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Vice Chancellor's Office

12th May, 1976

Dear Henry,

I return herewith the Fisher-Hardy correspondence you left with me, together with the following notes. If my notes are not clear I should be happy to discuss the details with you.

Fisher describes in his first letter [16.10.41] two examples of a 'perfect difference set', for the moduli 73 and 7. Generally, if $a_1, a_2, \ldots a_{k+1}$ are $k + 1$ distinct integers, then all differences $a_i - a_j$ $(i \neq j)$ give $k(k + 1)$ integers. If these $k(k + 1)$ are all different modulo v, where $v = k(k + 1)$, then $\{a_1, \ldots a_{k+1}\}$ is called a perfect difference set modulo v, since every integer modulo v (other than o) occurs exactly once as a difference $a_i - a_j$. For example, with $v = 7$, the set $\{1, 2, 4\}$ is a perfect difference set, since the 6 possible differences are

$$\begin{array}{ll} 2 - 1 \equiv 1, & 1 - 2 \equiv -1 \equiv 6 \\ 4 - 1 \equiv 3, & 1 - 4 \equiv -3 \equiv 4 \\ 4 - 2 \equiv 2, & 2 - 4 \equiv -2 \equiv 5. \end{array}$$

The elements of the set happen to be powers of 2. Similarly, Fisher remarks that the 9 distinct powers of 2 modulo 73 viz $\{1, 2, 4, 8, 16, 32, 64, 55, 37\}$ give a perfect difference set modulo 73.

Hardy looked at the direct generalization of these two examples, namely that the powers of 2 gave a perfect difference set modulo v for appropriate v; and he found that such a generalization is false.

However Hardy does not seem to have considered the broader problem of determining perfect difference sets (if they exist) for other values of k and $v = k(k + 1) + 1$. Fisher's guess in the final paragraph of his letter of 1/11/41 is indeed correct, and his hint that there may be a perfect difference set when k is a power of a prime is also correct. In fact, these results had just been established and published by J. Singer, 'A theorem in finite projective geometry and some applications to number theory', Trans. Amer. Math. Soc. 43 (1938), 377-385. (A detailed account of Singer's results is given in Marshall Hall, Combinatorial Theory (Blaisdell, 1967).)

It is interesting that the problem of establishing precisely which values of k allow a perfect difference set is still unsolved, and Singer's results give almost all the known information (i.e. no-one has yet found a perfect difference set for a value of k other than a prime or a prime power).

Yours sincerely,

Eric.

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