# Optical Flow Estimation in the Presence of Fast or Discontinuous Motion

by

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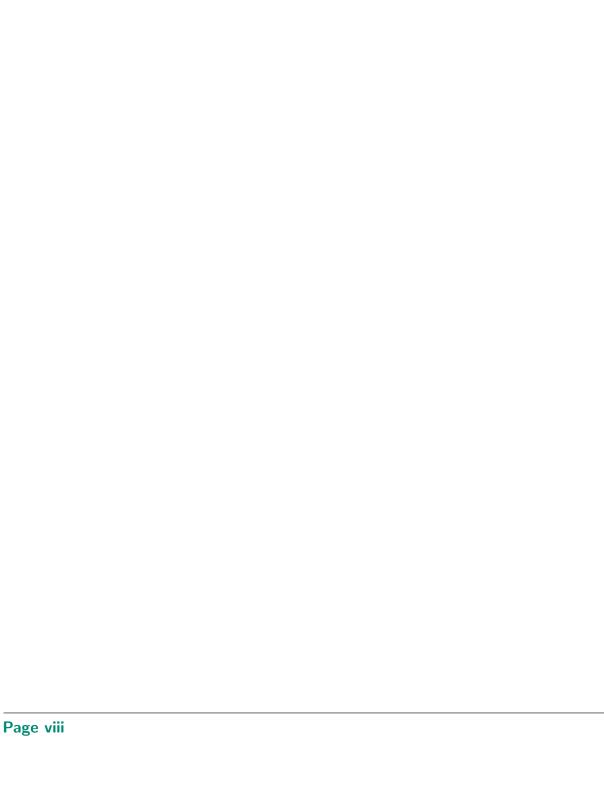
### **Abstract**

This thesis focuses on the computation of optical flow, i.e., the motion perceived from a sequence of gradually changing images, as an estimate for the 2D velocity of the scene. Due to the large variety and high complexity of the motion types existing in practice, motion recovery requires the estimation process to be highly adaptive. This thesis investigates how to select and combine the reasoning rules, namely the optical flow constraints, according to the type of motion information detected. Moreover, the thesis extends optical flow computation to fast rotation, an important, frequent and challenging motion type, which has not been addressed much in the literature.

The thesis starts by proposing various measures, based on theory as well as heuristics, for motion inconsistency detection. This facilitates selecting only the optical flow constraints that are valid for each pixel. While this selection benefits pixels affected by inconsistent motion, the combination of different constraints also enhances flow recovery for pixels that have consistent motion.

Two frameworks are designed for the combination of flow constraints. One utilizes motion segmentation; and the other is close in spirit to Expectation-Maximization. Within these frameworks, new constraints are formulated and tested. Furthermore, the adaptive reasoning is generalized from translational motion to motion that includes fast rotation. The key concept that enables this generalization is the use of intrinsic directions in differential geometry.

Experimental results on a variety of benchmark sequences have demonstrated the ability of the proposed methods to improve the performance of existing techniques in several situations, including strong motion discontinuities and fast rotational motion.



# **Statement of Originality**

This work contains no material that has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published written by another person, except where due reference has been made in the text.

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## **Notations**

### Image Intensity and Geometry

X,Y,Z	the world coordinate of a 3D point
<i>x</i> , <i>y</i>	spatial coordinates in the image plane
t	temporal coordinate in an image sequence
E(x, y, t)	image intensity function
$\vec{d}$	a general direction or the isophote (edge) direction, depending on the context
$\vec{n}$	the edge normal direction
$\mathcal{N}()$	the set of neighbours of a pixel
k	the kth pixel in a local patch, unless otherwise specified
$a^{(k)}$	the "a" of the kth pixel in the local patch
$a^{(c)}$	the "a" of the patch center
и	the horizontal component of the flow vector
v	the vertical component of the flow vector
$\vec{v}$	the flow vector, i.e, $\vec{v} = \begin{bmatrix} u & v \end{bmatrix}^T$
$ec{V}$	the homogeneous flow vector, i.e, $\vec{V} = \begin{bmatrix} u & v & 1 \end{bmatrix}^T$

