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Dear Fisher,

I have just looked through Bayes's paper again, and I cannot see anything that suggests a frequency interpretation to me. He unfortunately does not give any examples of what he means by the value of an expectation, but I see nothing in his remarks inconsistent with Frank Ramsey's idea of comparing the desirability of different courses of action. (Ramsey does not appear to mention Bayes - or me - and he is as naive about causality as Russell ; but he makes some good points.) There would be little point in ~~xxxxxxxx~~ defining probability in terms of expectation if the latter was then going to be defined in terms of frequency - it would be simpler to define probability in terms of frequency straight away, and expectation would become a superfluous notion. Note too the remark on p. 410 : "It should be carefully remembered that these deductions suppose a previous total ignorance of nature". This is, I think, clear enough that Bayes was not interpreting a prior probability as a statement of frequency.

The chief difficulty of Bayes's presentation is, I think, in his Prop.1, where he assumes expectations additive. This needs a good deal more analysis, which Ramsey has supplied. I mean that the value of two dinners on successive nights may be additive, but that of two on the same night may be less than that of one, at any rate if the morning after is taken into account. So further restrictions are needed before the definition will work - it must be clear that the expectations added do not interfere with one another. I am not sure when the Weber-Fechner law was invented ; if Bayes

had known of it I think that he would have taken precautions ; but personally I think it much simpler to take probability as a primitive idea and define expectation in terms of it when occasion arises.

The first definite identifiaation of probability and frequency that I know is by R.Leslie Ellis, in Camb.Phil.Trans. 1843 ; Keynes refers also to Cournot in the same year. Laplace in using the m/n ratio always sticks in the proviso that all the events must be equally probable, thus making probability a primitive idea, and this is also in Todhunter's Algebra, from which I got my first ideas. In a lot of later works this has been left out - e.g. by Neyman and Levy. Thus the Weldon-Pearson experiments on the bias of dice and the usefulness of Tippett's numbers as against personal choice to get a random sample become pointless - matters on which we should agree perfecly.

I notice that the same volume of the Phil.Trans, as contains Bayes's paper also contains a proof by him of the divergence of Stirling's series ; I suppose this was the first.

I have been reading or rereading a good deal of mathematical logic during the last few weeks and am beginning to wonder whether a good deal of pure mathematics is not inductive. Whitehead and Russell, Ramsey, and Wittgenstein all seem to agree that a number is a class of classes of distinguishable individuals, and therefore, one would think, that the existence of classes up to a certain number is an empirical proposition depending on the number of individuals in the world, which may well be finite. In that case there would be no such real number as $\sqrt{2}$, since the notion of a limit breaks down. The only possibility is seems to be that mathematics is either wholely idealist or that it is developed as science indicates that it may be wanted. ~~Then would be~~ Apparently even consistency

cannot be proved. I sounded Littlewood about this and he shares my doubts. Hitherto I have regarded pure mathematics as fundamental and probability as using its rules, but essentially as a separate idea ; but the alternative that pure mathematics is merely a special case of probability is going to need serious examination. At any rate on Wittgenstein's arguments it seems perfectly possible that the whole logic of infinity, i.e. theorems of limits & continuity, is just bunk, and nobody can apparently see anything wrong with them. I don't see much prospect of solving the puzzle myself but I think that I may manage to limit the questions to a few definite alternatives.

By the way you said somewhere that the variation of observations from a formula is just as essential a part of the law as the formula itself, but I cannot trace the reference when I want it. It is a most important point from the logical and philosophical point of view.

Yours sincerely,

Harold Jeffreys