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Dec. 19, 1935.

Dear Dr Fisher,

Professor Whittaker has informed me that you had asked for certain of my offprints. I have sent these this evening under a separate cover. One or two of them you may already have. The one on least squares and orthogonal polynomials might have been improved had I noticed in due time a slight numerical simplification, namely that in the working tables each column has usually an integer H.C.F., > 1, which may be cancelled throughout, its square being cancelled from the divisor at the foot of the column; so that, e.g. the working table for $n = 10$, p. 6 ..., reduces to

$n = 10.$

1	−9	6	−42	18	−6
	2	−4	56	−40	20
		1	−35	45	−35
			10	−30	40
				10	−30
					12
10	330	132	8580	2860	780

decidedly smaller numbers. Of course on the machine it would not make much difference.

There is another offprint in the press, and I shall send it also when it comes out. Its interest is mainly theoretical. The principal result is that the bivariate correlation function in fourfold sampling with replacement has the form

$$\phi(x, y; \delta) = \phi(x, y; 0)\left\{1 + \frac{\delta}{pp'qq'} G_1(x; p) G_1(y; p') + \frac{\delta^2}{2!\,(pp'qq')^2} G_2(x; p) G_2(y; p') + \cdots\right\},$$

p_{11}	p_{10}	p
p_{01}	p_{00}	q
p'	q'	1

$\delta = p_{11} - pp'$.

where $\phi(x, y; 0)$ is the corresponding <u>uncorrelated</u> function for the same marginal p, p', q, q', and the G's are polynomials

$$G_r(x) = x^{(r)} - rp(n-r+1)x^{(r-1)} + \binom{r}{2}p^2(n-r+2)^{(2)}x^{(r-2)} - \cdots + (-p)^r n^{(r)},$$

with $x^{(r)} = x(x-1)\cdots(x-r+1)$; having orthogonal properties

$$\sum_0^n \binom{n}{x} p^x q^{n-x} G_r(x) G_s(x) \begin{cases} = 0, & r \neq s \\ = r!\, n^{(r)} (pq)^r, & r = s \neq 0. \end{cases}$$

Regressions and higher moment arrays come out readily from the orthogonal properties. If replacement is not allowed, University of Edinburgh

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there are corresponding polynomials

$$V_r(x) = x^{(r)} - r\,\frac{(n-r+1)(Np-r+1)}{N-2r+2}\,x^{(r-1)} + \binom{r}{2}\frac{(n-r+2)^{(2)}(Np-r+2)^{(2)}}{(N-2r+3)^{(2)}}\,x^{(r-2)} - \cdots + (-)^r\,\frac{n^{(r)}(Np)^{(r)}}{(N-r+1)^{(r)}},$$

which possess analogous properties in respect of the hypergeometric frequency function, and lead to a similar fourfold correlation function.

I do not know whether you will have any offprints to spare of your own papers (in Phil. Trans. and Camb. Phil. Soc.) on the foundations of theoretical statistics; but if by chance you were able to spare me a copy, I should be very grateful indeed.

I am,
Yours sincerely,
A. C. Aitken.

December 30th, 1935

Dear Dr Aitken,

Thanks for your letter. I was very glad to get your offprints, some of which I had not before read.

About offprints of mine, I should like you to have a selection of all which would be useful to you, and I have many left though some of the earlier ones are exhausted. I believe the best plan would be for you to look through the bibliography printed in the latest 5th. edition of "Statistical Methods" and send me a list of those which you would like to have.

Yours sincerely,