February 26, 1940

Dear Professor Fréchet,

Thanks for your note of February 20th. I think logically the fiducial argument proceeds in three stages, setting aside for the moment my usual cautions about using the whole of the information.

1) A continuous distribution is found for T for samples of a given size drawn from a population having parameter θ. θ is then also a parameter of this distribution of T.

2) a relation is established between the true value and any percentile point T_p of the distribution of T. We shall suppose that this also establishes a univalent inverse relationship from which, given T_p, θ may be found. It is then true for all samples of the given size (or otherwise specified by ancillary statistics) that the inequality T exceeds θ will occur with given frequency when T and θ are mutually related as defined above.

3) In these circumstances <u>I think it proper</u> to refer to p as the fiducial probability that θ is less than T. This as it stands is a definition of the phrase <u>fiducial probability</u>. I

believe it is, properly speaking, a probability, measuring as it does the relative frequency of one out of two or more well defined outcomes of a well defined procedure. I think it may be described properly as the admission of a new principle, if this phrase means, as I suppose it does, the thinking of a given situation in an unfamiliar way. Alternatively, I have no objection to regarding stages 1) and 2) as logical deductions, and No. 3) as an arbitrary definition. The definition is, however, a matter of choice and not a matter of chance.

The outstanding difference from inverse probability lies in the population of events of which the particular one, to which the probability refers, is regarded as a member. In the case of inverse probability this population is that of all samples of a given size, selected to have a given value for the estimate T, drawn by chance from a population which has itself been drawn by chance from a super-population having a given specification in respect of the distribution of the parameter θ.

The population of events referred to in fiducial probability consists of all samples of a given size drawn from any population defined by some value or other θ. It is obvious that the frequency of a given event in members of this last population may be unequal to the frequency of the same event in the population considered in the theory of inverse probability.

I hope this will do something to clear up this rather knotty problem.

Yours sincerely,