A. C. A

March 1, 1939

Dear Aitken,

Some little time ago I completed the distribution of the quantic roots by evaluating the constant term, which comes to

π^{ip}

$$\frac{\gamma^{\gamma_0 p}}{\frac{p-2}{2}!\,\frac{p-3}{2}!\cdots 0!}$$

Dr Hsu of this College tells me that he has constructed a complete proof of the distribution, and I have offered to publish it in the same number of the Annals as my own paper. I ~~ought~~ write to let you know this, in order that you may not give more time to the problem than it deserves for its intrinsic interest to you. Dr Hsu seems to be a good hand at matrix algebra, but I have not yet seen how he handles the problem.

Yours sincerely,

Mathematical Institute
16 Chambers Street
Edinburgh, 1

Mar. 2, 1939.

Dear Fisher,

The constant in your distribution of quantic roots was bound to be, as you write,

$$\frac{\pi^{\frac{1}{2}p}}{\left(\frac{p-2}{2}\right)!\left(\frac{p-3}{2}\right)!\ \ldots\ 0!}\ .$$

Thank you for writing to tell me that the investigation is complete. I am sorry I was of no particular help, but I look forward to reading your paper, and Hsu's demonstration. (Hsu, as I know from correspondence and conversation with him, is well versed in matrix algebra, especially in such statistical applications as these.)

I was never able even to put pen to paper to evaluate that Jacobian differential,

but only to think of it sometimes for a few minutes before going to sleep at night. In those cogitations I could see, from considerations of symmetry and homogeneity that integration would give a constant, involving a power of π and Γ-functions.

Your new distribution, a kind of multiple Beta one, is clearly most important, as well as most elegant, and I congratulate you on the insight by which you divined it.

At some time I must solve for my own satisfaction a matrix problem suggested by it, namely to express the discriminant

$$\{(\theta_1-\theta_2)(\theta_1-\theta_3)\cdots(\theta_{p-1}-\theta_p)\}^2$$

of the characteristic equation $|A-\theta I|=0$ of a matrix A of order p, in terms of the elements a_{ij} of A. The result for ordinary algebraic equations is known, and is classical! The matrix extension must exist.

Yours sincerely,
A.C. Aitken