June 2, 1939

Dear Bliss

I have been putting some work in on the data you send respecting the effect of depth of paraffin upon retardation of growth of the root of <u>Lactuca</u> seedlings at different lengths of exposure to X-rays.

The data are really extraordinary in respect of the much higher precision with which the different experiments agree at the higher dosage rates, whether this is due to absorption by parrafin or not. Probably the most instructive thing I have done with the data can be illustrated by the enclosed copies of working sheets.

At this stage I adopted weights calculable from <u>x</u>, which itself is the log exposure, using the formula

$$\log w = 1.75272\ x$$

and subtracting .580 in the second half of each block. This gave me the working weights

1.395	.3659
4.700	1.234
9.562	2.509
15.84	4.156
23.43	6.147

of which the second column bears a constant ratio a little more than a quarter to the first.

Then for each half block I calculated S(wy) and $\overline{y}$, obtaining the analysis of the 20 half blocks

	Degrees of Freedom	Sums of Squares
Blocks	9	1.27676
Depth	1	4.28845
Depth x Blocks	9	.55834
	19	6.12355

The values of S(wy) for treatments are obtained simply by multiplying the column totals by the weights. Here I get:

1st half	2nd half	total	mean
6.139395	.8810872	7.0204822	.3986872
28.279900	5.251904	33.531804	.5650793
69.343624	13.006656	82.350280	.6822159
125.43696	25.027432	150.464392	.7524725
196.81200	39.678885	236.490885	.7995770

These give the two further items

	Degrees of Freedom	Sums of Squares
Exposure	4	5.33574
Exposure x Depth	4	.00662
	8	5.34236

Also, using the above totals to obtain S(wxy), it appears that the linear component in exposure takes up 5.31576, leaving only .01998 for the three non-linear degrees of freedom. The value of S(wx^2) is 16.10833, and of S(wxy) 9.25354, so that the regression is .5744568, and, as the difference in $\overline{y}$ due to depth of parrafin is .1938136, the equivalent difference in log. exposure x is .3373859, very like your value .3315.

You will notice that I have taken out 27 D.F. for possible effects and left only 72 for error. Essentially this only differs from your analysis by taking out the contribution of depth x blocks, which is significantly large. My residual S.S. is .78124. Owing to my arbitrary weights I cannot compare this directly with yours, but below I give two comparable analyses, in each of which the mean square error for these 72 D.F. is reduced to unity:

Analysis with weights

	D.F.	S.Ss.	Mean Square
Blocks	9	117.67	13.074
Depth	1	395.23	395.229
Exposure	4	491.75	122.937
Exposure x depth	4	.61	.153
Blocks x depth	9	51.46	5.717
Error	72	72.00	1.000

Analysis without weights

	D.F.	S.Ss.	Mean Square
Blocks	9	98.14	10.905
Depth	1	165.87	165.873
Treatment	4	368.26	92.066
Treatment x depth	4	.47	.117
Blocks x depth	9	26.64	2.960
Error	72	72.00	1.000

It would appear, if I have made no mistake, and I am deliberately sending you all the material for checking, if you want to do so, that all the causes of differentiation in growth rate are accentuated by the weighting, or, in other words, that the increased precision which might be hoped for has been effected. This is the first evidence that the weights are any good. The questions: could they be better? and are they good enough? seem to me more difficult. I imagine that the latter is in the affirmative if, with admittedly improved weights, all conclusions of importance are altered to only a slight extent. To try and trace what is happening, I have further subdivided the S.Ss. for error from the different columns of the table. The subdivision appears to be right, as the total checks when each S.S. is multiplied by its appropriate weight. You will see that $S(y - Y)^2$ decreases in both series with length of exposure, but this might not be very strong corroberation of the value of the weights, since, if one weighted unequally members of series in reality equally variable, one would certainly cut down the residuals from the more heavily weighted members. I therefore consider the series of S.Ss. after multiplication by the weights used. If the weights were perfectly fitted, this series should show zero regression on x, where, in the determination of the regression, the diffe.ent exposures are weighted equally.

