2 Sycamore Terrace,
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April 13, 1931.

Dr R.A.Fisher,
Rothamsted Experimental Station,
Harpenden.

Dear Dr Fisher,

I have now had ample opportunity for trying out those methods of interpolation, and am in a position to give you an account of my experience on an "Archimedes" machine. It is very strongly impressed upon me by my findings, as compared with the remarkable times you have achieved, that the type of machine used makes a vital difference in such tests as these. I do not propose just yet to take up the question on the "Millionaire", since too persistent application on the other has caused some insomnia, and I have no desire to incur a breakdown such as I sustained some years ago through prolonged (and fatuously useless) computations. But it may not be amiss if I contrast the technique of the "Archimedes" with that of the "Millionaire", in order that the results I give may find explanation.

You are aware that on the "Millionaire" one multiplies or divides by plugging in to the required hole, 5 say, and turning the crank once only, the carriage itself moving to the next position. On the "Archimedes" it is necessary to turn the crank the requisite number of times, here 5, with the right hand and then with the left hand lift and drop the carriage into the next position. One can, of course, save a number of turns quite often by using the nines-complement of the digit and reversing into subtraction with the left thumb. Even so, while for example to multiply by 87654321 on the "Millionaire" requires only 8 revolutions, on the "Archimedes" at the very least 22 are required, $1\overline{1235}4321$, and though the recording dial keeps one right, the eye must watch it. On the other hand, the handle of the "Archimedes" is small and on light bearings, easy to turn by a pure wrist action, while during the turnings the eye is often free to move and pick up the next entry required to be set up. There is however no possible doubt that in multiplication the "Millionaire" has the "Archimedes" hopelessly outclassed in 8-digit work. When we turn to the formation of 2nd, 4th and 6th differences the case is somewhat different; 2nd differences are formed in the usual way by 'first plus third minus twice middle' in succession, and 6ths from 2nds by 'first plus 6 times third plus fifth minus 4 times fourth and second', and the rapid turning and the fact that the carriage does not move are advantages. Here, I imagine, the "Archimedes" is certainly the equal of the "Millionaire", at any rate not far behind it. To these reasons, and one or two others bearing on convenience of position, I attribute the fact that my times for Everett without differences and for Jordan, though much reduced from my earlier efforts, remain nearly equal.

Impressed by your remark that Mr Murray had found much time taken up by the copying down of tabular values, interpolates and differences, I resolved in the first place to undergo extensive practice in this branch, and attained high speeds. These are typical:

Copying.

Copying.

8 consecutive entries of 10-digit numbers	35-40 secs.
8 .... 8-digit	30 secs.
2 .... 8-digit	8 secs.
100 entries of 6 or 7-digit entries in columns. (Table, p.74, "Statistical Methods for Research Workers")	4min. 45 secs.
Pairs of central tabular values and even differences up to the 6th	15-20 secs.

Copying and Differencing. ("Archimedes")

To copy 2 central tabular values, to difference and copy 6 second, 4 fourth, 2 sixth diffs.	200-210 secs.
To do the same, but computing only two 4th diffs., and the 6ths from 2nds by (1,-4,6,-4,1)	175-180 secs.
To copy 8 consecutive 8-digit values, difference and copy 6 second, 4 fourth, 2 sixth diffs.	225-230 secs.
To make a linear interpolation with 5-digit θ from two 8-digit results	70-75 secs.
To copy down Everett coefficients (contracted where necessary) and interpolate on written data up to 6th diffs. (190, 200, 190, 195, 200,..)	195 secs.
To copy 2 central values and six 4-digit second diffs. and compute and write down paired even diffs up to 6th.	55-60 secs.
The same, but with 5-digit second diffs.	75-80 secs.
To memorise and write down a number of 35 digits (Memorise, 30 secs)	45 secs.

After preliminaries of this kind I proceeded to actual interpolations, doing the steps separately for practice at first and afterwards working straight ahead on lists prepared ready. Times were taken to the nearest 5 second point after the last figure was written down. In brackets are given typical sequences. Each interpolation was paired and checked by another in which the data were written in reverse order and a complementary value of θ used.

Everett 4-point, differences given.

Central data and coefficients copied. 70-80 sec.

