26th July, 1956.

My dear Chester,

The problem you sent me looks to me very like the topic I discussed in "The Design of Experiments" under the heading of Section 64 ('Wider Tests based on the Analysis of Variance'). That is to say one should not, I fancy, be satisfied with a neat statement of fiducial limits for any one particular parameter, without regard to the other.

If one were to apply the method of that section, it would go somewhat like this:

The sum of squares for 2 degrees of freedom, which you call the first term, is, I fancy, $\frac{(83.502)^2}{18} + \frac{(2165.474)^2}{18}$. Had you been considering not the hypothesis that these two components should be zero, but that your measures of them should have true values α and β, this sum of squares would be replaced by $\frac{(\alpha-83.502)^2 + (\beta-2165.474)^2}{18}$. To decide how large this expression could be without ~~ever~~ over-turning the hypothesis, consider your error for 19 degrees of freedom, which has a mean square 938.84, a natural logarithm 6.83393, and one-half the natural logarithm 3.41696, to which can be added the tabular z for 5% significance

with 2 and 19 degrees of freedom, namely .6295, bringing the total to 4.0465, which is the natural logarithm of 57.2 approximately. The variance for 2 degrees of freedom would then be $(57.2)^2 = 3271.84$ and the sum of squares for the 2 degrees of freedom would be 6543.7, so multiplying by 18 one would have $(\alpha-83.502)^2 + (\beta-2165.474)^2 < 117786$, if the hypothesis is not to be contradicted at the 5% level.

Graphically this puts the hypothetical first harmonic pair of terms within a circle, or rather a series of concentric circles, for different levels of significance. I think I should be inclined to express the apparent precision of the observations in some such graphical form, rather than to particularize as to the fiducial limits within which the phase angle and the amplitude separately lie. Of course, for the phase angle separately I should use the method of the ratio of two means, as in Section 62.1, while I suppose the amplitude treated in isolation from the phase angle would be dealt with exactly by Mahalanobis's D^2 (of which, perhaps, tables have been worked by R. C. Bose), but as you cannot help having a multiplicity of hypotheses in view, it seems to me more rational to treat them all simultaneously.

I hope all this is some use to you. The work of that Committee with which I was charged at the Quitandinha is essentially completed and we have a Report, to which almost everyone has

given their approval, ready to be sent to the Bureau when I have collected all their names, for whatever happens I do not intend to start it all again! I believe it will give essentially what you wanted, though in a somewhat different way from any which had been specifically proposed.

Sincerely yours,