

St John's College
Cambridge
1938 March 5.

Dear Fisher,

I heard yesterday that you were back ; I hope you found India enjoyable. Meanwhile I have got out a number of things that may interest you. There seem to be a lot of cases where very good approximations to maximum likelihood should exist and be attainable without terrific trouble, even when sufficient statistics do not exist. In dealing with errors of observation I found that it is possible to take the laws of types II and VII in such forms that the location, scale, and index parameters are uncorrelated to the first order, so that in the m.l. equations the cross terms represent only sampling errors and will give only errors of order n^{-1} if they are dropped. Thus it is possible to proceed by iteration. After getting the normal law solution as a first approximation it took about $1\frac{1}{2}$ pages of arithmetic to get the index, and about the same to correct the other things. The forms are :

$$y = \frac{(m-\frac{1}{2})!}{\{2\pi(m+\frac{1}{2})\}^{\frac{1}{2}}(m-1)! \sigma} \left\{ 1 - \frac{m^2(x-a)^2}{2(m+\frac{1}{2})^2 \sigma^2} \right\}^{m-1} \quad \text{for } m > 1;$$
$$y = \frac{m!}{\{2\pi(m-\frac{1}{2})\}^{\frac{1}{2}}(m-\frac{1}{2})! \sigma} \left\{ 1 + \frac{m^2(x-a)^2}{2(m-\frac{1}{2})^2 \sigma^2} \right\}^{-m}$$

Four-figure accuracy was ample. I got what may be a workable approximation to a similar method when there is asymmetry, but did not use it since the evidence for asymmetry in the actual cases did not seem appreciable.

A slight modification of minimum χ^2 should give a fairly easy solution when all the ranges considered are occupied by observations. I think that something could still be done when there are long tails, by using grouping as far as possible and special treatment at the

tails, but I have not pushed it. I enclose a spare proof of what I have done. It seems to me that with these dodges it should always be possible to get the m.l. solution without retaining a large number of figures in the computations.

I read your 'Design of Experiments' recently with much interest. It leads me to query your disparagement of your intuition in the note at the end of ^{my} ~~xxxx~~ paper in the Annals. At any rate wherever my methods, which are really based on the Grammar of Science, lead to criteria differing from K.P.'s, you seem to have recommended something either identical with mine or so similar that hardly anybody would notice the difference. By the way I find that χ^2 at small groups is just hopeless in any form. Yates's modification is an improvement, but not ~~xxxx~~ enough. The proper test is always the ratio of the m.l. departure to its standard error.

By the way where did you get the identification of a prior probability with a frequency in the external world? K.P.'s habit of calling his tables of probabilities frequencies suggests that he identified them; but I haven't found anything analogous in the work of anybody else that has used inverse probability. It is however always hard to be sure that a particular statement is not in Laplace somewhere.

Yours sincerely,

Harold Jeffreys