(70, 75, 80, 75, 75, 70, ...)

Everett 4-point, without diffs.

(130, 115, 110, 115, 120, 130, 125, ..) 125 sec.

Jordan 4-point.

(130, 125, 130, 125, 125, 120, ..) 125-130 s.

(Linear interpolations 90-95, mult. 35)

[Thus I found hardly any difference between the Everett 4-point without differences and the Jordan. As a matter of fact I prefer the Jordan, which shows a smaller variation; and it was very convenient in Chappell's tables to find the requisite coefficient in the early column. If I were allowed to use non-routine methods I might cut the Jordan time considerably.]

Everett 6-point, differences given.

(120, 125, 130, 110, 125, ...) 120-130 s.

(Actual mult^n; 70, 60, 70, 55, 65, ..)

Everett 6-point, without diffs.

(155+ 65, 150 + 70, 150 + 65, 155 + 70,..) 220 secs.

Jordan 6-point.

Copying and linear interpolation, 120, differencing 35, mult^{n} 60, with little variation. 215 sec.

[Here again there is not much difference, but if you gave me a little latitude I might bring Jordan nearer your own time of 180 sec.]

Everett 8-point, diffs. given

15-20 sec + 190-200 sec. 210-220s.

Everett 8-point, without diffs.

(175-185 + 190-200) 370-385s.

Jordan 8-point.

210-215 + 75 + 75 360-365s.

[Here Jordan appears to advantage, though I had to give up my final tests on Everett owing to strain. I believe I could reduce the Jordan further, possibly to 350s.]

I have not worked out percentages in the way you suggest, because it is perfectly clear that these would be heavily influenced by the type of machine and idiosyncrasies of the computer. Further the speeds at which I performed these tests is much in excess of normal requirements on the "Archimedes", and I should not care to sustain them long. But I think you will see that I have devoted a good deal of strenuous practice to Jordan's method.

Now that the trials are done with, taking in all I suppose 30 hours at least during the past several days, I should like to say that your letters, particularly your second, made me realise that I had misunderstood Dr Wishart's original request to Professor Whittaker. There were two references in his letter to the policy of printing even differences, and the cost; and in his next letter, to myself, he wrote that "while it has been our usual procedure to recommend the printing of even differences with a table, it was suggested that this would be unnecessary if the Jordan formula were brought prominently forward". Naturally I took it that the emphasis lay on the question whether the Jordan formula was better than the Everett with differences provided, and that the question of how it compared with Everett without differences was subsidiary. In such tests as I was able to make, most of them being carried out on the afternoon preceding the Tuesday morning when I made final tests, and some at home on a miniature "Brunsviga", it was clear that in 6-point interpolation on the "Archimedes" or "Brunsviga" at least, Jordan's method took at least twice as long as Everett with differences, and that Everett without differences was comparable with Jordan. Had it occurred to me for a moment that, to quote your own phrase, "the immediate question is whether Jordan's method is to be discredited among English computers as of no practical service when differences are not available", I should have confined my remarks to the first point only, and should have begged for time to make finer discrimination in regard to the second. Not that I think for a moment that to rank a new method as comparable in speed with a generally praised old method is any expression of depreciation; on the contrary I have rarely used Everett myself, disliking the double line of data, and so far do I feel from belittling the Jordan, for the ingenious simplicity of which I have a sincere admiration, that I have incorporated it into my class lecture-notes and have passed it on to Dr Lidstone and Mr D.C.Fraser. We can be pretty sure it will soon be a household word among actuaries, and will have its merits thoroughly brought into the light. It never occurred to me that the remarks of an isolated individual, recorded in a minute, would be more than a featherweight among the opinions of more experienced English computers, but they were not irresponsibly made, and it should now be clear to you that I always took the matter seriously.

The question of relative efficiency on the "Millionaire" interests me very much, but I must postpone it for the present. Some time I shall certainly do it, and write to you.

Just one little point arising out of your last letter. It is often necessary, you write, to compute for θ = say ·355 and also for ·356. Is that in order to compute an intermediate value, say ·3552793 ?

I do not know the "Marchant" machine at all, but it is probably different both from the "Archimedes" and the "Millionaire". All our machines are about fifteen years or more old.

Yours sincerely,

A. C. Aitken.