St John's College
Cambridge.

1938 June 6.

Dear Fisher,

As this thing follows rather closely on your work I feel inclined to submit it for the Annals. I think it is mostly just a tidying up of loose ends, but there are a few minor practical points in it, e.g. on p.13. On p.4 I think that I am really flogging a dead horse, but people still won't let it be buried. Bartlett and E.S.Pearson wrote to me lately accusing me of claiming knowledge inaccessible to them, and I am rather tired of it.

I think that the kind of argument on p.10 would be useful inverted when direct distributions are wanted. E.g. take a set of measures; we have

$$P(dx_1 \dots dx_n \mid x, \sigma, h) = (2\pi\sigma^2)^{-\frac{1}{2}n} \exp\left\{-\frac{n}{2\sigma^2}[(x-\bar{x})^2 + s^2]\right\} dx_1 \dots dx_n \quad (1)$$

which on integration will lead to a definite distribution for the probability of $\bar{x}$ and s given x and σ; and since the total probability of x and s must be 1 it must be of the form

$\cancel{P(dx_1 \dots dx_n \mid x,}$

$$P(d\bar{x}\, ds \mid x, \sigma, h) = \frac{1}{\sigma^2} f\left(\frac{x-\bar{x}}{\sigma}, \frac{s}{\sigma}\right) d\bar{x}\, ds \qquad (2)$$

But with

$$P(dx\,d\sigma \mid h) \propto dx\,d\sigma/\sigma, \qquad (3)$$

(1) gives

$$P(dx\,d\sigma \mid \theta h) \propto \sigma^{-(n+1)} \exp\left[-\frac{n}{2\sigma^2}\{\quad\}\right] dx\,d\sigma \qquad (4)$$

the other factor being independent of x and σ; hence

$$P(dx\,d\sigma \mid \theta h) \cancel{\propto} = \phi(n)\, s^{n-1} \sigma^{-(n+1)} \text{————} dx\,d\sigma \qquad (5)$$

But (2) or (3) gives

$$P(dx\,d\sigma \mid \bar{x}, s, h) \propto f\left\{\frac{x-\bar{x}}{\sigma}, \frac{s}{\sigma}\right\} \frac{dx\,d\sigma}{\sigma^3} \qquad (6)$$

$$= \kappa(n)\, s^3 f\left[\frac{x-\bar{x}}{\sigma}, \frac{s}{\sigma}\right]\frac{d\bar{x}\,ds}{\sigma^2} \qquad (7)$$

Comparing (5) & (7) {since (4) shows that $\bar{x}$ & s are sufficient statistics}

$$s^{n-2}\sigma^{-n+2}\exp\left[-\frac{n}{2\sigma^2}\{(x-\bar{x})^2+s^2\}\right] = \phi(n)\, f\left(\frac{x-\bar{x}}{\sigma}, \frac{s}{\sigma}\right) \qquad (8)$$

$$\therefore\ P(d\bar{x}\,ds \mid x,\sigma,\kappa) = \lambda(n)\,\frac{s^{n-2}}{\sigma^n}\exp\left[-\frac{n}{2\sigma^2}\left((x-\bar{x})^2+s^2\right)\right]d\bar{x}\,ds. \qquad (9)$$

Thus the form is got while the multiple integrations are short-circuited. (3) of course really cancels, but ~~it is the only one that works~~ a higher index would make (4) diverge at $\sigma \to \infty$ for n = 2, and for n = 1 there is no problem.

One W.O.Storer here got the z distribution as a posterior probability last term, as the probability distribution of the standard deviation of a new set of observations, a previous set being the data ; it is exactly your form. The same applies to the prediction of a set of means. The kind of argument I used for 'Student' suggested to me that this should be another case where a direct distribution should go over with no alteration at all, and it does.

By the way do you approve of my expression 'standard variation ?' I prefer to restrict 'deviation' to deviations of observed quantities from the mean ; otherwise qualifications are needed to show which we are talking about. I don't like 'standard error' - the word error makes Dingle see red - because in the gravity problem and your plot yields the error of measurement is a very small part of the whole variation that is treated as random. Equally I don't like either 'true value' or 'population parameter' but cannot see anything really satisfactory. How about 'estimand'?

Yours sincerely,

Harold Jeffreys.