</b>	<b>Photonic Crystal Fibre</b>
<b>PhC</b>	<b>Photonic Crystal</b>
<b>PTBM</b>	<b>Photonic Tight-Binding Model</b>
<b>PMMA</b>	<b>Poly-Methyl Meth-Acrylate</b>
<b>PEI</b>	<b>Poly-Ethyl-Imide</b>
<b>PES</b>	<b>Poly-Ethyl-Sulphonate</b>
<b>SPARROW</b>	<b>Stratified Planar Anti-Resonant Reflecting Optical Waveguide</b>
<b>(c/p)TMM</b>	<b>(cylindrical/planar) Transfer Matrix Method</b>



# Physical Constants

All quantities here and within adopt the *mks* system of units.

Speed of Light in Vacuum	$c$	$=$	$2.99792458 \times 10^8 \text{ ms}^{-1}$	(defined)
		$=$	$1/\sqrt{\epsilon_0\mu_0}$	
Vacuum Permeability	$\mu_0$	$=$	$4\pi \times 10^{-7} \text{ NA}^{-1}$	(defined)
Vacuum Permittivity	$\epsilon_0$	$=$	$(\mu_0 c^2)^{-1}$	(defined)
		$\approx$	$8.854187817 \dots \times 10^{-12} \text{ Fm}^{-1}$	



# Symbols

$\epsilon$	$\equiv \epsilon_r \epsilon_o$	permittivity	$\text{Fm}^{-1}$
$\epsilon_r$	$\equiv \epsilon / \epsilon_o$	relative permittivity	unitless
$\mu$	$\equiv \mu_r \mu_o$	permeability	$\text{NA}^{-1}$
$\mu_r$	$\equiv \mu / \mu_o$	relative permeability	unitless
$n$	$\equiv \sqrt{\epsilon_r}$	refractive index	unitless
$\lambda$		free-space wavelength	m
$k$	$\equiv 2\pi / \lambda$	free-space wavenumber	$\text{m}^{-1}$
$k_i$	$\equiv n_i k$	intra-material wavenumber ( $i^{\text{th}}$ material)	$\text{m}^{-1}$
$\beta$		longitudinal wavenumber / propagation constant	$\text{m}^{-1}$
$\tilde{n}$	$\equiv \beta / k$	modal effective refractive index	unitless
$P$		power	W ( $\text{Js}^{-1}$ )
CL	$\equiv \frac{20}{\ln(10)} \text{Im} \{ \beta \}$	modal confinement loss	dB/m
$\nu$		frequency	Hz ( $\text{s}^{-1}$ )
$\omega$	$\equiv 2\pi\nu$	angular frequency	$\text{rads}^{-1}$