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# ROTHAMSTED EXPERIMENTAL STATION

(LAWES AGRICULTURAL TRUST)

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HARPENDEN
HERTS

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Professor R. A. Fisher, F.R.S.,
Galton Laboratory,
Gower Street,
London, W.C.4.

Dear Dr. Fisher,

Some time ago you suggested to me the expression

$$I = \int_{-\infty}^{\infty} S^2 \frac{(n+1)(x-\mu)}{ns^2+(x-\mu)^2} \prod \left\{ns^2+(x-\mu)^2\right\}^{-\frac{n+1}{2}} d\mu$$

$$\div \int_{-\infty}^{\infty} \prod \left\{ns^2+(x-\mu)^2\right\}^{\frac{n+1}{2}} d\mu$$

for the information about $\mu$. For two samples only, the contour integration presents no difficulty for $n_1$, $n_2$ both odd, but the algebraic expressions for the residues become very lengthy as $n_1$, $n_2$ increase.

I was interested in the suggestion you made, in connection with this problem, that in particular cases we could obtain a test of significance by calculating the likelihood function L for a number of values of $\mu$ and finding 5% levels for $\mu$ from L itself. Would this procedure be applicable to the least square fitting of equations of the form e.g. $Y = 1 - e^{-\beta x}$, where the equation is not effectively linear in the parameter $\beta$ ? You pointed out once in a lecture that the "linear regression" theory no longer holds and I have often wondered how far it is out for small samples and how one could make an exact test of significance if wanted in any particular case. Could one use

$$L \propto e^{-\frac{1}{2} S\{y - \alpha(1 - e^{-\beta x})\}^2}$$

calculate L for different values of $\beta$ and reject values at both tails. It is not clear to me what relation this procedure would have to the sampling distribution of the least squares estimate of $\beta$ . I would be grateful for your advice on this point.

Yours sincerely,

W.G. Cochran.