18th January 1935.

Dear Ashby,

I have been looking at the problem of the distribution of points scattered over a set of 25 squares, and find that if the number of points is fixed, e.g. 30, then the distribution of the number of empty squares is determinate.

The distribution is not very well known and may be expressed by saying that the frequency of having r empty squares is:
$$\frac{25!}{r!(25-r)!}\,\Delta^{25-r}\,0^{s} \div 25^{s}$$

In this expression the Δ to the power of 25 - r reppesents 25-r repetitions of the operation of taking the first differences of the series $0^{s}, 1^{s}, 2^{s}$, ... of the powers of natural numbers, and with numbers of the size you want is rather tedious.

There is no particular difficulty, however, about getting the means and variances, and if you desire them, the higher moments of distributions.

The mean is
$$25\left(\frac{24}{25}\right)^{s}$$

and the variance is
$$25\left\{24\left(\frac{23}{25}\right)^{s} - 25\left(\frac{25}{24}\right)^{2s} + \frac{24}{25}^{s}\right\}$$ $\frac{24}{25}$

For example, for s=30 I find the average number of empty squares to be 7.3464, with a variance 2.5561, and the values for any other value of *s* can be calculated similarly. You might like to compare these with the variances in your observed series for any group of observations in which the total number of plants is the same.

Yours sincerely,

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