For the second half of the experiment, it appears then that the weighting has been about right, while the indication is that in the first half it might, with advantage, have been even steeper than the values used. If you have time you might find it instructive to take a higher value, e.g., 2 instead of 1.75, for my weighting formula and see if the expected consequences are realised, namely, greater equality in the first half, and perhaps a rising sequence in the second half.

Notice that I do not use weights based merely on the variation of values at each exposure, because this will necessarily include block differences, if these are real, as they seem to be, and this inclusion will be very much more important with the more precise values at long exposures than with the less precise values at short exposures. Hence, one might expect that your values of $\frac{1}{s^2}$ would be in lower ratios than the true precisions attained at different exposure length. Obviously, however, when block effect is not very great, as in this case, they provide a useful start.

Yours sincerely,

June 2, 1939

Copy of working sheet - depth dose

I adopt weighting calculable from x

$$\text{Log } w = 1.75272\, x$$

subtracting the constant .580 in the second half of each block

	first half		second half			
w	S(wy)	$\bar{y}$	S(wy)	$\bar{y}$	total S(wy)	
1.395	41.397678	.753685	9.8926873	.686425	51.2903653	
4.700	39.605194	.721051	6.7531785	.468583	46.3583725	
9.562	44.436103	.809003	8.9242488	.619228	53.3603519	
15.84	44.526408	.810647	8.6733339	.601818	53.1997419	
23.43	46.865331	.853229	8.0626491	.559444	54.~~927801~~	9279801
.3659	42.208914	.768455	7.6983700	.534168	49.9072840	
1.234	41.381470	.753390	8.7778762	.609071	50.1593462	
2.509	38.941388	.708966	6.2986654	.437046	45.2400534	
4.156	43.969000	.800499	9.0176671	.625710	52.9866671	
6.147	42.680393	.777038	9.7472878	.676336	52.4276808	
69.3389	426.011879	.7755965	83.8459642	.5817829	509.8578432	.73531285

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Copy for Dr C.I.Bliss of working sheet (depth-dose)

values of (y - Y)

	.12581	.05421	-.00929	-.01699	-.00309	
-	.14355	-.02215	.01135	.00565	.00455	
	.00349	-.01911	.02039	-.00931	.00159	
	.05085	.00925	.00875	-.01095	-.00105	
-	.09873	-.00033	.05017	.01347	-.02363	
-	.15696	-.06156	-.01106	.02524	.00914	
	.13211	.06051	-.05299	-.01269	.01021	
-	.07747	-.08007	-.02457	.02673	.01263	
	.01700	.02840	-.01010	-.01580	.00810	
	.14746	.03086	.01736	-.00534	-.01844	
Total	.00001	.00001	.00001	.00001	.00001	
$S(y-Y)^2$	.118905	.0195014	.0071618	0.0024993	4.00134520	S(y-Y)²
$wS(y-Y)^2$	.16593	.09166	.06848	.03959	.03152	x w .39718

Steeper weighting suggested

Values of (y - Y)

	.17190	.05510	-.00770	.01250	-.02580	
-	.04268	-.13448	-.14328	-.00708	.09262	
-	.21767	.01153	-.01527	-.00407	.01863	
	.20056	.07476	.05096	-.01284	-.03814	
-	.03954	-.04534	-.11114	.05306	.02076	
-	.34386	-.07266	.00154	.02574	.01544	
	.02997	.03717	.04437	-.02143	-.01273	
	.03000	.02020	.03740	-.03440	.00340	23
	.06440	-.00740	.07180	-.00600	-.02730	
	.14694	.06114	.07134	-.00546	-.04576	
Total	.00002					
$S(y-Y)^2$	.266315	.0397606	.0493851	.0055741	3.0147218	S(y-Y)²
$w(y-Y)^2$	.09744	.04906	.12391	.02317	.09049	x w .38407

.78125

weighting about right