### **Derivatives**

$E_x$	first order partial derivatives of $E(x, y, t)$ with respect to variable $x$
$E_{xx}$	second order partial derivatives of $E(x, y, t)$ with respect to variable $x$
Ė	total temporal derivative when compact notation is needed, $\dot{E} = \frac{dE}{dt}$
$\nabla$	spatial gradient vector
$\nabla_3$	spatial-temporal gradient vector
$\nabla_d$	gradient vector in an oriented coordinate system
Δ	Laplacian, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$
$\operatorname{div}\left(\left[\begin{array}{c}a_1\\a_2\end{array}\right]\right)$	divergence div $\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \frac{\partial}{\partial x}(a_1) + \frac{\partial}{\partial y}(a_2)$
$\operatorname{div}_d\left(\left[\begin{array}{c}a_1\\a_2\end{array}\right]\right)$	directional divergence $\operatorname{div}_d(\left[\begin{array}{c}a_1\\a_2\end{array}\right])=\frac{\partial}{\partial \bar{d}}(a_1)+\frac{\partial}{\partial \bar{n}}(a_2)$
$\partial_{ec{d}}$	directional derivative in direction $\vec{d}$ , i.e., $\frac{\partial}{\partial \vec{d}}$

#### **Notations**

the spatial Hessian matrix 
$$\begin{bmatrix} E_{xx} & E_{xy} \\ E_{xy} & E_{yy} \end{bmatrix}$$

H<sub>3D</sub> the spatio-temporal Hessian matrix  $\begin{bmatrix} E_{xx} & E_{xy} & E_{xt} \\ E_{xy} & E_{yy} & E_{yt} \\ E_{xt} & E_{yt} & E_{tt} \end{bmatrix}$ 

S<sub>2D</sub> the spatial Structure tensor  $\begin{bmatrix} \sum E_x^2 & \sum E_x E_y \\ \sum E_x E_y & \sum E_y^2 \end{bmatrix}$ 

S<sub>3D</sub> the spatio-temporal Structure tensor  $\begin{bmatrix} \sum E_x^2 & \sum E_x E_y \\ \sum E_x E_y & \sum E_y^2 \end{bmatrix}$ 
 $\begin{bmatrix} \sum E_x E_x & \sum E_x E_y & \sum E_x E_t \\ \sum E_x E_y & \sum E_y E_t \end{bmatrix}$ 

### **Vector and Matrix Operation**

<u>0</u>	a matrix whose elements are all zeros
$\vec{a}$	"a" is a vector
$ec{X}$	the motion parameter vector in the local system $A\vec{X} = \vec{b}$
$\ \ _1$	the $l_1$ norm of a vector
$\ \ _2$	the $l_2$ norm of a vector
$[]^T$	the transpose of a vector or a matrix
$\left\langle \vec{a},\vec{b}\right angle$	inner product of vector $\vec{a}$ and $\vec{b}$
A*B	the convolution of $A$ and $B$
$A^+$	the pseudo-inverse of matrix $A$
$N\left(A\right)$	the null space of matrix $A$
col()	the column space
$span(\vec{a}_1,\cdots,\vec{a}_n)$	the space spanned by basis vectors $\vec{a}_1, \dots, \vec{a}_n$
$\perp$	orthogonal
$G_{\sigma}$	Gaussian smoothing kernel of standard deviation $\sigma$
$K_{rac{\partial}{\partial ec{d}}}$	directional differencing filter
$A \sim B$	matrix <i>A</i> can be transformed to <i>B</i> by elementary matrix operations

#### Miscellaneous

Œ the error functional  $\alpha_1, \cdots, \alpha_6$  affine motion model parameters a small positive number to prevent the denominator from being 0 Lagrange multiplier λ the *i*th eigenvalue or singular value of a matrix  $\lambda_i$ weight ω index of the iteration stage, unless otherwise specified τ the axes of the locally oriented coordinate frame  $\zeta,\eta$ Tthreshold, unless otherwise specified the refinement or increase of a in an iteration process  $\delta a$ R the set of real numbers  $R^+$ the set of positive real numbers