

March 30, 1942

Dear Jeffreys,

You are quite right that the problem you mention is very near to that of Behrens, but owing to the additional degree of freedom interpreted now as additional evidence on the values of s_1 and s_2 , the significance test tabulated for Behrens' test is not exactly applicable. The analysis is, however, very similar in form, and I was tempted to try to tabulate values for your problem, and only deterred by their being one more (4 instead of 3) parameter with which the table must be entered. Tabulation would, therefore be rather extravagantly elaborate, and the use of effective approximations based on the asymptotic approach of Student's to the Normal distribution is all the more important.

I have discussed this approximation rather fully in the enclosed paper. By-the-way, as the Annals is now very hard put to it to continue, and has received so far no promise of help from the Rockefeller Fund held by the Royal, do you not think your Department, or some libraries connected with it, could be induced to subscribe for the journal?

The fundamental relations used are set ~~up~~ out in Section 2, and the asymptotic approach in Section 3. Your case differs from

Behrens in that, though both depend on a well determined line in the plane of t_1 and t_2 . Behrens is concerned with the total frequency integrated on one side of the line, whereas you have to deal with the section of the frequency surface cut by the line itself. The maximal likelihood solution is the mode of this section, and to assign fiducial limits to this estimate one is concerned to determine the points on the lines which cut off $\frac{1}{2}\%$, or $2\frac{1}{2}\%$, or whatever you want of the sectional area at each tail, i.e., at p. 152 you would put $y = d$ in the expression for t_1 and t_2 and integrate for x only. I imagine the analysis will be about as difficult as what I have done on this and the next page, but you may not need so many approximative terms as I have introduced. At least, using definite values from the start, you are in a good position to see where to stop, whereas with algebraic investigation the relative importance of different terms is different in different parts of the table.

In the new edition of Statistical Tables also I am putting table 6 of this paper, together with Sukhatmé's tables and an illustration of the sort of application I should think appropriate in physical or astronomical work. I should be very glad to hear what you think of these two parallel attempts, especially from the point of view of logical exposition, since such a disturbance has been raised about this aspect of the problem.

Yours sincerely,