IOWA STATE COLLEGE
OF AGRICULTURE AND MECHANIC ARTS
AMES, IOWA

Statistical Laboratory
December 8, 1938

Dear Dr. Fisher:

I have been lecturing here on some of the uses of the semi-invariants, and worked through a simplified case of the approximation to the z-distribution for high n_1, n_2, as given in your paper with Cornish: "Moments and Cumulants in the Specification of Distributions". It occurred to me that the formulae which you give in "Statistical Methods" and in the Tables for the 1% and 0.1% values of z might be improved.

Your formulae are

$$z = \frac{\xi}{\sqrt{h-1}} - \left(\frac{\xi^2+2}{6}\right)\left(\frac{1}{n_1}-\frac{1}{n_2}\right)$$

where ξ is a normal deviate with unit S.E.

This is approximately

$$z = \frac{\xi}{\sqrt{h}} + \frac{\xi}{2h\sqrt{h}} - \frac{(\xi^2+2)}{6}\left(\frac{1}{n_1}-\frac{1}{n_2}\right)$$

The term $\xi/2h\sqrt{h}$ is an approximation to the term $(\xi^3+3\xi)/12h\sqrt{h}$ in the more accurate expression given on p. 14 of your paper with Cornish. The approximation will be good if ξ^2+3 is near 6.

For the 5% point, $\xi = 1.6449$, $\xi^2+3 = 5.706$
" " 1 " " $\xi = 2.3263$, $\xi^2+3 = 8.412$
" " 0.1 " " $\xi = 3.0902$, $\xi^2+3 = 12.549$

Thus the term $\xi/2h\sqrt{h}$ becomes progressively too small. A better approximation would be to take (h − 1.4) instead of (h − 1) for the 1% and (h − 2) for the 0.1% point.

For $n_1 = 24$, $n_2 = 60$, the respective values of z are

	Your formula	Mine	Correct
P = ·1 %	·3923	·3948	·3946
P = 0·1 %	·4875	·4954	·4956

The improvement is I think worth having, as both formulae are equally easy to calculate.

I am having a very interesting time here. The weather has been glorious. I understand that it's the finest autumn within living memory.

My best wishes to you all for an enjoyable Xmas.

Yours sincerely,

W G Cochran.