26th February 1934.

Dr. H. Jeffreys,
St. John's College,
Cambridge.

Dear Jeffreys,

Thanks for your letter. I am glad you think the differences in our points of view do not go so deep as one might judge, as it would be a pity if we occupied the space offered by the Royal in imitating the Kilcanny cats.

I myself feel no difficulty about the ratio of two quantities, both of which increase without limit tending to a finite value, and think personally that this limiting ratio may be properly spoken of as the ratio of two infinite values when their mode of tending to infinity has been properly defined. I gave a rigorous statement on this point in the Proc. Cam. Phil. Soc., at the begining of a paper entitled "The Theory of Estimation". The proposition there stated is not difficult to prove, and I cannot see that it leaves any ambiguity as to the meaning of frequency ratios in infinite populations. The question has been discussed in other terms by Von Mies, but his definitions applied, I believe, only when the populations are denumerable and unimportant, but unnecessary restrictions, seeing that we often use probabilities proportional to lengths or areas.

I do not object to the generalisation to all <u>probabilities</u> of the laws appropriate to the games of chance, but I do think, and indeed claim came to have shown, that there are also logical situations in which a rigorous statement of the nature of uncertainty in our uncertain inferences is expressible not in terms of probability, but in terms of likelihood, a quantity which does not obey these laws. The derivation of probability statements from statements involving likelihood, in the special cases where such derivation is possible, interests me greatly, and seems the right starting point for exploring the almost unknown field of the relations between probability and likelihood.

Here is an example of the kind of problem in this connection which puzzles me. A man makes genetical tests on a number of plants from a wild population. He finds three of them may be called a, two more b, two more c, and one each of d, e, and f. His data thus consists of the partition

$(3\,2^2\,1^3)$

of the number 10.

Let him know, or be willing to assume that different types occur in the population with frequencies in the ratio $1:r:r^2:r^3 \ldots\ldots$. For any value of r he can calculate the probability of getting his observed partition, supposing his sample has been chosen really at random. He thus knows the likelihood of all values of r, but unless he has prior knowledge as to the distribution of r, I think you will agree with me that he does not know the probability of r exceeding any assigned value. For each value of r there is, I think, a calculable probability that the next plant to be tested will be of a type not hitherto found, and this probability will I suppose, increase from 0 to 1 as r increased from 0 to 1. What sort of information has he then about this probability?

You would, I think, approach the problem introducing some sort of prior knowledge, which would make this probability definite though it would depend on the prior knowledge introduced, but you would not be unwilling to conceive that a rational being might happen to lack the prior knowledge of the kind introduced, and yet you would, I suppose, be unwilling to assert that no amount of experience without such prior knowledge could give him any guidance as to whether to expect new types or not. It looks as though some sorts of rational inference require both the concepts of probability and likelihood in a rigorous statement of the nature of their uncertainty.

Yours sincerely,